Modifying gravity, black holes, and gravity wave constraints

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Plan

- Modification of gravity: an introduction
  - Observation and theory: From local to cosmological scales.
  - Modification of gravity and basic rules
  - Scalar tensor theories, the question of frames, self tuning
  - Horndeski's theorem
  - Self tuning dark energy
- Black holes and no hair
  - The BBMB solution
  - A no hair theorem
  - Hair recovery
  - example solutions
- Gravitational wave observations and constraints on scalar tensor theories
  - Horndeski revisited
  - The question of frames
  - Constraints from neutron star boundaries
  - The allowed theories and disformed black holes

Summary so far

- Scalar tensor theory with 2nd order EOM: Horndeski theory-parametrized by 4 free functions.
- Can go beyond Horndeski... More on this later on
- Vacua in Horndeski can be non trivial and give dark energy without a cosmological constant. Non trivial vacua lead to time dependent scalars even for flat spacetime.
- Self tuning solutions for flat and de Sitter spacetimes. Can we connect dark energy vacua to black hole solutions?
- Black holes have no hair but, no hair theorems are not valid for time dependent spacetimes.

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...and shift symmetric theories

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Let us now look at a specific no hair theorem for static and spherically symmetric spacetimes...

...and shift symmetric theories

No hair [Hui, Nicolis] [Sotiriou, Zhou] [Babichev, CC, Lehébel]

Static no hair theorem

Consider shift symmetric Horndeski theory with G_2, G_3, G_4, G_5 arbitrary functions of X. We have a Noether current J^μ which is conserved, $\nabla_\mu J^\mu = 0$.

We now suppose that:

1. spacetime and scalar are spherically symmetric and static,
$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dK^2, \; \phi = \phi(r)$$
2. spacetime is asymptotically flat, $\phi' \to 0$ as $r \to \infty$ and the norm of the current J^2 is finite on the horizon,
3. there is a canonical kinetic term X in the action,
4. and the G_i functions are such that their X-derivatives contain only positive or zero powers of X.

Under these hypotheses, ϕ is constant and thus the only black hole solution is locally isometric to Schwarzschild.

Most interesting part of no go theorem: Breaking any of these hypotheses leads to black hole solutions!

Theorem can be extended for star solutions. [Lehébel et al.]

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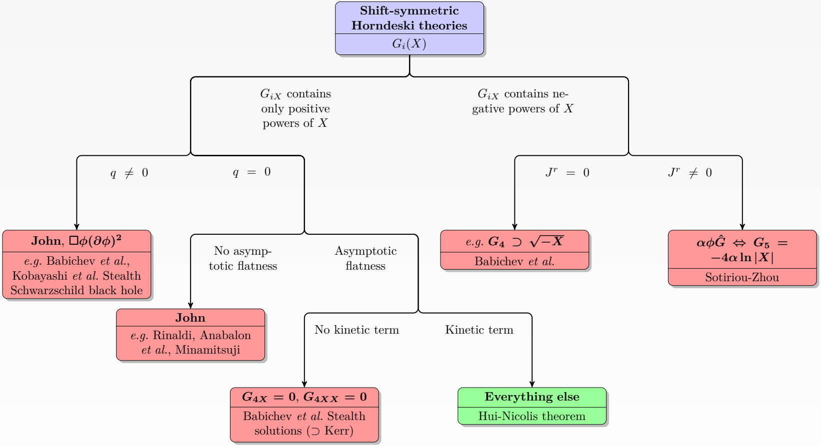
Hair versus no hair [figure: Lehébel]



Introducing time dependence, $q \neq 0$

Spherical symmetry certainly does not impose staticity (not like GR).

- Furthermore, for self accelerating or self tuning solutions one has a time dependence for the scalar in FRW coordinates
- In spherical symmetry this leads to a time and radially depending scalar already for flat spacetime.
- So let us allow time dependence for the scalar while keeping for a static and spherically symmetric spacetime.

