Electron acceleration by random DC electric fields

Anastasios Anastasiadis, Loukas Vlahos and Manolis K. Georgoulis

Section of Astrophysics, Astronomy and Mechanics,

Department of Physics, University of Thessaloniki, GR-54006 Thessaloniki, Greece

Received ______________________; accepted ______________________

ABSTRACT

We present a global model for the acceleration of electrons in the framework of the *statistical flare* model (Vlahos et al. 1995). In this model, solar flares are the result of an internal Self-Organised Critical (SOC) process in a complex, evolving, and highly inhomogeneous active region. The acceleration of electrons is due to localised DC electric fields which are closely related with the energy release process in the active region. Our numerical results regarding the kinetic energy distribution of the accelerated electrons show a power-law or an exponential-law behaviour depending upon the maximum trapping time of the energetic particles inside the acceleration volume.

Subject headings: Sun: flares – Sun: particle acceleration

1. Introduction

It is documented that the energy released during solar flares is separated in three parts: (1) intense localised heating, (2) particle acceleration and (3) mass flows. It is also clear that a successful model for the radiation emitted during an eruptive energy release in the sun cannot be constructed without consideration of the global aspects of the energy release process (Priest 1992).

A large number of observations on the spatio-temporal evolution of solar flares have been collected (for reviews see: Dennis 1985, 1988; Ramaty & Murphy 1987; Vilmer 1987; Pick et al. 1990) and their interpretation lead to a variety of proposals for acceleration processes such as MHD waves, shock waves, DC electric fields, double layers, coherent acceleration processes etc. (for reviews see: Heyvaerts 1981; de Jager 1986; Forman et al. 1986; Scholer 1988; Vlahos 1989; Vlahos et al. 1989; Melrose 1990).

In the existing literature the transport of the energetic particles was decoupled from the acceleration processes in most cases. A few attempts have been made to reconcile the acceleration and the transport of energetic particles (e.g. Miller & Ramatty 1989; Ryan & Lee 1991; Hamilton & Petrosian 1992). All of the above approaches have no clear connection between the acceleration mechanism and the energy release process.

Recent observations of flare radio emission (Kuijpers et al. 1981; Benz 1985; Benz & Aschwanden 1992; Vilmer 1993; Trottet 1994; Aschwanden et al. 1995) suggest that the energy released during solar flares is fragmented. Parker (1988, 1989) proposed that a large number of small unobserved energy releases (10^{24} ergs up to 10^{26} ergs) called "nanoflares" may provide essential energy input to the corona. This proposal places great emphasis on the magnetic inhomogeneities in the solar corona and changes our views of studying the "explosive" and "quiet" phenomena on the solar surface.

Following the above ideas, a number of qualitative models for the energy release in solar flares have been developed (for reviews see Vlahos 1993; 1996 and papers appearing in van den Oord 1994). These models revealed the necessity to study and understand the global behaviour of the evolution of active regions. Two approaches can be used for this purpose:

(1). MHD simulations (e.g. Galsgaard & Nordlund 1996; Einaudi et al. 1996). According to these models, random shearing motions of the magnetic field lines at the photospheric boundary lead to the formation of a number of current sheets inside the active region, where magnetic reconnection occurs. The dynamical evolution of these current sheets drives the creation of supersonic or super-Alfvénic flows and the formation of smaller current sheets (see for e.g. Fig. 1 of Galsgaard & Nordlund 1996). The spatial and temporal distribution of the current concentrations is highly intermittent, suggesting the application of statistical theories for the solar activity.

(2). Cellular Automata simulations. Lu & Hamilton (1991) and Lu et al. (1993) were the first to apply cellular automata simulations to solar flares. They showed that the energy released inside the active region may well be a result of an internal self-organisation process. The continuous loading of the active region with new magnetic flux produces several magnetic discontinuities. Simple rules were applied for the redistribution of magnetic fields and the release of magnetic energy in these discontinuities. According to this scenario the solar flare is not a simple superposition of many smaller energy releases (e.g. microflares), since energy released locally, affects the entire structure of the flaring region by redistributing the magnetic field, creating new instabilities in an avalanche -like manner.

