COSMIC INFLATION

and

THE CURVATON HYPOTHESIS

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The Hot Big Bang

Cosmological Principle: The Universe on large scales is Homogeneous and Isotropic

CMB observations: The Universe is spatially Flat

Flat FRW: \( ds^2 = -dt^2 + a^2(t)(dr^2 + r^2d\Omega^2) \)

\( a(t) \): Scale Factor (parameterizes Universe evolution)

Successes of the Hot Big Bang

- Hubble expansion: \( v \sim H_0 r \)
- Age of the Universe: \( t_0 \approx 14 \text{ Gyrs} \)
- Big Bang Nucleosynthesis (BBN)
- Cosmic Microwave Background (CMB)

Problems of the Hot Big Bang

- Initial conditions (singularity & expansion)
- Matter over Anti-matter (Baryogenesis)
- Flatness Problem (\( \Omega \equiv \rho/\rho_c = 1 \): repulsor)
- Horizon Problem (superhorizon CMB correlations)
- Curvature perturbations \( \Rightarrow \) Large Scale Structure formation & CMB temperature anisotropy

Compelling solution: INFLATION
**Cosmic Inflation**

**Inflation:** A period of accelerated expansion in the Early Universe

Friedmann equation: \[ H^2 = \frac{\rho}{3m_p^2} \]

[Geometrical units: \( c = \hbar = 1 \) & \( 8\pi G = m_p^{-2} \)]

\[ H(t) \equiv \frac{\dot{a}}{a} \] : Hubble parameter

\( \rho \simeq \Lambda_{\text{eff}} m_p^2 \Rightarrow a \propto e^{H t} \) De-Sitter expansion

During inflation: \( H \simeq \text{constant} \)

**Solving the problems of the Hot Big Bang**

- **Expansion:** Inflation provides boost
- **Horizon Problem:** Superluminal expansion
  \( \Rightarrow \) superhorizon correlations
- **Flatness Problem:** Curvature = Inflated away
- **LSS & CMB Anisotropy:** Particle Production

  Typical realization: **Inflationary Paradigm**
The Inflationary Paradigm

The Universe undergoes accelerated expansion due to being dominated by the potential density of a scalar field

The dynamics of the Universe

Universe content = collection of perfect fluids

\[(T^\nu_\mu)_{ij} = \text{diag}(\rho_i, p_i, p_i, p_i) \; \text{with} \; p_i = w_i \rho_i\]

\[w_i: \text{barotropic index} \quad w_m = 0 \; : \text{matter} \quad w_\gamma = \frac{1}{3} \; : \text{radiation}\]

Independent fluids: \(\nabla_\mu (T^{\mu\nu}) = 0 \Rightarrow\)

\[d(a^3 \rho_i) = -p_i d(a^3) \iff \dot{\rho}_i + 3H(\rho_i + p) = 0\]

\[\Rightarrow \rho \propto a^{-3(1+w_i)} \quad \begin{cases} \rho \propto a^{-3} & \text{Matter} \\ \rho \propto a^{-4} & \text{Radiation} \end{cases}\]

\[H(t) = \frac{2t^{-1}}{3(1+w)} \quad \& \quad \rho = \frac{4}{3(1+w)^2} \left(\frac{m_P}{t}\right)^2\]

\[\rho_\gamma = \frac{\pi^2}{30} g_* T^4 \Rightarrow \text{Adiabaticity: } T \propto a^{-1}\]

Raychadhuri: \[\frac{\ddot{a}}{a} = -\frac{\rho + 3p}{6m_P^2} \Rightarrow \ddot{a} > 0 \iff w < -\frac{1}{3}\]

\[w = -1 \Rightarrow \rho = -\text{const.} \Rightarrow a \propto \exp(Ht)\]
Scalar fields in Cosmology
\[ \mathcal{L} = \frac{1}{2} g^{\mu \nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) \]

\[ \Rightarrow \quad T^{\mu \nu} = \partial_{\mu} \phi \partial_{\nu} \phi - g^{\mu \nu} \mathcal{L} \]

