COSMIC INFLATION

and

THE CURVATON HYPOTHESIS

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The Hot Big Bang

Cosmological Principle: The Universe on large scales is Homogeneous and Isotropic

CMB observations: The Universe is spatially Flat

Flat FRW : $ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\Omega^2)$

$a(t)$: Scale Factor (parameterizes Universe evolution)

Successes of the Hot Big Bang

- Hubble expansion: $v \simeq H_0 \, r$
- Age of the Universe: $t_0 \simeq 14$ Gyrs
- Big Bang Nucleosynthesis (BBN)
- Cosmic Microwave Background (CMB)

Problems of the Hot Big Bang

- Initial conditions (singularity & expansion)
- Matter over Anti-matter (Baryogenesis)
- Flatness Problem ($\Omega \equiv \rho/\rho_c = 1$: repulsor)
- Horizon Problem (superhorizon CMB correlations)
- Curvature perturbations $\Rightarrow$ Large Scale Structure formation & CMB temperature anisotropy

Compelling solution: INFLATION

Cosmic Inflation

Inflation: A period of accelerated expansion in the Early Universe

Friedmann equation: $H^2 = \rho / 3m_P^2$

[Geometrical units: $c = \hbar = 1$ & $8\pi G = m_P^{-2}$]

$H(t) \equiv \dot{a}/a$: Hubble parameter

$\rho \simeq \Lambda_{\rm eff}\, m_P^2 \Rightarrow a \propto e^{Ht}$ De-Sitter expansion

During inflation: $H \simeq$ constant

Solving the problems of the Hot Big Bang

- **Expansion:** Inflation provides boost
- **Horizon Problem:** Superluminal expansion $\Rightarrow$ superhorizon correlations
- **Flatness Problem:** Curvature = Inflated away
- **LSS & CMB Anisotropy:** Particle Production

Typical realization: Inflationary Paradigm

The Inflationary Paradigm

The Universe undergoes accelerated expansion due to being dominated by the potential density of a scalar field

The dynamics of the Universe

Universe content = collection of perfect fluids

$$(T^{\nu}_{\mu})_i = \text{diag}(\rho_i, p_i, p_i, p_i) \;\text{ with }\; \underline{p_i = w_i \rho_i}$$

w_i: barotropic index $\begin{array}{ll} w_m = 0 & : \text{matter} \\ w_\gamma = \frac{1}{3} & : \text{radiation} \end{array}$

Independent fluids: $\nabla_\mu (T^{\mu\nu})_i = 0 \Rightarrow$

$$d(a^3 \rho_i) = -p_i d(a^3) \Leftrightarrow \boxed{\dot{\rho}_i + 3H(\rho_i + p) = 0}$$

$$\Rightarrow \;\; \rho \propto a^{-3(1+w_i)} \quad \begin{cases} \rho \propto a^{-3} & \text{Matter} \\ \rho \propto a^{-4} & \text{Radiation} \end{cases}$$

$$H(t) = \frac{2t^{-1}}{3(1+w)} \quad \& \quad \rho = \frac{4}{3(1+w)^2}\left(\frac{m_P}{t}\right)^2$$

$$\rho_\gamma = \frac{\pi^2}{30} g_* T^4 \;\Rightarrow \text{Adiabaticity} : T \propto a^{-1}$$

$$\text{Raychadhuri} : \frac{\ddot{a}}{a} = -\frac{\rho + 3p}{6m_P^2} \Rightarrow \boxed{\ddot{a} > 0 \Leftrightarrow w < -\frac{1}{3}}$$

$$w = -1 \Rightarrow \rho = -\text{ const.} \Rightarrow a \propto \exp(Ht)$$

Scalar fields in Cosmology

$$\mathcal{L} = \tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi)$$

$$\Rightarrow \quad T^{\mu\nu} = \partial^\mu \phi \, \partial^\nu \phi - g^{\mu\nu} \mathcal{L}$$