Several qualitative attempts to study the problem of particle acceleration in the framework of a concrete proposal for a fragmented energy release process have been made in the past (e.g. Holman 1985; Khan 1989; Haerendel 1994). Anastasiadis & Vlahos (1991, 1994) proposed a model for the acceleration of particles (electrons and ions) by an ensemble of shock waves. In this model the energy was assumed to be released by means of many localised, small - scale explosive phenomena which are the drivers of a number of shock fronts (small - scale, short - lived discontinuities; see simulations by Cargill et al. 1988, or Galsgaard & Nordlund 1996).

In this article we present a model for electron acceleration based on a random number of localised electric fields, which are closely associated with the energy released during solar flares. We are interested in the global characteristics of the energy release process and the acceleration of high energy electrons. The approach we are going to use for the energy release process is presented in § 2., while in § 3. the description of the acceleration model is given. In § 4. we present our results and finally we summarise our work and discuss some open problems which require further study.

2. Energy release process and the statistical flare

Solar flares are associated with active regions, where the magnetic fields are intense and highly concentrated. It is believed that active regions are highly inhomogeneous and complex dynamical systems, as the magnetic field emerging from the convection zone is subject to random shuffling from the photosphere. Due to these motions the magnetic field lines interact randomly and create a large number of current sheets in different sites. The sizes of those current sheets vary over a wide range.

In order to simulate the above physical picture, in a series of recent articles , Vlahos et.al. (1995), Georgoulis et al. (1995) and Georgoulis & Vlahos (1996), have developed a cellular automaton model, based on the concept of Self-Organized Criticality (Bak et al. 1987; Bak et al. 1988; Tang & Bak 1988). The main goal was to explore the origin of the frequency distribution of solar flares, the problem of coronal heating by nanoflares and the variability of the occurrence frequency of solar flares. In these articles "flares" are not considered as isolated localised explosions but rather as randomly appearing clusters of reconnection sites in a highly inhomogeneous topology. The active regions are continuously in a "flaring" state but the spatio-temporal evolution of the explosions changes randomly inside the active region. Vlahos et al. (1995) named this type of flaring activity "*statistical flare*" and used a cellular automaton model, in order to describe the energy release process.

The basic rules of the statistical flare model are:

1. A 3D cubic lattice represents an active region. To each grid point a random value of magnetic field is assigned (initial loading).

2. Randomly selected grid points gain increments δB, selected also randomly out of the following power-law probability distribution function (loading)

$$P(\delta B) \sim (\delta B)^{-5/3} \tag{1}$$

where $P(\delta B)$ is the probability of occurrence of a certain increment δB. Notice that this distribution is a power-law where the exponent is a multiple of 1/3. Such exponents occur frequently in phenomenological theories of turbulence due to Kolmogorov's second refined hypothesis (Kluiving 1995). Since the increments δB in eq. (1) are supposed to be the outcome of a highly turbulent process in the convection zone, the particular form of the exponent (i.e. that it is a multiple of 1/3) is loosely explained. It is clear however, that the connection between eq. (1) and the turbulent convection zone should be analyzed further (see Galsgaard 1996; Georgoulis & Vlahos 1997).

3. The continuous loading generates local discontinuities which excite currents ($\vec{\nabla} \times \vec{B} \propto \vec{J}$). If a certain critical threshold B_{cr} is exceeded, the topology is considered

unstable and is relaxed by redistributing magnetic field to the neighborhood and releasing energy (relaxation process).

Thus the ambient magnetic gradients, developed due to loading activity, give rise to local instabilities which are relaxed by releasing the excess magnetic energy through a large number of reconnection events. The amount of the released energy during the restructuring of the magnetic field at a single grid point i is given by the relation (Georgoulis et al. 1995; Georgoulis & Vlahos 1996)

$$\epsilon_{ri} \sim \left(B_i - \frac{6}{7} B_{cr} \right)^2 \tag{2}$$

where B_i is the local magnetic field at the i-grid point. This energy is treated as the magnetic energy release during magnetic reconnection.

By performing a number of iterations, we are able to obtain spatio-temporal information on the energy released (in arbitrary units). In Fig.1a we demonstrate the clustering of current sheets of all sizes for a small time interval. The biggest cluster is presented in Fig. 1b. The corresponding energy release time series are given in Fig. 1c and Fig. 1d respectively. Notice that the temporal evolution of the energy released for the cluster of Fig. 1b mimics very closely the well known behaviour of an isolated flare. The cluster undergoes many dramatic changes during a few time steps. It is obvious that a big cluster is surrounded by many smaller clusters which play an important role for the energy released from the active region during a "flare", and the subsequent acceleration of energetic particles.