But is this consistent with respect to the field equations:

$$\mathcal{E}_\phi = 0, \qquad \mathcal{E}_{\mu\nu} = 0$$

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The question of time dependence, $qt + \psi(r)$

Consistency theorem [Babichev, CC, Hassaine]

Consider :
-an arbitrary shift symmetric Horndeski theory $\phi \to c + \phi$
-a scalar-metric ansatz $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dK^2$, $\phi = qt + \psi(r)$ with $q \neq 0$.
The unique solution to the scalar field equation $\mathcal{E}_\phi = 0$ and the "matter flow" metric equation $\mathcal{E}_{tr} = 0$ is given by $J^r = 0$.

- We are killing two birds with one stone.
- The current now reads, $J^\mu J_\mu = -h(J^t)^2 + (J^r)^2/f$ and is regular. Time dependence renders no hair theorem irrelevant.
- If $J^r = 0$ allows $\phi' \neq 0$ solutions then we may construct hairy solutions.
- This is where the higher order nature of Horndeski theory is essential!!

General solution

Consider, $L = R - \eta(\partial\phi)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\Lambda$ For static and spherically symmetric spacetime.

The general solution of theory L for static and spherically symmetric metric and $\phi = \phi(t, r)$ is given as a solution to the following third order algebraic equation with respect to $\sqrt{k(r)}$:

$$(q\beta)^2 \left(1 + \frac{r^2}{2\beta}\right)^2 - \left(2 + (1 - 2\beta\Lambda)\frac{r^2}{2\beta}\right) k(r) + C_0 k^{3/2}(r) = 0$$

All metric and scalar functions given with respect to k and $\phi = qt + \psi(r)$.

For general shift symmetric G_2, G_4 the result can be extended, [Kobayashi, Tanahashi]

Let us now give some specific examples for the different cases...

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Scalar with constant velocity $q \neq 0$

Consider the action,

$$S = \int d^4x \sqrt{-g} \left[\zeta R - 2\Lambda - \eta (\partial\phi)^2 + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right] \ldots,$$

- Scalar field equation and conservation of current,

$$\nabla_\mu J^\mu = 0, \quad J^\mu = (\eta g^{\mu\nu} - \beta G^{\mu\nu}) \partial_\nu \phi.$$

- Take $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$, and $\phi = \phi(t, r)$ then
- $\phi = \psi + qt$ while $\mathcal{E}_{tr} = -\frac{q^2 J^r}{f} \longrightarrow J^r = 0$ solves both equations...
- $\beta G^{rr} - \eta g^{rr} = 0$ ie. $f = \frac{(\beta + \eta r^2) h}{\beta (rh)'}$ or $\phi' = 0$

For a higher order theory $J^r = 0$ does not necessarily imply $\phi = const.$

$J^r = 0$ means that we kill primary hair since, $\nabla_\mu J^\mu = 0 \rightarrow \sqrt{-g}(\beta G^{rr} - \eta g^{rr}) \partial_r \phi = c$

- We now solve for the remaining field eqs...

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Solving the remaining EoM

- From (rr)-component get ψ'

$$\psi' = \pm \frac{\sqrt{r}}{h(\beta+\eta r^2)} \left(q^2 \beta(\beta+\eta r^2) h' - \frac{\zeta\eta + \beta\Lambda}{2} (h^2 r^2)' \right)^{1/2}.$$

- and finally (tt)-component gives $h(r)$ via,

$$h(r) = -\frac{\mu}{r} + \frac{1}{r} \int \frac{k(r)}{\beta+\eta r^2} dr,$$

with

$$q^2 \beta(\beta+\eta r^2)^2 - \left(2\zeta\beta + (\zeta\eta - \beta\Lambda) r^2\right) k + C_0 k^{3/2} = 0,$$

Any solution to the algebraic eq for $k = k(r)$ gives full solution to the system!

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Asymptotically flat limit : $\Lambda = 0$, $\eta = 0$

- Consider $S = \int d^4x\sqrt{-g}\,[\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]$
- Algebraic equation to solve: $q^2\beta^3 - 2\zeta\beta k + C_0 k^{3/2} = 0 \rightarrow k = constant$!
- $f(r) = h(r) = 1 - \mu/r$
- $\phi_\pm = qt \pm q\mu\left[2\sqrt{\frac{r}{\mu}} + \log\frac{\sqrt{r}-\sqrt{\mu}}{\sqrt{r}+\sqrt{\mu}}\right] + \phi_0 \ldots$
- Consider $v = t + \int (fh)^{-1/2}dr$ then $ds^2 = -hdv^2 + 2\sqrt{h/f}\,dvdr + r^2 d\Omega^2$
  Regular chart for horizon, EF coordinates
- $\phi_+ = q\left[v - r + 2\sqrt{\mu r} - 2\mu\log\left(\sqrt{\frac{r}{\mu}} + 1\right)\right] + \text{const}$
- Scalar regular at future black hole horizon.

