# Modifying gravity, black holes, and gravity wave constraints

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COST EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

# Plan

- Modification of gravity: an introduction
  - Observation and theory: From local to cosmological scales.
  - Modification of gravity and basic rules
  - Scalar tensor theories, the question of frames, self tuning
  - Horndeski's theorem
  - Self tuning dark energy
- Black holes and no hair
  - The BBMB solution
  - A no hair theorem
  - Hair recovery
  - example solutions
- Gravitational wave observations and constraints on scalar tensor theories
  - Horndeski revisited
  - The question of frames
  - Constraints from neutron star boundaries
  - The allowed theories and disformed black holes

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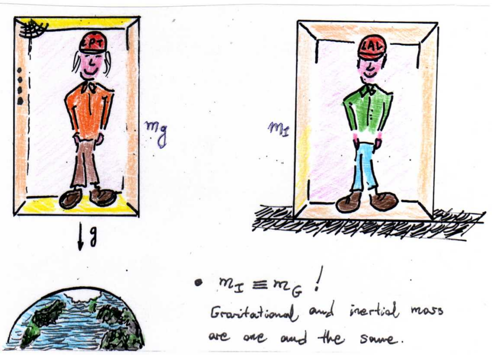

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- This means:
  -Gravity is a local condition of spacetime
  -Gravity sees all (including vacuum energy!)
  -In Newtonnian gravity $m_I$ and $m_G$ happen to be the same, in GR it is a founding principle

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- $g_{\mu\nu}$:field variable $\xrightarrow{\partial}$ $\Gamma^{\gamma}_{\alpha\beta}$, frame $\xrightarrow{\partial}$ $R^{\sigma}_{\mu\nu\rho}$ dynamics
- $G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$, EoM are second order EPD's with respect to the field variable.
- GR is a classical or an effective theory of gravity. At energy scales where curvature is large it has to be UV-completed to a quantum theory of gravity...
- Here we will concentrate on classical or IR modifications of GR.

- Theoretical consistency: In 4 dimensions, consider $\mathcal{L} = \mathcal{L}(\mathcal{M}, g, \nabla g, \nabla\nabla g)$. Then Lovelock's theorem in $D = 4$ states that GR with cosmological constant is the unique metric theory emerging from,

$$S_{(4)} = \int_{\mathcal{M}} d^4x \sqrt{-g^{(4)}}\,[R - 2\Lambda]$$

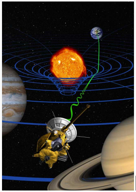
giving,

  - Equations of motion of $2^{\mathrm{nd}}$-order (Ostrogradski no-go theorem 1850!)
  - given by a symmetric two-tensor, $G_{\mu\nu} + \Lambda g_{\mu\nu}$
  - and admitting Bianchi identities.

*Under these hypotheses GR is the unique massless-tensorial 4 dimensional theory of gravity!*

# Observational data

- Experimental consistency:
  -Excellent agreement with solar system tests and strong gravity tests on binary pulsars
  -Observational breakthrough GW170817: Non local, $40Mpc$ and strong gravity test from binary neutron stars. $c_T = 1 \pm 10^{-15}$

Time delay of light  Planetary tajectories 


 Neutron star binary

# Q: What is the matter content of the Universe today?

Assuming homogeinity-isotropy and GR

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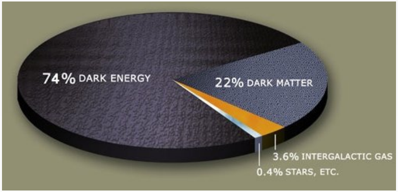


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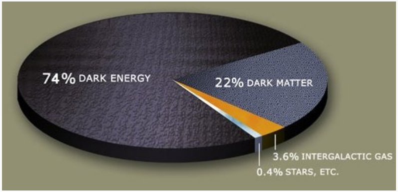


A: -Only a 4% of matter has been discovered in the laboratory. We hope to see more at LHC. But even then...

If we assume only ordinary sources of matter (DM included) there is disagreement between local, astrophysical and cosmological data.

Simplest way out: Assume a tiny cosmological constant $\rho_\Lambda = \frac{\Lambda_{obs}}{8\pi G} = (10^{-3} eV)^4$, ie modify Einstein's equation by,

$$G_{\mu\nu} + \Lambda_{obs} g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Cosmological constant introduces $\sqrt{\Lambda}$ and generates a cosmological horizon
- $\sqrt{\Lambda}$ is as tiny as the inverse size of the Universe today, $r_0 = H_0^{-1}$
- Note that $\frac{\text{Solar system scales}}{\text{Cosmological Scales}} \sim \frac{10\text{ A.U.}}{H_0^{-1}} = 10^{-14}$
- Typical mass scale for neutrinos...
- Theoretically the cosmological constant should be huge!