Homogeneous Scalar Field $\Rightarrow$ Perfect fluid $T^{\mu\nu}$:

$$\boxed{\begin{array}{l} p_\phi = \rho_{\rm kin} - V \\ \rho_\phi = \rho_{\rm kin} + V \end{array}} \quad \text{where} \quad \rho_{\rm kin} \equiv \tfrac{1}{2}\dot{\phi}^2 \ \& \ V = V(\phi)$$

Klein-Gordon: $\boxed{\ddot{\phi} + 3H\dot{\phi} + V' = 0}$

Inflation by a scalar field

$$w_\phi < -\tfrac{1}{3} \Leftrightarrow \rho_{\rm kin} < \tfrac{1}{2} V$$

$\rho_{\rm kin} \ll V \Rightarrow w_\phi \simeq -1 \Rightarrow$ (quasi) de-Sitter Inflation

$\rho_{\rm kin} \ll V \Rightarrow$ Slow Roll: $3H\dot{\phi} \simeq -V'$

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{1}{2} m_P^2 \left(\frac{V'}{V}\right)^2 \ \& \ \eta \equiv m_P^2 \frac{V''}{V}$$

Slow Roll conditions: $\epsilon, |\eta| \lesssim 1 \Rightarrow$ Inflation

$\boxed{\phi = \text{Inflaton Field}}$

Inflation terminates when: $\rho_{\rm kin} \sim V$

Reheating: After the end of Inflation the inflaton oscillates around its VEV. These coherent oscillations correspond to massive particles, which decay into the thermal bath of the Hot Big Bang

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + V' = 0 \Rightarrow \boxed{T_{\rm reh} \simeq \sqrt{\Gamma m_P}} > T_{\rm BBN}$$

Particle Production

All non-conformally invariant, effectively massless fields are gravitationally generated during inflation

(non-conformal invariance $\Rightarrow$ ϕ couples to the metric)

$m < H$ $\Leftrightarrow$ Compton wavelength > Horizon ($\sim H^{-1}$)

The quantum fluctuations of a light field are able to reach and exit the Horizon during inflation without becoming suppressed by the uncertainty principle

$$\left.\begin{array}{c} \boxed{\Delta\mathcal{E}\cdot\Delta t \sim 1} \\ \\ \left.\begin{array}{c} \Delta\mathcal{E} \sim \rho_\phi \times \Delta V \\ \Delta t \sim H^{-1}\ \&\ \Delta V \sim H^{-3} \\ \rho_\phi \sim [\partial(\delta\phi)]^2 \sim (H\delta\phi)^2 \end{array}\right\} \end{array}\right\} \Rightarrow \quad \boxed{\delta\phi = \frac{H}{2\pi}} \quad \text{Hawking} - T$$

Particle Horizon during Inflation	$\Leftrightarrow$	Event Horizon of inverted Black Hole

Particle production in inflation $\Rightarrow$ Bath of Hawking-T
$\Rightarrow$ Massive fields become Bolzmann suppressed

Superhorizon Evolution

After Horizon exit: fluctuation $\Rightarrow$ classical object

Klein – Gordon : $\boxed{(\ddot{\delta\phi}) + 3H(\dot{\delta\phi}) + m^2(\delta\phi) = 0}$

$$\Rightarrow\ \delta\phi \simeq \frac{H}{2\pi}\left[e^{-\frac{1}{3}(\frac{m}{H})^2 H\Delta t} - \frac{1}{9}\left(\frac{m}{H}\right)^2 e^{-3H\Delta t}\right] \approx H/2\pi$$

perturbation freezes $\Rightarrow$ scale-invariant spectrum

The Curvature Perturbation

$$H^2(\vec{x},t) = \frac{\rho(\vec{x},t)}{3m_P^2} + \frac{2}{3}\nabla^2\mathcal{R}(\vec{x},t)$$ (Friedmann)

$$\Rightarrow 2H\delta H_k = \frac{\delta\rho_k}{3m_P^2} + \frac{2}{3}\left(\frac{k}{a}\right)^2\mathcal{R}_k \quad \text{where} \quad R^{(3)} = 4\left(\frac{k}{a}\right)^2\mathcal{R}$$