In the next section, we use the evaluated energy - release time series (corresponding to the entire active region and not the one related to a single cluster) for the acceleration of test particles. We study the interaction of particles with a number of different clusters which surround the main event. The energy - release time series obeys a well-defined double power-law frequency distribution (see Fig.1 of Georgoulis & Vlahos 1996), it exhibits a scale-invariant behaviour and it encloses a self-similar nature (Vlahos et al. 1995; Georgoulis et al. 1995). Due to the self-similar structure of the time series, the acceleration will follow a similar behaviour even if we used the internal structure of an isolated cluster. It turned out that, due to self-similarity, our results are not affected by the size of the simulation box, provided that the box is not too small (not smaller than $50 \times 50 \times 50$).

3. Model for electron acceleration

We propose here a model for the acceleration of electrons in a dynamically evolving active region. Our acceleration model is based on the interaction of electrons with a number of Reconnecting Current Sheets (RCS). A basic ingredient for the estimation of the energy gained by the electrons is the strength of the electric field inside the RCS and their effective acceleration length along this field.

We assume that the magnitude of the electric field inside each RCS is comparable to the convective electric field, due to the ambient plasma flow and the local magnetic field. If we assume that the flow velocity of the plasma inside the flaring region is of the order of the Alfvén speed v_A, then the convective electric field is given by the relation :

$$E = | - \frac{\vec{v_A} \times \vec{B}}{c} | \approx \frac{B^2}{c(4\pi n m_i)^{1/2}} \tag{3}$$

where B is the magnetic field, c is the speed of light, n is the ambient plasma density and m_i is the proton mass. From the above relation we can find that the convective electric field in Volts cm^{-1} is

$$E(t) \approx 2.184 \; 10^3 \; B^2(t) \; n^{-1/2} \tag{4}$$

Litvinenko (1996) performed a similar calculation in order to find a relation between the magnetic field and the electric field in a RCS, using a self-consistent calculation for the collisionless case.

We consider now that the ambient plasma parameters inside our flaring region are: the density $n = 10^{10}$ cm^{-3}, the temperature $T = 10^7$ K, corresponding to an electron thermal velocity $V_e = 1.23 \times 10^9$ cm s^{-1} and a kinetic energy $E_T = 430$ eV. Using these typical values, we find that the Dreicer electric field is $E_D \approx 6 \times 10^{-5}$ Volts cm^{-1}.

The SOC model, which was described in the previous section, connects the magnetic field strength $B(t)$ at a given time t with the derived energy release time series $\epsilon_r(t)$, as $\epsilon_r(t) \sim B^2(t)$ (see eq. (2)). We thus can estimate the ratio of the convective electric field to the Dreicer field as

$$\frac{E(t)}{E_D} \approx 3.64\ 10^7\ n^{-1/2}\ \epsilon_r(t) \tag{5}$$

Using the above relation, we are able to construct a new time series ($E(t)$) which represents the electric fields at each RCS, associated with the energy release sites. This new time series exhibits the same characteristics as the energy release time series. Notice that for our model parameters ($n = 10^{10}$ cm^{-3} and $\epsilon_r(t) \geq 0$) the electric field inside the RCS is larger than the Dreicer field. This fact is consistent with a number of arguments presented by Litvinenko (1996). These highly localised and strong electric fields will excite plasma instabilities. The particle, interacting with the electric field and the plasma waves, will depart from the RCS with a final energy gain or loss, which for our purpose will be simulated by a "virtual" electric field (see below).

Each electron of the ambient Maxwellian distribution

$$f(v) = \frac{n}{(2\pi)^{1/2} V_e} \, exp\left(-\frac{v^2}{2V_e^2} \right) \tag{6}$$

with initial velocity in the range $2 \leq (v/V_e) \leq 5$, remains inside the flaring region for a different number of time steps (N_j), selected randomly from an interval (1, N_{max}). The parameter N_{max} is the maximum number of time steps that the injected electron distribution remains inside the acceleration volume. Thus, N_{max} is a measure of the maximum trapping time for the energetic particles. This trapping time corresponds to the environment of the local random electric fields and not to the possible large scale trapping of the electrons due to magnetic field convergence and/or their pitch angle scattering.