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- Vacuum energy fluctuations are at the UV cutoff of the QFT $\Lambda_{vac}/8\pi G \sim m_{Pl}^4 \ldots$
- Vacuum potential energy from spontaneous symmetry breaking $\Lambda_{EW} \sim (200\,GeV)^4$
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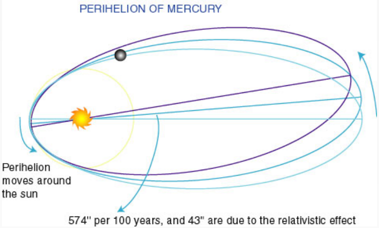


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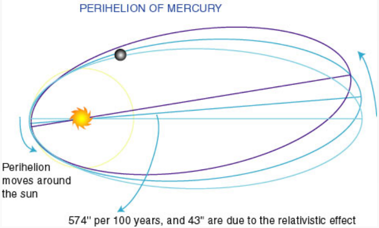


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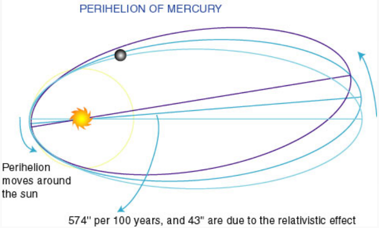


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# General issues to deal with

- Since GR is unique we need to introduce new and genuine gravitational degrees of freedom!
- They generically must not lead to higher derivative equations of motion. Additional degrees of freedom can lead to ghosts and vacuum is unstable (Ostrogradski theorem 1850 [Woodard 2006]). Since [Gleyzes et al] we know that higher derivative EOM do not always lead to ghosts. What is essential is the number of propagating dof.
- Matter does not directly couple to novel gravity degrees of freedom. Matter sees only the metric and evolves in metric geodesics. As such EEP is preserved and space-time can be put locally in an inertial frame.
- Novel degrees of freedom need to be screened from local gravity experiments. Need a well defined GR local limit (Chameleon [Khoury 2013], Vainshtein [Babichev and Deffayet 2013]).
- Exact solutions are essential in modified gravity in order to understand strong gravity regimes and novel characteristics. Need to deal with no hair paradigm, absence of GR black hole theorems etc.
- A modified gravity theory should tell us something about the cosmological constant problem and in particular how to screen an a priori enormous cosmological constant. Self tuning and self acceleration.

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# Possible modified gravity theories

- Assume extra dimensions : Extension of GR to Lovelock theory with modified yet second order field equations. Braneworlds DGP model RS models, Kaluza-Klein compactification, String theory and holography.
- Graviton is not massless but massive! dRGT theory and bigravity theory.
- 4-dimensional modification of GR:
  - Scalar-tensor theories, $f(R)$, Galileon/Hornedski theories $\to$ Beyond Horndeski and DHOST theories.
  - Vector-tensor theories
- Lorentz breaking theories: Horava gravity, Einstein Aether theories
- Theories modifying geometry: inclusion of torsion, choice of geometric connection

# Scalar-tensor theories

- are the simplest modification of gravity with one additional degree of freedom
- Admit a uniqueness theorem due to Horndeski 1973 and extended to DHOST theories [Langlois et.al] [Crisostomi et.al.]
- contain or are limits of other modified gravity theories.
- (Can) have insightful screening mechanisms (Chameleon, Vainshtein)
- Include terms that can screen classically a big cosmological constant or give self accelerating solutions. Need a non trivial scalar field.
- Have non trivial hairy black hole solutions even around non trivial self accelerating vacua
- Theories are strongly constrained from gravity waves.

# Jordan-Brans-Dicke theory [Sotiriou 2014]

## Simplest scalar tensor theory

$$S_{\rm BD} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(\varphi R - \frac{\omega_0}{\varphi}\nabla^\mu\varphi\nabla_\mu\varphi - m^2(\varphi-\varphi_0)^2\right) + S_m(g_{\mu\nu},\psi)$$

- $\omega_0$ Brans Dicke coupling parameter fixing scalar strength
- $\varphi = \varphi_0$ constant gives GR black hole solutions (with a cosmological constant) but spherically symmetric solutions are not unique (and not GR)!
- For spherical symmetry we find,

$$\gamma \equiv \frac{h_{ij}|_{i=j}}{h_{00}} = \frac{2\omega_0 + 3 - exp\left[-\sqrt{\frac{2\varphi_0}{2\omega_0+3}}mr\right]}{2\omega_0 + 3 + exp\left[-\sqrt{\frac{2\varphi_0}{2\omega_0+3}}mr\right]}$$

- where $\gamma = 1 + (2.1 \pm 2.3)\times 10^{-5}$ from local tests.
- $\omega_0 > 40000$ in the absence of potential.
- Need a more complex version in order to screen the scalar mode locally and to obtain hairy black holes.$\rightarrow$ Higher order dervative theories.

# Jordan versus Einstein frame

Jordan frame is the physical frame. Matter couples only to metric and the weak equivalence principle is satisfied.

$$S_{Jordan} = \frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\left(\varphi\tilde{R} - \frac{\omega(\varphi)}{\varphi}\tilde{\nabla}^{\mu}\varphi\tilde{\nabla}_{\mu}\varphi - V(\varphi)\right) + S_m(\tilde{g}_{\mu\nu},\psi)$$

Consider a conformal transformation: $\tilde{g}_{ab} = \Omega^2(x)\, g_{ab}$ :

$$\int d^4x\sqrt{-g}\, R = \int d^4x\sqrt{-\tilde{g}}\,(\tilde{R}\,\Omega^2 + 6\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega), \qquad \Phi = \Phi(\varphi;\Omega)$$

and the action transforms into

$$S_{Einstein} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R - \nabla^{\mu}\Phi\nabla_{\mu}\Phi - U(\Phi)) + S_m(g_{\mu\nu},\Phi,\psi)$$

- The action is GR like, but,
- Matter couples to metric and scalar!
- Non physical frame or Einstein frame. Matter in free-fall does not follow $g_{\mu\nu}$ geodesics!
- Frames are equivalent mathematically and physically as long as we know how matter couples to the metric.

# $f(R)$: a higher order metric theory?

- $f(R)$ is a scalar tensor theory in disguise

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_m(g_{\mu\nu},\psi)$$

metric field eqs are:

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - [\nabla_\mu\nabla\nu - g_{\mu\nu}\Box]f'(R) = 8\pi G T_{\mu\nu}$$

Clearly 4th-order equations in $g_{\mu\nu}$. Ghosts? Not really because,

- $$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[\varphi R - V(\varphi)\right] + S_m(g_{\mu\nu},\psi)$$

where $V(\varphi) \equiv f(\phi) - \phi f'(\phi)$ and $\varphi = f'(\phi)$. Hence BD with $\omega = 0$. Higher derivative theory can be written in terms of a lower derivative scalar tensor theory.
Order of field eqs does not necessarily dictate number of DoF and hence presence of ghosts. One has to identify the number and physical nature of degrees of freedom. Theories such as $f(R)$ are degenerate theories evading Ostrogradski theorem.

# Self-Tuning idea

- Expected value of the cosmological constant is enormous compared to the observed value
- Weinberg's no go theorem states that we cannot have a Poincare invariant vacuum with $\Lambda \neq 0$
- Question: Can we break Poincare invariance for some additional field (not the metric)?
- Keep $g_{\mu\nu} = \eta_{\mu\nu}$ locally but allow for $\phi \neq constant$.
- Can we have a portion of flat spacetime whatever the value of the cosmological constant...
- and without fine-tuning any of the parameters of the theory?
- Toy model theory of self-tuning scalar field.
- Can I have geometric de Sitter acceleration independent of the vacuum cosmological constant? Can dark energy be driven by a dynamical source?
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- Keep $g_{\mu\nu} = \eta_{\mu\nu}$ locally but allow for $\phi \neq \textit{constant}$.
- Can we have a portion of flat spacetime whatever the value of the cosmological constant...
- and without fine-tuning any of the parameters of the theory?
- Toy model theory of self-tuning scalar field.
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- In order to elaborate on such ideas (self acceleration, self tuning, screening etc) we need to introduce higher derivative theories.

# Self-Tuning idea

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# Galileons/Horndeski [Horndeski 1973]

**What is the most general scalar-tensor theory**

with second order field equations [Horndeski 1973]?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x \sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$\begin{aligned}
L_2 &= K(\phi, X), \\
L_3 &= -G_3(\phi, X)\Box\phi, \\
L_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
L_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}$$

the $G_i$ are free functions of $\phi$ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- In fact same action as covariant Galileons [Deffayet, Esposito-Farese, Vikman]. Galileons are scalars with Galilean symmetry for flat spacetime.

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- Examples: $G_4 = 1 \longrightarrow R$.
  $G_4 = X \longrightarrow G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$.
  $G_3 = X \longrightarrow$ "DGP" term, $(\nabla\phi)^2\Box\phi$
  $G_5 = lnX \longrightarrow$ gives GB term, $\hat{G} = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - 4R^{\mu\nu}R_{\mu\nu} + R^2$
  Action is unique modulo integration by parts.

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- Horndeski theory admits self accelerating vacua with a non trivial scalar field in de Sitter spacetime. A subset of Horndeski theory self tunes the cosmological constant.