$R^{(3)}$: spatial curvature
$\mathcal{R}$: curvature perturbation
$\mathcal{P}_\mathcal{R}$: power spectrum
k : comoving momentum scale

$$\langle\mathcal{R}^2\rangle = \int_{1/L}^{aH}\mathcal{P}_\mathcal{R}(k)\frac{dk}{k} \quad (1/L \sim aH) \Rightarrow \langle\mathcal{R}^2\rangle \sim \mathcal{P}_\mathcal{R}$$

$$\delta_k \equiv \frac{\delta\rho_k}{\rho} = -\frac{2}{5}\left(\frac{k}{aH}\right)^2\mathcal{R}_k$$ density perturbation

$$\delta_H \equiv \sqrt{\langle\delta^2\rangle}\,\Big|_{k=aH} \Rightarrow \delta_H(k) = \frac{2}{5}\sqrt{\mathcal{P}_\mathcal{R}(k)}$$

Sachs-Wolfe effect: CMB light is redshifted when crossing overdensities

$$\text{Sachs} - \text{Wolfe}: \left.\frac{\delta T}{T}\right|_{\text{CMB}} = -\frac{1}{5}\mathcal{R}_{\text{LS}} = \frac{1}{2}\frac{\delta\rho}{\rho} \simeq 10^{-5}$$ (COBE)

$$\mathcal{P}_\mathcal{R}(k) \propto k^{n_s-1} \Rightarrow n_s(k) - 1 \equiv \frac{d\ln\mathcal{P}_\mathcal{R}}{d\ln k}$$ spectral index

WMAP: $n_s = 0.99 \pm 0.04$

Inflaton Hypothesis

The field responsible for the curvature perturbation spectrum is the same field that drives the dynamics of Inflation

The inflaton has to be a light field

Perturbations on the value of the inflaton $\Rightarrow$

Inflation is terminated at different times at different regions of space

$$\left.\begin{array}{r} \rho \propto t^{-2} \\ t \sim H^{-1} \\ \delta\phi \sim \dot{\phi}\delta t \end{array}\right\} \Rightarrow \mathcal{R} \sim \frac{\delta\rho}{\rho} \sim \frac{\delta t}{t} \sim \frac{\delta\phi}{\dot{\phi}t} \sim \left(\frac{H}{\dot{\phi}}\right)\delta\phi$$

$$\left.\begin{array}{r} \mathcal{R}_k = -\left(\frac{H}{\dot{\phi}}\right)\delta\phi_k \\ \sqrt{\mathcal{P}_\phi(k)} = \frac{H}{2\pi}\Big|_{k=aH} \end{array}\right\} \Rightarrow \boxed{\sqrt{\mathcal{P}_\mathcal{R}(k)} = \frac{H}{2\pi\dot{\phi}}}$$

$$\left.\begin{array}{r} \delta_H = \frac{2}{5}\sqrt{\mathcal{P}_\mathcal{R}} = \frac{2}{5}\mathcal{R} \\ 3H\dot{\phi} \simeq -V' \end{array}\right\} \Rightarrow \boxed{\frac{\delta\rho}{\rho} = \frac{1}{5\sqrt{3}\pi}\frac{V^{3/2}}{m_P^3|V'|}}$$

COBE: $\delta_H = 2 \times 10^{-5} \Rightarrow \boxed{V_{\rm inf}^{1/4} = 0.027\epsilon^{1/4}m_P} \sim$ GUT

$\boxed{n_s = 1 + 2\eta - 6\epsilon} \Rightarrow |\eta| < 0.01$ (WMAP)

Under the inflaton hypothesis the CMB imposes stringent constraints on Inflation model-building, which rule out many models, otherwise well motivated by particle physics