Each injected electron enters into the acceleration volume and interacts successively with $N_j \leq N_{max}$ randomly selected elements of the electric field time series. It is clear that the random selection of the N_j for each electron leads to a random selection of the number of RCS, which it will experience. Following this approach, electrons with the same initial kinetic energy E_k are neither entering into the acceleration volume at the same time and at the same position, nor are they escaping from the same place at the same instant.

We include the case of energy losses for the electrons, by assuming that their interaction with the associated electric field can be accomplished either in phase or out of phase. This selection is also random. Notice that these kinds of losses are energy independent and are due to the electric field at each RCS. We do not consider the effect of Coulomb collisions in our model, which are energy dependent losses, as we are initialising the tail of the Maxwellian distribution and thus the collision frequency is small.

We define an "acceleration coefficient" α which is selected randomly to vary between zero and one at each electron - RCS interaction. With this coefficient, we intend to simulate the fact that only a portion of the energy released goes to the particle acceleration process; the rest of this energy heats the ambient medium of the flaring region. Thus an electron interacts with a portion of the electric field ("virtual field") allocated at a RCS. The variation of the "acceleration coefficient" at a single electron - RCS interaction, represents also the fact that each electron travels along the RCS' electric field a different distance, and thus the effective acceleration length Δl is not constant. In this way the variability of Δl can also be assigned to the coefficient α.

Under the above assumptions, the kinetic energy change of a single electron in eV, due to the interaction with an electric field $E(t)$, at a given time t, is given by the relation:

$$\Delta E_k \ = \pm \ \alpha \ \ e \ E(t) \ \Delta l \tag{7}$$

where the plus (minus) sign corresponds to in (out of) phase interaction, e is the electron charge ($e = -1$) and $\Delta l = 10^3$ cm is the maximum effective length of the current sheet where the electric filed $E(t)$ is located.

The only free parameter for our model is the maximum number of time steps (N_{max}) that the electron distribution, moving inside the flaring region, will complete before escaping from the box and thus the maximum number of RCS it can experience. In the following section, we present our results concerning the final energy distribution of accelerated electrons by performing a parametric study with respect to our free parameter.

4. Results

According to our model, an electron performs a "free flight" between electric fields of variable strength, which exhibit a power-law frequency distribution. At each interaction, an electron can gain or loose energy in a "nonsystematic" way.

In order to justify this, the temporal evolution of the velocity (normalised to the thermal one) of a single electron is presented in Fig. 2a and the part of the electric field time series with which this particular particle interacts is shown in Fig. 2b. Note that the sign of the velocity in Fig. 2a corresponds to the direction of motion. A rescaled detail of Fig. 2a and Fig. 2b are given in Fig. 2c and Fig. 2d respectively, in order to illustrate the self-similar character of the acceleration scaling.

As the electric field exhibits large variations locally, the energy gain of a single particle undergoes dramatic changes locally as well. It is then difficult to define a mean acceleration rate along the path of the electron with small variations around this mean. We believe that this process is not a normal diffusion (i.e. normal random walks, Brownian motion). In highly non-linear systems, where the accelerators are localised and have irregular strength, new methods of analysis have been developed lately, using the Lévy - flight statistical approach (e.g. Shlesinger et al. 1995; Klafter et al. 1996). Developing diffusion equations for particles moving inside electric fields with fractal spatio-temporal characteristics is currently a very challenging problem.

We are interested in the electron population of the distribution given by eq. (6), with initial velocity range $2 \leq (v/V_e) \leq 5$. We normalise the injection distribution in such a way that $f(5V_e) = 1$ and follow numerically the kinetic energy change of 1000 electrons per injected velocity bin.

The final kinetic energy distributions of the accelerated electrons (those with $E_k > E_T = 430$ eV), for different realisations of the model, based on the interval $(1,\ N_{max})$ are given in Fig. 3. These are averaged distributions over 10 sample runs with the same parameters. Results for N_{max} = 500, 2000, 5000, 10000, 20000 and 40000, are presented in Fig. 3(a-f).