Schwarzschild geometry with a non-trivial regular scalar field.

Exterior geometry for star

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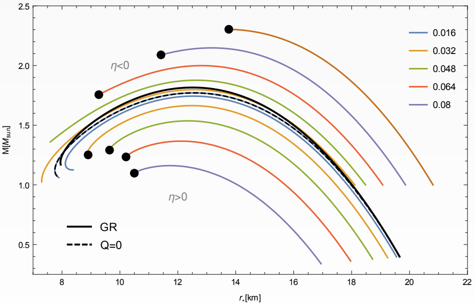
Star solutions [Cisterna, Delsate, Rinaldi], [Maselli, Silva, Minamitsuji, Berti]

- Consider $S = \int d^4x\sqrt{-g}\,[\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]$
- Take stealth solution for exterior and consider PF matter for interior with ρ and P that does not couple to scalar.
- $J^r = 0$, and therefore $G^{rr} = 0$ which effects star interior.
- For fixed star radius $\beta > 0$ ($\beta < 0$) gives heavier (lighter) stars than GR.
- No GR limit for $q \to 0$



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- Consider $S = \int d^4x\sqrt{-g}\,[\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]$
- Take stealth solution for exterior and consider PF matter for interior with ρ and P that does not couple to scalar.
- $J^r = 0$, and therefore $G^{rr} = 0$ which effects star interior.
- For fixed star radius $\beta > 0$ ($\beta < 0$) gives heavier (lighter) stars than GR.
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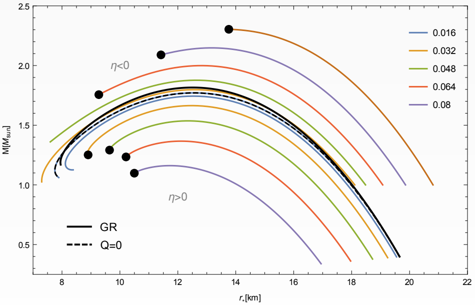

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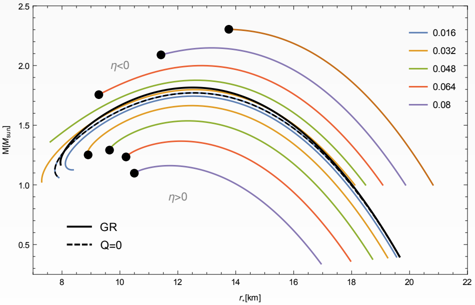

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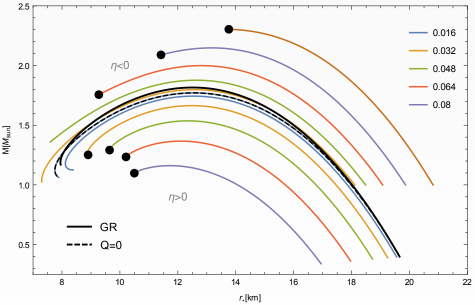

Self tuning de Sitter black hole

- $$S = \int d^4x \sqrt{-g} \left[\zeta R - 2\Lambda - \eta \left(\partial\phi\right)^2 + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