Curvaton Hypothesis

The field responsible for the curvature perturbation spectrum is a field σ other than the inflaton (curvaton)

The curvaton has to be a light field

$$\mathcal{R} \equiv -H\frac{\delta\rho}{\dot\rho}$$

curvature perturbation on foliage of spatially flat hypersurfaces

$$\left.\begin{array}{r}\delta\rho = \sum_i \delta\rho_i \\ \dot\rho_i = -3H(\rho+p)_i\end{array}\right\} \Rightarrow \mathcal{R} = \sum_i \frac{1+w_i}{1+w}\left(\frac{\rho_i}{\rho}\right)\mathcal{R}_i \Rightarrow$$

$$\mathcal{R} \simeq \left(\frac{\rho_\sigma}{\rho}\right)\mathcal{R}_\sigma \quad \text{where} \quad \mathcal{R}_\sigma \simeq \delta\sigma = \frac{H}{2\pi}$$

WMAP: $(\rho_\sigma/\rho) \geq 9 \times 10^{-3}$

$\mathcal{R}$ depends on the evolution after the end of inflation

[KD, G.Lazarides, D.H.Lyth & R.R.deAustri, 2003]

- During inflation σ is frozen with $V(\sigma) \ll V_{\rm inf}$
- After inflation σ unfeezes when $m_\sigma \sim H(t)$
- After unfreezing σ oscillates around its VEV
- σ (nearly) dominates the Universe imposing its $\mathcal{R}$
- σ decays into thermal bath before BBN

$$n_s = 1 + 2\tilde\eta - 2\epsilon \quad \text{with } \tilde\eta \equiv m_P^2\,(\partial_{\sigma\sigma}V \,/\, V)$$

Merits of the Curvaton Hypothesis

- **COBE relaxes to upper bound:** $V_{\rm inf}^{1/4} < {\rm GUT}$
- **No problem with** $\eta \sim 1 \Leftrightarrow m_\phi \sim H_{\rm inf}$
- **Predicts:** $n_s \approx 1$ **(WMAP** $\surd$**)**

Inflation is liberated by the Curvaton Hypothesis [KD & D.H. Lyth, 2002]

The curvaton may be associated with low energy (TeV) physics and can be easily accommodated into simple extensions of the standard model

- **righthanded sneutrino (neutrino masses)**
  [A.Hebecker, J.March-Russel & T. Yanagida, 2003]
  [K.Hamaguchi, M.Kawasaki, T.Moroi & F.Takahashi, 2003]
- **MSSM (& NMSSM) flat direction**
  [K.Enqvist, S.Kasuya & Mazumdar, 2003]
  [M.Bastero-Gil, V.DiClemente & S.F.King, 2003]
- **Peccei-Quinn field (solves strong CP)**
  [KD, G.Lazarides, D.H.Lyth & R.R.deAustri, 2003]
- **Pseudo Nambu-Goldstone Boson (e.g. string axion)**
  [KD, D.H.Lyth, A.Notari & A.Riotto, 2003]

The curvaton can link cosmological observations with collider experiments

Observational signatures

- **Non-Gaussianity** $f_{\rm NL} \sim 0.1$ **(SDSS & 2dF detectable)**
- **Isocurvature (anti)correlated with** $\mathcal{R}$ **(smoking gun)**

Quintessential Inflation

$\Omega_m < \Omega_0 \Rightarrow \exists$ Dark Energy $\Omega_\Lambda \simeq 2\,\Omega_m$

SN-Ia (& SDSS) $\Rightarrow$ acceleration $\Rightarrow w_\Lambda < -\frac{1}{3}$

$\boxed{\Lambda\text{CDM}}$: $\rho_\Lambda = \Lambda m_P^2 =$ constant & $w_\Lambda = -1$

BUT: $\rho_\Lambda \sim \rho_0 \Rightarrow \Lambda \sim 10^{-120} m_P^2$!!