The kinetic energy distributions for the high energy electrons (with $(E_k/E_T) > 10^3$) initially exhibit a well - defined power-law behaviour for N_{max} = 500, 2000 and 5000. The power-law index has a value of -1.64 initially and as the maximum trapping time increases it becomes flatter, reaching -1.44 for N_{max}= 5000. The relative error in the calculation of the above indices is less than 3 %. Further increase of the trapping time affects the shape of the final distribution, which begins to diverge from the power-law behaviour and an exponential tail is starting to develop. For N_{max}=10000 the shape of the distribution deviates significantly from either power- or exponential-law behaviour. Finally, further increase of N_{max} leads to the emergence of an exponential-law. The electron distribution in this stage becomes a "thermal" one with an "effective temperature" of the order $10^{5-6} \times E_T$. The energetic electrons in this case, remain longer time intervals inside the acceleration volume and try to reach an equilibrium with the mean energy associated with the random electric fields.

We note that if we limit the energy range of interest to the interval $10^3 \leq (E_k/E_T) < 10^5$, then the energy distributions for the case of $N_{max} \geq 10000$ can also be fitted with power-laws. The power-law index has a value of -0.91 initially (for N_{max}=10000) and again as the maximum trapping time increases it bocomes flatter, reaching -0.48 for $N_{max} = 40000$. It is clear that for the case of $N_{max} \geq 10000$ the shape of the the energy distribution depends strongly upon the considered energy range.

The maximum energy reached by the electrons is of the order of several MeV. This value is connected with the selection of the maximum effective acceleration length Δl, which on the other hand does not influence the overall behaviour of the distributions. There is a close connection between the structure and the nature of the energy release ($\epsilon_r(t)$) or the electric field ($E(t)$) time series and the resulting energy distributions. This is obvious as, if we assume a random type time series (white noise) then our distributions do not exhibit any kind of structure (power or exponential type). In Fig. 4 the final kinetic energy distribution, averaged over 10 sample runs, for the case of random time series and for $N_{max} = 500$ is presented.

5. Summary and Discussion

The statistical flare model suggests a new way for the study of eruptive phenomena in the solar atmosphere. It incorporates two basic elements: (1) the fragmentation of the energy release process and (2) the global aspects of the energy flow from the convection zone to the outer parts of the stellar atmosphere. Considering active regions as driven dissipative systems with many interacting elements changes our strategy for the study of acceleration and transport of high energy particles. Such an approach for the acceleration of energetic particle was used in the past (see Anastasiadis & Vlahos 1991; 1994). The acceleration of particles was due to the presence of a number of shock waves distributed randomly inside an active region, with no consideration of the spatio -temporal evolution of the energy release process. In the present work the concept of Self - Organised Criticality (SOC) is used in order to overcome the lack of understanding the way and the amount of the energy released during solar flare events.

We propose, in this study, that electrons are accelerated by randomly placed DC electric fields inside an evolving active region. We use the statistical flare model to simulate the spatio-temporal evolution of the energy released in active regions and subsequently we trace electrons inside this highly structured environment.

The electric fields form inside the energy release region a spatio - temporal structure with fractal geometry. The injected electrons move inside this environment sensing either an electric field with varying strength locally or flying long distances without any interaction. This movement departs radically from the well known random walk with uniformly placed electric fields or with a very strong local electric fields. The free parameter in this study is the maximum allowed number of interactions of each electron with the electric fields (N_{max}). Assuming that it takes a certain mean time for each electron - electric field interaction, then by increasing N_{max} we increase, on the average, the time that the injected distribution remains inside the energy release region (the trapping time). It is clear that N_{max} is only a rough measure of the trapping time.

Our main conclusions are:

1. There is a close connection between the turbulent driver inside the convection zone (power-law in the loading process), the frequency distribution of flares (power-law behaviour) and energy distribution of accelerated particles.

2. The well - known debate between several types of solar flares, i.e. "nonthermal" with a power-law behaviour for the electrons above tens of keV, "thermal" with an exponential distribution or "hybrid" seem to have a clear relation with the maximum trapping time of the energetic particles inside the active region. For short trapping time the distribution of energetic electrons exhibits a well - defined power-law behaviour. As the trapping time increases, the power-law distribution becomes flatter until an exponential tail is starting to develop (transition region). Further increase of the trapping time leads to exponential distribution for the energetic electrons.

It is obvious that the differences between our estimated energy distributions and the ones derived by the analysis of the observational data are due to the fact that no transport of the electrons is taken into account in this article. Our numerically calculated energy distribution can be considered as an injection/ source distribution in a diffusion-transport equation, which studies the global evolution of the accelerated particles.