Homogeneous Scalar Field ⇒ Perfect fluid \( T^{\mu \nu} \):
\[ \begin{align*}
p_\phi &= \rho_{\text{kin}} - V \\
\rho_\phi &= \rho_{\text{kin}} + V
\end{align*} \]

where \( \rho_{\text{kin}} \equiv \frac{1}{2} \dot{\phi}^2 \) & \( V = V(\phi) \)

Klein-Gordon: \[ \ddot{\phi} + 3H \dot{\phi} + V' = 0 \]

Inflation by a scalar field
\[ w_\phi < -\frac{1}{3} \Leftrightarrow \rho_{\text{kin}} < \frac{1}{2} V \]

\( \rho_{\text{kin}} \ll V \Rightarrow w_\phi \sim -1 \Rightarrow \) (quasi) de-Sitter Inflation

\( \rho_{\text{kin}} \ll V \Rightarrow \) Slow Roll: \[ 3H \dot{\phi} \sim -V' \]

\[ \epsilon \equiv -\frac{\dot{H}}{H^2} \approx \frac{1}{2} m_P^2 \left( \frac{V'}{V} \right)^2 \quad \& \quad \eta \equiv m_P^2 \frac{V''}{V} \]

Slow Roll conditions: \( \epsilon, |\eta| \leq 1 \Rightarrow \) Inflation

\[ \phi = \text{Inflaton Field} \]

Inflation terminates when: \( \rho_{\text{kin}} \sim V \)

Reheating: After the end of Inflation the inflaton oscillates around its VEV. These coherent oscillations correspond to massive particles, which decay into the thermal bath of the Hot Big Bang

\[ \ddot{\phi} + 3H \dot{\phi} + \Gamma \dot{\phi} + V' = 0 \Rightarrow T_{\text{reh}} \approx \sqrt{\frac{m_P}{\Gamma}} > T_{\text{BBN}} \]
Particle Production

All non-conformally invariant, effectively massless fields are gravitationally generated during inflation

(non-conformal invariance \(\Rightarrow\) \(\phi\) couples to the metric)

\(m < H \Leftrightarrow\) Compton wavelength > Horizon (\(\sim H^{-1}\))

The quantum fluctuations of a light field are able to reach and exit the Horizon during inflation without becoming suppressed by the uncertainty principle

\[
\begin{align*}
\Delta \mathcal{E} \cdot \Delta t & \sim 1 \\
\Delta \mathcal{E} & \sim \rho_\phi \times \Delta V \\
\Delta t & \sim H^{-1} \quad \& \quad \Delta V \sim H^{-3} \\
\rho_\phi & \sim [\partial (\delta \phi)]^2 \sim (H \delta \phi)^2
\end{align*}
\]

\[\Rightarrow \quad \delta \phi = \frac{H}{2\pi}\]

Hawking \(- T\)

Particle Horizon \(\Leftrightarrow\) Event Horizon of inverted Black Hole during Inflation

Particle production in inflation \(\Rightarrow\) Bath of Hawking-\(T\)

\(\Rightarrow\) Massive fields become Boltzmann suppressed

Superhorizon Evolution

After Horizon exit: fluctuation \(\Rightarrow\) classical object

Klein – Gordon:

\[
(\ddot{\delta \phi} + 3H\dot{\delta \phi} + m^2(\delta \phi) = 0)
\]

\[\Rightarrow \quad \delta \phi \sim \frac{H}{2\pi} \left[ e^{-\frac{1}{3}\left(\frac{m}{H}\right)^2H\Delta t} - \frac{1}{9} \left(\frac{m}{H}\right)^2 e^{-3H\Delta t} \right] \approx \frac{H}{2\pi}
\]

perturbation freezes \(\Rightarrow\) scale-invariant spectrum
The Curvature Perturbation

\[
H^2(\bar{x}, t) = \frac{\rho(\bar{x}, t)}{3m_p^2} + \frac{2}{3}\nabla^2 \mathcal{R}(\bar{x}, t) \quad \text{(Friedmann)}
\]