  ... $q^2\beta(\beta+\eta r^2)^2 - \left(2\zeta\beta + (\zeta\eta - \beta\Lambda) r^2\right) k + C_0 k^{3/2} = 0$
- $f = h = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta} r^2$ de Sitter Schwarzschild!
- $\psi' = \pm \frac{q}{h} \sqrt{1-h}$ and $\phi(t,r) = q\,t + \psi(r)$
- The effective cosmological constant is not the vacuum cosmological constant. In fact,
- Self tuning relation : $q^2\eta = \Lambda - \Lambda_{eff} > 0$
- Hence for any $\Lambda > \Lambda_{eff}$ fixes q, integration constant.
- where $\Lambda_{eff} = -\frac{\eta}{\beta}$ is fixed by effective theory.
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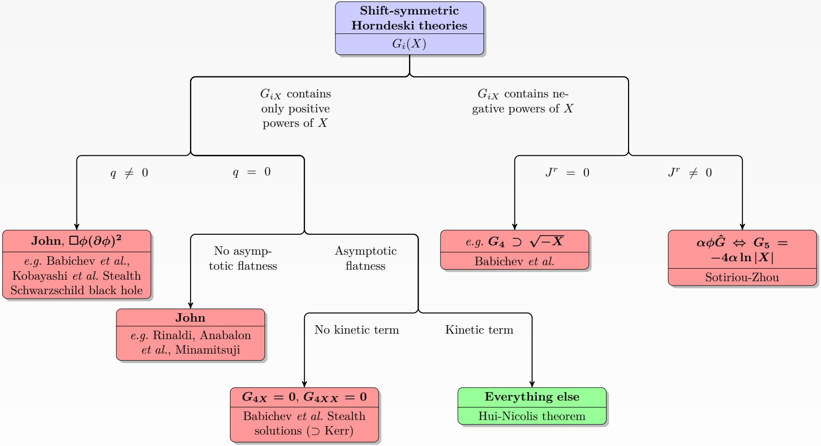
Hair versus no hair [Lehébel]



The special case of the Gauss-Bonnet invariant

[Sotiriou, Zhou] [Duncan et.al] [Mavromatos et.al]

The Gauss-Bonnet term, $\hat{G} = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - 4R^{\mu\nu}R_{\mu\nu} + R^2$, is a topological invariant in 4 dimensions.
Variation with respect to the metric gives the 4 dim Lovelock identity, $H_{\mu\nu} = -2P_{\mu cde}R_{\nu}{}^{cde} + \frac{g_{\mu\nu}}{2}\hat{G} = 0$. If we couple to scalar then $\phi\hat{G}$ ceases to be trivial. It can be obtained in Horndeski theory via $G_5 \sim \ln X$
The theory

$$\mathcal{L}^{\rm GB} = \frac{R}{2} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + \alpha\phi\hat{G}$$

is non trivial and shift symmetric. Here, $\hat{G}$ (is independent of ϕ) and acts as a source to the scalar which cannot be set to zero.

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- $\Box\phi + \alpha\hat{G} = 0$
- Numerical solution can be found where the scalar and mass integration constants are fixed so that the solution is regular at the horizon.

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- $\Box\phi + \alpha\hat{G} = 0$
- The mass of the black hole has a minimal size fixed by the GB coupling α. The singularity is attained at positive r.

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- $\Box\phi + \alpha\hat{G} = 0$
- Solutions with $q \neq 0$ and regular Noether current are in a different branch and are singular.

Conformally coupled scalar field

- Consider a conformally coupled scalar field ϕ revisited:

$$S[g_{\mu\nu}, \phi, \psi] = \int_{\mathcal{M}} \sqrt{-g} \left(\frac{R}{16\pi G} - \frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi - \frac{1}{12} R \phi^2 \right) \mathrm{d}^4 x + S_m[g_{\mu\nu}, \psi]$$

- **Invariance of the EOM of ϕ under the conformal transformation**

$$\begin{cases} g_{\alpha\beta} \mapsto \tilde{g}_{\alpha\beta} = \Omega^2 g_{\alpha\beta} \\ \phi \mapsto \tilde{\phi} = \Omega^{-1} \phi \end{cases}$$

- There exists a black hole geometry with non-trivial scalar field and secondary black hole hair.
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BBMB completion [CC, Kolyvaris, Papantonopoulos and Tsoukalas]

- We would like to combine the above properties in order to obtain a hairy black hole.
- Consider the following action, $S(g_{\mu\nu}, \phi, \psi) = S_0 + S_1$ where
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$$S_0 = \int dx^4 \sqrt{-g} \left[\zeta R + \eta \left(-\frac{1}{2}(\partial\phi)^2 - \frac{1}{12}\phi^2 R \right) \right]$$
  and
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$$S_1 = \int dx^4 \sqrt{-g} \left(\beta G_{\mu\nu} \nabla^\mu \Psi \nabla^\nu \Psi - \gamma T^{BBMB}_{\mu\nu} \nabla^\mu \Psi \nabla^\nu \Psi \right) ,$$
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- Scalar field equation of S_1 contains metric equation of S_0.
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- Scalar ϕ has an additional branch regular at the "horizon"