The scope of our study was to answer the following question: Can the SOC derived fragmentation of the energy release reproduce the observed inferred energy distributions? We have shown that this is possible. Many questions still remain open and will be addressed in a future study: (a) A more detail connection of our study with the time evolution of the energy release process (b) Including transport of energetic particles in our model.

We believe that these concepts are just in the beginning of their development. A new and highly promising field of research, which differs from the localised acceleration approach, is now emerging. The study of the gross characteristics of the global system as well as the detailed study of the local characteristics are necessary for a complete understanding of a complex problem such as the dynamics of an active region.

We are grateful to Prof. J. Kuijpers, Dr. G. Trottet, Dr. R. Kluiving and Dr. H. Isliker for their useful comments and discussions on particle acceleration models in solar flares. This work was supported by the Greek General Secretary of Science and Technology PENED programme.

REFERENCES

Anastasiadis, A., & Vlahos, L. 1991, A&A, 245, 271

Anastasiadis, A., & Vlahos, L. 1994, ApJ, 428, 819

Aschwanden, M. J, Montello, M., Dennis, B. R., & Benz, A. O. 1995, ApJ, 440, 394

Bak, P., Tang, C., & Wiesenfeld, K. 1987, Phys. Rev. Lett., 59, 381

Bak, P., Tang, C., & Wiesenfeld, K. 1988, Phys. Rev. A., 38, 364

Benz, A. O. 1985, Sol. Phys., 96, 357

Benz, A. O., & Aschwanden, M. J. 1992, in Lecture Notes in Physics, Vol. 399, ed. Z. Svestka et al., (Berlin:Springer-Verlag), 106

Cargill, P. J., Goodrich, C. C., & Vlahos, L. 1988, A&A, 189, 254

de Jager, C. 1986, Space Sci. Rev., 44, 43

Dennis, B. R. 1985, Sol. Phys., 100, 465

Dennis, B. R. 1988, Sol. Phys., 118, 49

Einaudi, G., Velli, M., Politano, H., & Pouquet, A. 1996, ApJ , 457, L13

Forman, M. A., Ramaty, R., & Zweibel, E. G. 1986, in Physics of the Sun Vol II, ed. P. A. Sturrock et al., (Dordrecht : Reidel), 249

Galsgaard, K. 1996, A&A, in press

Galsgaard, K., & Nordlund, A. 1996, J. Geophys. Res., 101, 13445

Georgoulis, M., & Vlahos, L. 1996, ApJ, 469, L135

Georgoulis, M., & Vlahos, L. 1997, ApJ, to be submitted

Georgoulis, M., Kluiving, R., & Vlahos L. 1995, Physica A, 218, 191

Haerendel, G. 1994, ApJS, 90, 765

Hamilton, R. J., & Petrosian, V. 1992, ApJ, 398, 350

Heyvaerts, J. 1981, in Solar Flare Magnetohydrodynamics, ed. E. Priest, (New York : Gordon and Breach), 429

Holman, G. D. 1985, ApJ, 293, 584

Khan, J. A. 1989, Proc. Astron. Soc. Australia, 8, 29

Klafter, J., Shlesinger, M. F., & Zumofen, G. 1996, Phys. Today, 49, No.2, 33

Kluiving, R. 1995, in Chance and Uncertainty, ed. H. W. Capel et al., (Amsterdam:Amsterdam University Press), 61

Kuijpers, J., van der Post, P., & Slottje, C. 1981, A&A, 103, 331

Litvinenko, Y. E. 1996, ApJ, 462, 997

Lu, E. T., & Hamilton, R. J. 1991, ApJ, 380, L89

Lu, E. T., Hamilton, R. J., McTierman, J. M., & Bromund, K. R. 1993, ApJ, 412, 841

Melrose, D. 1990, Australian J. Phys., 43, 703

Miller, J. A., & Ramaty, R. 1989, ApJ, 344, 973

Parker, E. N. 1989, Sol. Phys., 121, 271

Parker, E. N. 1988, ApJ, 330, 474

Pick, M., Klein, K. L., & Trottet, G. 1990, ApJS , 73, 235

Priest, E. R. 1992, in Lecture Notes in Physics, Vol. 399, ed. Z. Svestka et al., (Berlin:Springer-Verlag), 15

Ramaty, R., & Murphy, R. J. 1987, Space Sci. Rev., 45, 213

Ryan, J. M., & