From Gravitation Theories to a Theory of Gravitation

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A theory of gravitation theories?

No axiomatic formulation of GR or any other gravity theory!

Possible advantages

- Deeper understanding of conceptual basis
- New insight in dealing with long-standing problems (*e.g.* Quantum Gravity)
- Experimental benefits: experiments test principles not theories
- Classification and discrimination among the numerous alternatives to GR

Maybe at least a set of physical principles — a meta-theory of gravity — as a first step?

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Various version of the EP

- *Weak Equivalence Principle (WEP)*:
  If an uncharged test body is placed at an initial event in spacetime and given an initial velocity there, then its subsequent trajectory will be independent of its internal structure and composition.
- *Einstein Equivalence Principle (EEP)*:
  (i) WEP is valid,
  (ii) the outcome of any local non-gravitational test experiment is independent of the velocity of the freely falling apparatus (Local Lorentz Invariance or LLI) and
  (iii) the outcome of any local non-gravitational test experiment is independent of where and when in the universe it is performed (Local Position Invariance or LPI).

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- *Strong Equivalence Principle (SEP)*:
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Subtle point about the EP

1. SEP $\Rightarrow$ GR??
2. What exactly is a "test particle"?
   - How small is it?
   - Can it be defined in all theories?
3. What is the relation of the EP and the variables used to describe the theory?

Main problem

EP is qualitative not quantitative: of little practical value.

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Metric Postulates

The metric postulates can be stated in the following way:

1. there exists a metric $g_{\mu\nu}$ (second rank non degenerate tensor).
2. $\nabla_{\mu} T^{\mu\nu} = 0$, where ∇_{μ} is the covariant derivative defined with the Levi-Civita connection of this metric and $T_{\mu\nu}$ is the stress-energy tensor of non-gravitational (matter) fields.

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Representation dependence!

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Questions raised

What is precisely the definition of $T_{\mu\nu}$?

- Reference to an action? Minimal coupling?
- Generalization of the special relativistic $T_{\mu\nu}$?
- A mixed definition?

What does "non-gravitational field" mean?

A field minimally coupled to gravity?
Counter example:
Scalar field in $\lambda\phi^4$ theory

$$S=\int d^4x\sqrt{-g}\left[\left(\frac{1}{2\kappa}-\xi\phi^2\right)R-\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi-\lambda\phi^4\right]$$

One loop quantization makes ξ non-zero!

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What is a theory?

Possible definitions (Wiktionary):

1. An unproven conjecture.
2. An expectation of what should happen, barring unforeseen circumstances.
3. A coherent statement or set of statements that attempts to explain observed phenomena.
4. A logical structure that enables one to deduce the possible results of every experiment that falls within its purview.
5. A field of study attempting to exhaustively describe a particular class of constructs.
6. A set of axioms together with all statements derivable from them.

Tentative definitions

Physical Theory

A coherent logical structure, preferably expressed through a set of axioms together with all statements derivable from them, plus a set of rules for their physical interpretation, that enable one to deduce and interpret the possible results of every experiment that falls within its purview.

Representation (of a theory) # 1

A finite collection of equations interrelating the physical variables which are used to describe the elements of a theory and assimilate its axioms.

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Representation (of a theory) # 2

A non-unique choice of physical variables between which, in a prescribed way, one can form inter-relational expressions that assimilate the axioms of the theory and can be used in order to deduce derivable statements.

The action of scalar-tensor theory

$$S = S^{(g)} + S^{(m)}\left[e^{2\alpha(\phi)}g_{\mu\nu}, \psi^{(m)}\right]$$

where

$$S^{(g)} = \int d^4x\,\sqrt{-g}\left[\frac{A(\phi)}{16\pi G}R - \frac{B(\phi)}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right]$$

- 4 unspecified functions A, B, V, and α
- Action describes class of theories
- Obvious redundancies; fixing leads to pin-pointing either the theory or the representation!
- Action formally conformally invariant

Fixing theory or representation

- Invariance under the transformation

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2(\phi) g_{\mu\nu}$$

  implies that fixing any of A, B, V, and α just corresponds to a choice of Ω.
- One can conveniently redefine the scalar ϕ as well

Outcome

Two of the four function can be fixed without choosing the theory! (freedom to choose clocks and rods)

Fixing the matter fields

One could even redefine ψ as

$$\tilde{\psi} = \Omega^s \psi$$

so that

$$S^{(m)} = S^{(m)}\left[\tilde{g}_{\mu\nu}, \tilde{\psi}\right]$$

Together with the choice $A = B = 1$ the action is

$$S = \int d^4x\, \sqrt{-g}\left[\frac{\tilde{R}}{16\pi G} - \frac{1}{2}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\phi}\tilde{\nabla}_\nu\tilde{\phi} - \tilde{V}(\tilde{\phi})\right] + S^{(m)}\left[\tilde{g}_{\mu\nu}, \tilde{\psi}\right]$$

GR + minimally coupled scalar field except $\tilde{\psi} = \tilde{\psi}(\tilde{\phi})$!!!

Jordan frame vs Einstein Frame

Jordan frame ($A = \phi$, $\alpha = 0$)

$$S = S^{(g)} + S^{(m)}\left[g_{\mu\nu}, \psi^{(m)}\right]$$

$$S^{(g)} = \int d^4x\,\sqrt{-g}\left[\frac{\phi}{16\pi G}R - \frac{B(\phi)}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right]$$

Einstein frame ($A = B = 1$)

$$S = S^{(g)} + S^{(m)}\left[e^{2\tilde{\alpha}(\phi)}\tilde{g}_{\mu\nu}, \psi^{(m)}\right]$$

$$S^{(g)} = \int d^4x\,\sqrt{-\tilde{g}}\left[\frac{1}{16\pi G}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\phi\tilde{\nabla}_\nu\phi - \tilde{V}(\phi)\right]$$

Energy Conservation

Stress-energy tensor:

Jordan frame

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S^{(m)}}{\delta g^{\mu\nu}}$$

Einstein frame

$$\tilde{T}_{\mu\nu} \equiv -\frac{2}{\sqrt{-\tilde{g}}}\frac{\delta S^{(m)}}{\delta \tilde{g}^{\mu\nu}}$$

$$\nabla_\mu T^{\mu\nu} = 0,$$

$$\tilde{\nabla}_\alpha \tilde{T}^{\alpha\beta} = -\tilde{T}\,\frac{\tilde{g}^{\alpha\beta}\tilde{\nabla}_\alpha\Omega}{\Omega}$$

Metric postulates not satisfied by $\tilde{T}_{\mu\nu}$ even though the two representation describe the same theory!!!

Free-fall trajectories

Considering a dust fluid in the Einstein frame with

$$\tilde{T}_{\alpha\beta} = \tilde{\rho}\,\tilde{u}_\alpha\tilde{u}_\beta$$

gives

$$\tilde{\nabla}_\alpha\left(\tilde{\rho}\,\tilde{u}^\alpha\tilde{u}^\beta\right) = \tilde{\rho}\,\frac{\tilde{g}^{\alpha\beta}\,\tilde{\nabla}_\alpha\Omega}{\Omega}$$

Projecting onto the 3-space orthogonal to $\tilde{u}^\alpha$ yields

$$\tilde{a}^\gamma = \delta^{\gamma\alpha}\frac{\partial_\alpha\Omega(\phi)}{\Omega(\phi)}$$

- No geodesic motion
- Always a force proportional to $\nabla^\mu\phi \Rightarrow$ No massive test particle in the Einstein frame!

Wrong stress-energy tensor?

Reconsider:

$$\bar{S}^{(m)} = \int d^4x\, \sqrt{-\tilde{g}} \left[-\frac{1}{2}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\phi}\tilde{\nabla}_\nu\tilde{\phi} - \tilde{V}(\tilde{\phi})\right] + \\ + S^{(m)}\left[e^{2\tilde{\alpha}(\tilde{\phi})}\tilde{g}_{\mu\nu}, \psi^{(m)}\right]$$

$$\bar{T}_{\mu\nu} \equiv -(2/\sqrt{-\tilde{g}})\delta\bar{S}^{(m)}/\delta\tilde{g}^{\mu\nu}$$

Field equations

$$\tilde{G}_{\mu\nu} = \kappa\bar{T}_{\mu\nu}$$

Bianchi identity

$$\tilde{\nabla}_\mu\tilde{G}^{\mu\nu} = 0 \Rightarrow \tilde{\nabla}_\mu\bar{T}^{\mu\nu} = 0$$

Wrong stress-energy tensor?

Not a solution!

- $\tilde{g}_{\mu\nu}$ is still not the metric whose geodesics coincide with free-fall trajectories
- $\bar{T}_{\mu\nu}$ does not reduce to the special relativistic SET when $\tilde{g}_{\mu\nu}$ is taken to be flat

$$\bar{T}_{\mu\nu} = \tilde{\nabla}_\mu\tilde{\phi}\tilde{\nabla}_\nu\tilde{\phi} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{\nabla}^\sigma\tilde{\phi}\tilde{\nabla}_\sigma\tilde{\phi} - \tilde{g}_{\mu\nu}\,\tilde{V}(\tilde{\phi}) + \tilde{T}_{\mu\nu}$$

Moral

Finding quantities that satisfy the metric postulates does not mean that they will be physically meaningful

Matter or Geometry?