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- Generically ST or SA vacua acquire a non trivial scalar field with flat or de Sitter metric.

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- This brings up the issue of time dependance which will be crucial for black holes. We will now briefly examine time dependent scalars and dark energy related issues.

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Find the most general scalar-tensor theory with self-tuning property:

-Admitting flat space time solution with a non trivial scalar

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-Without fine tuning the parameters of the theory.

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\begin{aligned}
\mathcal{L}_{john} &= \sqrt{-g}V_{john}(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \\
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Fab 4 terms

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Fab 4 terms

- All are scalar-curvature interaction terms stemming from Lovelock theory. They are the unique interaction terms yielding second order field equations.
- Theory depends on 4 arbitrary potentials.
- Fab 4 terms can self-tune the cosmological constant for flat spacetime. At the absence of curvature Fab 4 terms drop out.
- Adding a standard kinetic term self tunes to de Sitter [Gubitosy, Linder]

# Example self tuning solution

Consider a slowly varying scalar field in the presence of an arbitrary cc in a time evolving universe,

- Flat spacetime: Milne metric $ds^2 = -dT^2 + T^2\left(\frac{d\chi^2}{1+\chi^2} + \chi^2 d\Omega^2\right)$...
- For simplicity take analytic expansion:

$$V_{john} = C_j\,,\, V_{paul} = C_p\,,\, V_{george} = C_g + C_g^1\,\phi\,,\, V_{ringo} = C_r + C_r^1\,\phi - \frac{1}{4}C_j\,\phi^2$$

- Friedmann equation reads,

$$c_j(\dot\phi H)^2 - c_p(\dot\phi H)^3 - c_g^1(\dot\phi H) + \rho_\Lambda = 0$$

  with matter source $\rho_\Lambda = \Lambda$, vacuum cosmological constant. Note that $\dot\phi H$ appear as homogeneous powers of time...
- Hence since $H = 1/T$ for Milne, taking $\phi = \phi_0 + \phi_1 T^2$ gives $c_j(\phi_1)^2 - c_p(\phi_1)^3 - c_g^1(\phi_1) + \rho_\Lambda = 0$ an algebraic constraint.
- Integration constant $\phi_1$ is fixed by the cosmological constant for arbitrary values of the theory potentials.
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the $G_i$ are free functions of $\phi$ and $X\equiv-\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX}\equiv\partial G_i/\partial X$.

- Horndeski theory includes Shift symmetric theories where $G_i$'s depend only on $X$ and $\phi\to\phi+c$.
  Associated with the symmetry there is a Noether current, $J^\mu$ which is conserved $\nabla_\mu J^\mu=0$.
  Presence of this symmetry permits a very general no hair argument

# So far...

- Even for a static spherically symmetric spacetime scalar field is to be time dependent if we are going to be in a non trivial branch of solutions
- Shift symmetric Horndeski theory provides a conserved Noether current.

# Black holes have no hair [recent review Herdeiro and Radu 2015]

During gravitational collapse...
Black holes eat or expel surrounding matter
their stationary phase is characterized by a limited number of charges
and no details
black holes are bald...

No hair arguments/theorems dictate under some reasonable hypotheses that adding degrees of freedom lead to singular solutions...

For example in vanilla scalar-tensor theories black hole solutions are GR black holes with constant scalar.

Warning : beyond GR Birkhoff's theorem is not valid.
Spherical symmetry thus does not guarantee staticity.
Scalar tensor black holes radiate monopole gravity waves.
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Let us now see a classical example of a hairy solution...

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# Example: BBMB solution

- Consider a conformally coupled scalar field $\phi$:

$$S[g_{\mu\nu},\phi,\psi]=\int_{\mathcal{M}}\sqrt{-g}\left(\frac{R}{16\pi G}-\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi-\frac{1}{12}R\phi^2\right)\mathrm{d}^4x+S_m[g_{\mu\nu},\psi]$$

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$$\begin{cases} g_{\alpha\beta}\mapsto\tilde{g}_{\alpha\beta}=\Omega^2 g_{\alpha\beta} \\ \phi\mapsto\tilde{\phi}=\Omega^{-1}\phi \end{cases}$$

- There exists a black hole geometry with non-trivial scalar field and secondary black hole hair.
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$$ds^2 = -\left(1 - \frac{m}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1 - \frac{m}{r}\right)^2} + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

with **secondary** scalar hair

$$\boxed{\phi = \sqrt{\frac{3}{4\pi G}}\frac{m}{r-m}}$$

- Geometry is that of an extremal RN.
  Problem:The scalar field is **unbounded** at $(r = m)$.
- A cosmological constant can cure this; [MTZ] family of solutions
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