alternative solution : The Universe at present undergoes a late period of inflation driven by a scalar field with $V_0 \sim \rho_0$

Quintessence: the fifth element after CDM, HDM, photons and baryons

Quintessential Inflation : Incorporates both Quintessence and the Inflaton in a single scalar

Merits:
- Avoid yet another scalar
- Single theoretical framework
- Fix initial conditions

Ingredients:
- Sterile inflaton (singlet)
- Gravitational Reheating
- Quintessential Tail

$V(\phi)$: Inflationary Plateau: $V_{\rm inf} \simeq$ constant
Quintessential Tail: $V(\phi \to \infty) \to 0$

After the end of Inflation

Gravitational Reheating : $\boxed{T_{\rm reh} = \alpha \frac{H_{\rm inf}}{2\pi}}$

Reheating efficiency: $\alpha \sim 0.1$ [L.H.Ford, 1986]
Instant Preheating: $\alpha \gg 1$ [G.Felder et al., 1999]

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{3\rho_{\rm kin}}{V} \Rightarrow V_{\rm end} \sim \rho_{\rm kin}^{\rm end} \gg T_{\rm reh}^4 \simeq \rho_\gamma^{\rm end}$$

Kination: [M.Joyce & T.Prokopec, 1998]

$$\rho \sim \rho_{\rm kin} \gg V \Rightarrow \boxed{w_{\rm kin} = 1 \Rightarrow \rho \propto a^{-6}}$$

$$\rho_\gamma \propto a^{-4} \Rightarrow \exists\ T_* : \rho_\gamma \sim \rho_{\rm kin} \ \boxed{T_* > T_{\rm BBN} \sim 1\ {\rm MeV}}$$

Hot Big Bang: $\phi \to \phi_F$ Freezeout at $V_F \simeq \Omega_\phi \rho_0$

COBE: $V_{\rm inf}^{1/4} \sim$ GUT $\Rightarrow \underline{V_{\rm inf}/V_F \gtrsim 10^{100}} \Rightarrow$

$V_{\rm end} =$ curved $\Rightarrow \eta \equiv m_P^2 V''/V \sim 1 \Rightarrow \underline{\underline{n_s \not\approx 1}}$ ×