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$$\phi(r) = \frac{c_0}{r} \; ,$$

$$\psi'(r) = \pm q \frac{\sqrt{mr - \frac{\gamma c_0^2}{12\beta}}}{r\, h(r)},$$

$$\beta\eta + \gamma(q^2\beta - \zeta) = 0 \; .$$

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- Scalar charge c_0 playing similar role to EM charge in RN
  Galileon Ψ regular on the future horizon

$$\psi = qv - q \int \frac{dr}{1 \pm \sqrt{1 - h(r)}}$$

So far...

- For $q \neq 0$ we can find solutions analytically for G_2, G_4 and otherwise numerically
- For $q = 0$ we need to source the scalar field equation breaking one of the hypotheses of the theorem [Babichev, CC, Lehébel]
- Slow rotation gives identical correction to GR. Stationary solutions not known except for stealth Kerr...
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Galileons/Horndeski [Horndeski 1973]

What is the most general scalar-tensor theory

with second order field equations [Horndeski 1973]?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x \sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$\begin{aligned}
L_2 &= G_2(\phi, X), \\
L_3 &= G_3(\phi, X)\Box\phi, \\
L_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
L_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}$$

the G_i are free functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- In fact same action as covariant Galileons [Deffayet, Esposito-Farese, Vikman]. Galileons are scalars with Galilean symmetry for flat spacetime.

Galileons/Horndeski [Horndeski 1973]

$$S_H = \int d^4x\sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$L_2 = G_2(\phi, X),$$
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  Action is unique modulo integration by parts.

Going beyond Horndeski [Gleyzes et.al], [Zumalacarregui et.al],[Deffayet et.al], [Langlois et.al], [Crisostomi et.al]

What is the most general scalar-tensor theory with three propagating degrees of freedom?

It is beyond Horndeski but not quite DHOST yet...

$$S_H = \int d^4x\sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)\ ,$$

where

$$L_2 = G_2(\phi, X), \qquad L_3 = G_3(\phi, X)\Box\phi,$$

$$L_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\,\epsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'},$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

$$+ F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}$$

where $XG_{5,X}F_4 = 3F_5\left[G_4 - 2XG_{4,X} - (X/2)G_{5,\phi}\right]$. Beyond Horndeski acquires one extra function. BH has similar SA and ST solutions.

Conformal and disformal relations [Bellido, Zumalacarregui]

How are theories mapped under conformal and disformal transformations?

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X) g_{\mu\nu} + D(\phi, X) \nabla_\mu \phi \nabla_\nu \phi$$

- Horndeski theory has G_2, G_3, G_4, G_5 free functions.
- For $C(\phi)$ and $D(\phi)$ we remain within Horndeski.
- However if we take a disformal $D(X)$ we jump to
- Beyond Horndeski (one more free function)
- Take a conformal $C(X)$ and jump to
- DHOST Type I (one more free function) [Langlois, Noui], [Crisostomi, Koyama]

In other words DHOST type I are all related to some Horndeski theory. Remaining DHOST theories are pathological [Langlois, Noui, Vernizzi]

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GW170817 constraints on scalar tensor theories [Creminelli, Vernizzi], [Ezquiaga, Zumalacarregui]

- The combined observation of a gravity wave signal from a binary neutron star and its GRB counterpart constraints $c_T = 1$ to a 10^{-15} accuracy.
- For dark energy the scalar field (ST or SA) is non trivial at such distance scales (40Mpc) and generically mixes with the tensor metric perturbations modifying the light cone for gravity waves.
- For Horndeski the surviving theory has free $G_2(\phi, X)$, $G_3(\phi, X)$, $G_4(\phi)$ and $G_5 = 0$.
- For beyond Horndeski we have $G_5 = 0, F_5 = 0, 2G_{4,X} + XF_4 = 0$ and theory,

$$L_{c_T=1} = G_2(\phi, X) + G_3(\phi, X)\Box\phi + B_4(\phi, X)\,{}^{(4)}R \\ - \frac{4}{X} B_{4,X}(\phi, X)(\phi^\mu \phi^\nu \phi_{\mu\nu}\Box\phi - \phi^\mu \phi_{\mu\nu}\phi_\lambda \phi^{\lambda\nu}) \,,$$

- For DHOST we just make a conformal transformation of the above, $G_2(\phi, X) G_3(\phi, X)$, $B_4(\phi, X)$, $C(\phi, X)$

Galileons/Horndeski [Horndeski 1973]

What is the most general scalar-tensor theory

with second order field equations [Horndeski 1973]?

Horndeski has shown that the most general action with this property is

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Physical and disformed frames

Most general scalar tensor theory with $c_T = 1$ minimally coupled to matter parametrized by G_2, G_3, B_4, C

$$L_{c_T=1} = G_2 + G_3 \Box\phi + B_4 C\, {}^{(4)}R - \frac{4B_{4,X}C}{X}\phi^\mu\phi^\nu\phi_{\mu\nu}\Box\phi + \left(\frac{4B_{4,X}C}{X} + \frac{6B_4 C_{,X}{}^2}{C} + 8C_{,X}B_{4,X}\right)\phi^\mu\phi_{\mu\nu}\phi_\lambda\phi^{\lambda\nu} + \frac{8C_{,X}B_{4,X}}{X}(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2 \,.$$

- Horndeski is related via a transformation

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X) g_{\mu\nu} + D(\phi, X)\nabla_\mu\phi\nabla_\nu\phi$$

  to the $L_{c_T=1}$ for given C and D.
- One can start with a $c_T \neq 1$ Horndeski theory and map it to a DHOST $c_T = 1$ theory for a specific function D.
- The former is what we could have called the Einstein frame respective to the latter, the Jordan frame...
- except that the metric is disformed in the procedure...
- The more the symmetry the better

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The physical frame and the disformed solution [Babichev, CC, GEFarèse, Lehébel]

- The theory

$$S=\int d^4x\sqrt{-g}\left[\zeta R-2\Lambda-\eta\left(\partial\phi\right)^2+\beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right],$$

is excluded or it is not in the physical frame.

- Solution: $f=h=1-\frac{\mu}{r}+\frac{\eta}{3\beta}r^2$, $\phi=qt\pm\frac{q}{h}\sqrt{1-h}$ with $\Lambda_{\text{eff}}=-\zeta\eta/\beta$.
- The physical frame is :

$$\tilde{g}_{\mu\nu}=g_{\mu\nu}-\frac{\beta}{\zeta+\frac{\beta}{2}\varphi_\lambda^2}\varphi_\mu\varphi_\nu.$$

- Indeed the $\tilde{g}_{\mu\nu}$ frame is a beyond Horndeski theory with $c_T=1$ for a cosmological background.
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Summary

- Starting from a no hair theorem we have seen how to construct hairy black holes.
- Similar theorem exists for neutron stars.
- Higher order terms essential for novel branches of black holes. Time dependance essential for regularity.
- One can construct solutions with EM fields and black hole solutions with primary hair by adding additional scalar fields
- Techniques for shift symmetric Horndeski can be extended to Maxwell-Proca theories.

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Slowly rotating solutions [Maselli, Silva, Minamitsuji, Berti]

Using the Hartle Thorne perturbative approximation in which frame-dragging is assumed linear in angular velocity

$$\mathrm{d}s^2 = -h(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2) - 2\omega(r) r^2 \sin^2\theta \mathrm{d}t\mathrm{d}\varphi,$$

We get an ode to linear order:

$$2(1-\beta X)\left[\omega'' + \frac{\omega'}{2}\left(\frac{f'}{f} + \frac{8}{r} - \frac{h'}{h}\right)\right] - 2\beta X'\omega' = 0$$

which agrees with GR for X constant.
What happens for $X \neq$ const.
We can integrate once,

$$(1-\beta X)\omega' = \frac{C_1\sqrt{k}}{r^4(1+\frac{r^2}{2\beta})}$$

but, one can show by using remaining field equations that correction is always identical to GR [Lehébel].

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