\[\Rightarrow 2H\delta H_k = \frac{\delta \rho_k}{3m_p^2} + \frac{2}{3} \left( \frac{k}{a} \right)^2 \mathcal{R}_k \quad \text{where} \quad R^{(3)} = 4 \left( \frac{k}{a} \right)^2 \mathcal{R} \]

\[R^{(3)} : \text{spatial curvature} \]
\[\mathcal{R} : \text{curvature perturbation} \]
\[\mathcal{P}_\mathcal{R} : \text{power spectrum} \]
\[k : \text{comoving momentum scale} \]

\[\langle \mathcal{R}^2 \rangle = \int_{1/L}^{aH} \mathcal{P}_\mathcal{R}(k) \frac{dk}{k} \quad (1/L \sim aH) \Rightarrow \langle \mathcal{R}^2 \rangle \sim \mathcal{P}_\mathcal{R} \]

\[\delta_k \equiv \frac{\delta \rho_k}{\rho} = -\frac{2}{5} \left( \frac{k}{aH} \right)^2 \mathcal{R}_k \quad \text{density perturbation} \]

\[\delta_H = \sqrt{\langle \delta^2 \rangle} \bigg|_{k=aH} \Rightarrow \delta_H(k) = \frac{2}{5} \sqrt{\mathcal{P}_\mathcal{R}(k)} \]

\textbf{Sachs-Wolfe effect: CMB light is redshifted when crossing overdensities}

\textbf{Sachs – Wolfe:} \quad \frac{\delta T}{T}_{\text{CMB}} = -\frac{1}{5} \mathcal{R}_{LS} = \frac{1}{2} \frac{\delta \rho}{\rho} \simeq 10^{-5} \quad \text{(COBE)}

\[\mathcal{P}_\mathcal{R}(k) \propto k^{n_s-1} \Rightarrow n_s(k) - 1 \equiv \frac{d \ln \mathcal{P}_\mathcal{R}}{d \ln k} \quad \text{spectral index} \]

\textbf{WMAP:} \quad n_s = 0.99 \pm 0.04
Inflaton Hypothesis

The field responsible for the curvature perturbation spectrum is the same field that drives the dynamics of Inflation

The inflaton has to be a light field

Perturbations on the value of the inflaton ⇒

\[ \rho \propto t^{-2} \quad t \sim H^{-1} \quad \delta \phi \sim \dot{\phi} \delta t \]

\[ \Rightarrow \quad \mathcal{R} \sim \frac{\delta \rho}{\rho} \sim \frac{\delta t}{t} \sim \frac{\delta \phi}{\dot{\phi} t} \sim \left( \frac{H}{\dot{\phi}} \right) \delta \phi \]

\[ \mathcal{R}_k = - \left( \frac{H}{\dot{\phi}} \right) \delta \phi_k \]

\[ \Rightarrow \quad \sqrt{\mathcal{P}_\phi(k)} = \frac{H}{2\pi} \bigg|_{k=aH} \]

\[ \delta H = \frac{2}{5} \sqrt{\mathcal{P}_R} = \frac{2}{5} \mathcal{R} \]

\[ 3H \dot{\phi} \sim -V' \]

\[ \Rightarrow \quad \frac{\delta \rho}{\rho} = \frac{1}{5\sqrt{3\pi} m_P^{3/2}} \frac{V^{3/2}}{|V'|} \]

COBE: \( \delta H = 2 \times 10^{-5} \Rightarrow \]

\[ V_{\text{inf}}^{1/4} = 0.027 \epsilon^{1/4} m_P \sim \text{GUT} \]

\[ n_s = 1 + 2\eta - 6\epsilon \Rightarrow |\eta| < 0.01 \quad (\text{WMAP}) \]

Under the inflaton hypothesis the CMB imposes stringent constraints on Inflation model-building, which rule out many models, otherwise well motivated by particle physics
Curvaton Hypothesis

The field responsible for the curvature perturbation spectrum is a field \(\sigma\) other than the inflaton (curvaton)

The curvaton has to be a light field

\[
\mathcal{R} \equiv -H \frac{\delta \rho}{\dot{\rho}}
\]

curvature perturbation on foliage of spatially flat hypersurfaces

\[
\delta \rho = \sum_i \delta \rho_i \quad \dot{\rho}_i = -3H(\rho + p)_i \implies \mathcal{R} = \sum_i \frac{1 + w_i}{1 + w} \left( \frac{\rho_i}{\rho} \right) \mathcal{R}_i
\]

\[
\mathcal{R} \sim \left( \frac{\rho_\sigma}{\rho} \right) \mathcal{R}_\sigma
\]

where \( \mathcal{R}_\sigma \sim \delta \sigma = \frac{H}{2\pi} \)

WMAP: \((\rho_\sigma/\rho) \geq 9 \times 10^{-3}\)

\(\mathcal{R}\) depends on the evolution after the end of inflation


- During inflation \(\sigma\) is frozen with \(V(\sigma) \ll V_{\text{inf}}\)
- After inflation \(\sigma\) unfreezes when \(m_\sigma \sim H(t)\)
- After unfreezing \(\sigma\) oscillates around its VEV
- \(\sigma\) (nearly) dominates the Universe imposing its \(\mathcal{R}\)
- \(\sigma\) decays into thermal bath before BBN

\[
n_s = 1 + 2\tilde{\eta} - 2\epsilon \quad \text{with} \quad \tilde{\eta} \equiv \frac{m_p^2}{\partial_{\sigma\sigma} V / V}
\]
Merits of the Curvaton Hypothesis

- COBE relaxes to upper bound: $V_{\text{inf}}^{1/4} < \text{GUT}$
- No problem with $\eta \sim 1 \Leftrightarrow m_\phi \sim H_{\text{inf}}$
- Predicts: $n_s \approx 1$ (WMAP $\checkmark$)

Inflation is liberated by the Curvaton Hypothesis [KD & D.H. Lyth, 2002]

The curvaton may be associated with low energy (TeV) physics and can be easily accommodated into simple extensions of the standard model

- righthanded sneutrino (neutrino masses)
  [K.Hamaguchi, M.Kawasaki, T.Moroi & F.Takahashi, 2003]
- MSSM (& NMSSM) flat direction
  [K.Enqvist, S.Kasuya & Mazumdar, 2003]
  [M.Bastero-Gil, V.DiClemente & S.F.King, 2003]
- Peccei-Quinn field (solves strong CP)
- Pseudo Nambu-Goldstone Boson (e.g. string axion)

The curvaton can link cosmological observations with collider experiments

Observational signatures

- Non-Gaussianity $f_{\text{NL}} \sim 0.1$ (SDSS & 2dF detectable)
- Isocurvature (anti)correlated with $\mathcal{R}$ (smoking gun)
**Quintessential Inflation**

$$\Omega_m < \Omega_0 \Rightarrow \exists \text{ Dark Energy } \Omega_\Lambda \approx 2 \Omega_m$$

SN-Ia ($\&$ SDSS) $\Rightarrow$ acceleration $\Rightarrow w_\Lambda < -\frac{1}{3}$

**$\Lambda$CDM**: $\rho_\Lambda = \Lambda m_p^2 = \text{constant } \& \ w_\Lambda = -1$

**BUT**: $\rho_\Lambda \sim \rho_0 \Rightarrow \Lambda \sim 10^{-120} m_p^2$ !!

**alternative solution**: The Universe at present undergoes a late period of inflation driven by a scalar field with $V_0 \sim \rho_0$

**Quintessence**: the fifth element after CDM, HDM, photons and baryons

**Quintessential Inflation**: Incorporates both Quintessence and the Inflaton in a single scalar

**Merits**: 
- Avoid yet another scalar  
- Single theoretical framework  
- Fix initial conditions

**Ingredients**: 
- Sterile inflaton (singlet)  
- Gravitational Reheating  
- Quintessential Tail

$V(\phi)$: Inflationary Plateau: $V_{\inf} \sim \text{constant}$

Quintessential Tail: $V(\phi \to \infty) \to 0$
After the end of Inflation

Gravitational Reheating: \[ T_{\text{reh}} = \alpha \frac{H_{\text{inf}}}{2\pi} \]

Reheating efficiency: \( \alpha \sim 0.1 \) [L.H.Ford, 1986]
Instant Preheating: \( \alpha \gg 1 \) [G.Felder et al., 1999]

\[ \epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{3\rho_{\text{kin}}}{V} \Rightarrow V_{\text{end}} \sim \rho_{\text{kin}}^{\text{end}} \Rightarrow T_{\text{reh}}^4 \sim \rho_{\gamma}^{\text{end}} \]

Kination: [M.Joyce & T.Prokopec, 1998]

\[ \rho \sim \rho_{\text{kin}} \gg V \Rightarrow w_{\text{kin}} = 1 \Rightarrow \rho \propto a^{-6} \]

\( \rho_{\gamma} \propto a^{-4} \Rightarrow \exists \ T_* : \rho_{\gamma} \sim \rho_{\text{kin}} \]

\( T_* > T_{\text{BBN}} \sim 1 \text{ MeV} \)

Hot Big Bang: \( \phi \rightarrow \phi_F \) Freezeout at \( V_F \simeq \Omega_\phi \rho_0 \)

COBE: \( V_{\text{inf}}^{1/4} \sim \text{GUT} \Rightarrow \frac{V_{\text{inf}}}{V_F} \gtrsim 10^{100} \Rightarrow \)

\( V_{\text{end}} = \text{curved} \Rightarrow \eta \equiv m_p^2 V''/V \sim 1 \Rightarrow n_s \not\approx 1 \times \eta\)-problem of Quintessential Inflation

Curvaton to the Rescue

The curvaton overcomes the \( \eta\)-problem
without negative side effects [KD, 2003]

\( V_{\text{inf}}^{1/4} < \text{GUT} \& n_s \neq n_s(\eta) \Rightarrow n_s(\eta \sim 1) \approx 1 \checkmark \)

Extra bonus: Dilutes gravitons & gravitinos
Reheating efficiency \( \alpha_{\text{eff}} \gg 1 \)
Modular Quintessential Inflation

\[ V(\phi) \sim \begin{cases} 
M^4 - \frac{1}{2}m^2\phi^2 \\
M^4 \exp(-b\phi/m_P)
\end{cases} \]

During Kination, \( \phi \) is oblivious of \( V(\phi) \)

\[ M \sim \sqrt{m_{3/2} m_P} \sim 10^{10.5}\text{GeV} \quad \alpha \sim 10^4 \]
\[ m \sim 1 \text{ TeV} \quad & \quad b \sim 4 \quad \Rightarrow \quad \phi_F \sim 10 M_P \]

\( M \): Gravity mediated supersymmetry breaking
\( \phi_F \): Strongly coupled Heterotic String Theory
Summary

- Despite its success, the Hot Big Bang is incomplete and suffers from a number of important problems.

- Inflation is the most elegant and compelling solution: Horizon, Flatness, LSS & CMB-anisotropy. (Also, Baryogenesis & Galactic Magnetic Fields)

- Typically, Inflation is realized using a scalar field (Inflaton) whose potential dominates the Universe.

- Inflation amplifies the quantum fluctuations of light fields, which, in turn, can generate a superhorizon scale-invariant spectrum of Gaussian curvature perturbations and explain LSS and CMB-anisotropy.

- Traditionally, the curvature perturbations are created by the Inflaton field (Inflaton Hypothesis). However, they may be due to some other field, unrelated to Inflation (Curvaton Hypothesis).

- The Curvaton liberates most models of Inflation and can be accommodated in simple extensions of the Standard Model (Inflaton may be obsolete).

- One example of a liberated model is Quintessential Inflation, which also accounts for the observed Dark Energy and current accelerated expansion.

- Realistic candidates for the Quintessential Inflaton are runaway fields such as the string moduli.