Example: Is ϕ a gravitational or a non-gravitational field?

- Jordan frame: Non-minimally coupled to gravity and minimally coupled to matter
  Seems gravitational!
- Einstein frame: Minimally coupled to gravity and non-minimally coupled to matter
  Seems non-gravitational!

How about vacuum?

$$\begin{aligned}\tilde{R}_{\alpha\beta} =& R_{\alpha\beta} - 2\nabla_\alpha\nabla_\beta\left(\ln\Omega\right) - g_{\alpha\beta}g^{\gamma\delta}\nabla_\gamma\nabla_\delta\left(\ln\Omega\right)\\ &+2\left(\nabla_\alpha\ln\Omega\right)\left(\nabla_\beta\ln\Omega\right) - 2g_{\alpha\beta}g^{\gamma\delta}\left(\nabla_\gamma\ln\Omega\right)\left(\nabla_\delta\ln\Omega\right)\end{aligned}$$

Vacuum solutions are mapped to non-vacuum solutions!

Matter or Geometry?

Can't we use Energy Conditions to characterize the fields?

Answer: Maybe, but this characterization would be representation dependent and this information would need to be carried as extra baggage

General point: mathematical laws always need rules for interpretation!

Example: Coupled oscillators

$$L=\frac{\dot{q}_1^2}{2}+\frac{\dot{q}_2^2}{2}-\frac{q_1^2}{2}-\frac{q_2^2}{2}+\alpha\, q_1 q_2$$

But using normal coordinates $Q_1\left(q_1,q_2\right), Q_2\left(q_1,q_2\right)$

$$L=\frac{\dot{Q}_1^2}{2}+\frac{\dot{Q}_2^2}{2}-\frac{Q_1^2}{2}-\frac{Q_2^2}{2}$$

$f(R)$ actions and field equations

Metric $f(R)$ gravity:

$$S_{met} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\, f(R) + S_M(g_{\mu\nu},\psi)$$

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu f'(R) + g_{\mu\nu}\Box f' = \kappa T_{\mu\nu}$$

Palatini $f(R)$ gravity:

$$S_{pal} = \frac{1}{2\kappa}\int d^4x\sqrt{-g} f(\mathcal{R}) + S_M(g_{\mu\nu},\psi)$$

$$f'(\mathcal{R})\mathcal{R}_{(\mu\nu)} - \frac{1}{2}f(\mathcal{R})g_{\mu\nu} = \kappa T_{\mu\nu}$$

$$\overset{\Gamma}{\nabla}_\lambda\left(\sqrt{-g}f'(\mathcal{R})g^{\mu\nu}\right) = 0$$

$f(R)$ gravity and Brans-Dicke theory

Introduction of an auxiliary scalar plus field redefinitions yields:

- Metric $f(R) \rightarrow \omega_0 = 0$ Brans-Dicke theory:

$$S_{met} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[\phi R - V(\phi)\right] + S_M(g_{\mu\nu}, \psi)$$

- Palatini $f(R) \rightarrow \omega_0 = -3/2$ Brans-Dicke theory:

$$S_{pal} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[\phi R + \frac{3}{2\phi} \nabla_\mu \phi \, \nabla^\mu \phi - V(\phi)\right] + S_M(g_{\mu\nu}, \psi)$$

Conclusions

- Problem not specific to conformal transformations
- In the $f(R)$ representations ϕ is not even there!

Eistein-Cartan(-Sciama-Kibble) theory

Description

- Theory with independent non-symmetric connection (zero non-metricity)
- Matter action depends on metric and connection
- Two objects describing matter fields: $T_{\mu\nu}$ and $\Delta^{\lambda}{}_{\mu\nu}$
- $T_{\mu\nu}$ is not divergence free

However

- $T_{\mu\nu}$ does not reduce to the SR SET at the suitable limit
- There exists a non-trivial combination of $T_{\mu\nu}$ and $\Delta^{\lambda}{}_{\mu\nu}$ that does
- This combination is divergence free with respect to a third connection!

Discussion

Conclusions:

- A theory should not be identified with its representation
- Each representation can be from convenient to misleading according to the application
- Literature is biased (or even wrong in some cases)
- Definitions and common notions such as the SET, gravitational fields or vacuum are representation dependent
- Abstract statement such as the EEP are representation independent
- Precise statement such as the metric postulate are not!

Discussion

Further comments:

- Problem not confined to conformal representations
- Measurable quantities are conformally invariant, (classical) physics is not!
- Notice the analogy with coordinate independence.
- All of the above predispose us towards specific theories
- Critical obstacle for further progress

Further understanding is essential to go beyond a trail-and-error approach to gravity theories