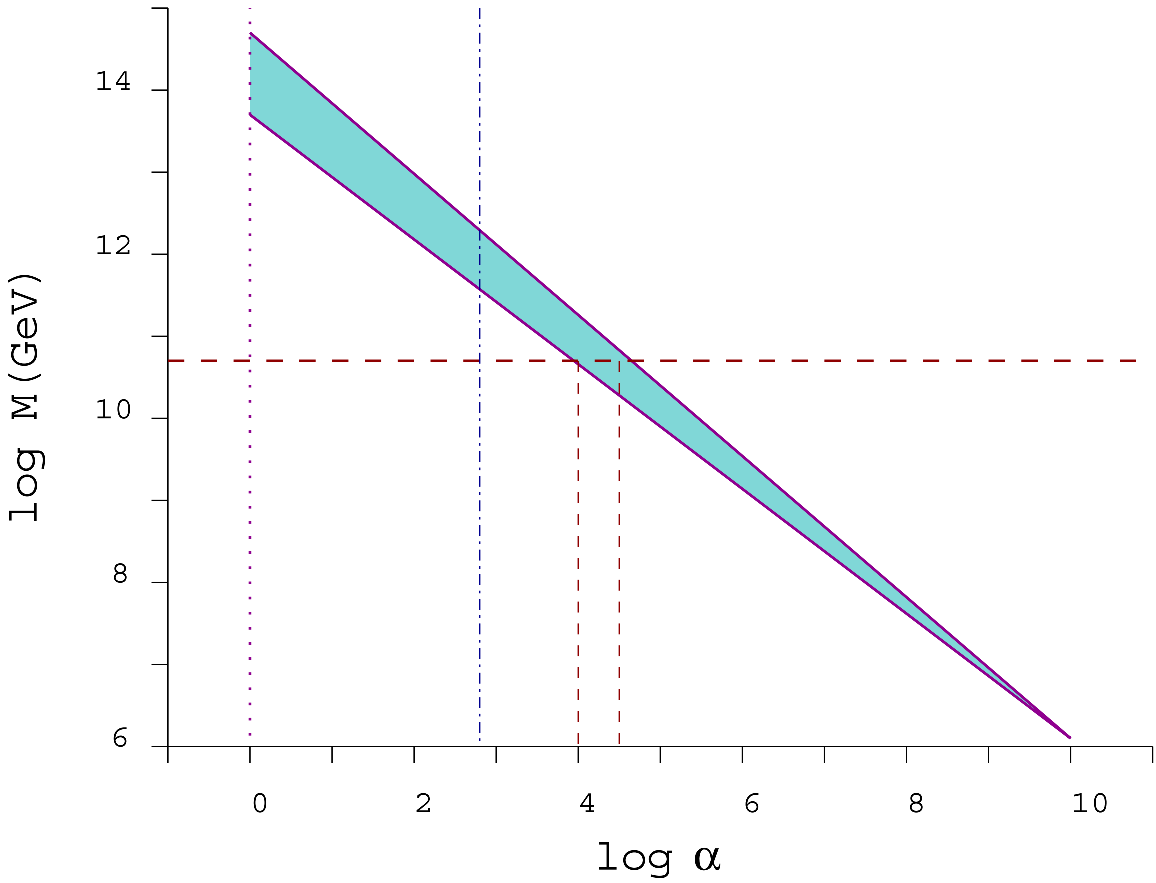
η-problem of Quintessential Inflation

Curvaton to the Rescue

The curvaton overcomes the η-problem
without negative side effects [KD, 2003]

$$V_{\rm inf}^{1/4} < {\rm GUT}\ \&\ n_s \neq n_s(\eta) \Rightarrow n_s(\eta \sim 1) \approx 1\ \checkmark$$

Extra bonus: Dilutes gravitons & gravitinos
Reheating efficiency $\alpha_{\rm eff} \gg 1$



Modular Quintessential Inflation

$$V(\phi) \simeq \begin{cases} M^4 - \frac{1}{2}m^2\phi^2 \\ \\ M^4 \exp(-b\phi/m_P) \end{cases}$$

During Kination ϕ is oblivious of $V(\phi)$

$$M \sim \sqrt{m_{3/2}m_P} \sim 10^{10.5}\text{GeV} \qquad \alpha \sim 10^4$$

$$m \sim 1 \text{ TeV} \quad \& \quad b \simeq 4 \qquad \Rightarrow \qquad \phi_F \simeq 10\ M_P$$

M: Gravity mediated supersymmetry breaking

ϕ_F: Strongly coupled Heterotic String Theory

Summary

- Despite its success, the Hot Big Bang is incomplete and suffers from a number of important problems
- Inflation is the most elegant and compelling solution: Horizon, Flatness, LSS & CMB-anisotropy. (Also, Baryogenesis & Galactic Magnetic Fields)
- Typically Inflation is realized using a scalar field (Inflaton) whose potential dominates the Universe
- Inflation amplifies the quantum fluctuations of light fields, which, in turn, can generate a superhorizon scale-invariant spectrum of Gaussian curvature perturbations and explain LSS and CMB-anisotropy
- Traditionally, the curvature perturbations are created by the Inflaton field (Inflaton Hypothesis). However, they may be due to some other field, unrelated to Inflation (Curvaton Hypothesis)
- The Curvaton liberates most models of Inflation and can be accommodated in simple extensions of the Standard Model (Inflaton may be obsolete)
- One example of a liberated model is Quintessential Inflation, which also accounts for the observed Dark Energy and current accelerated expansion
- Realistic candidates for the Quintessential Inflaton are runaway fields such as the string moduli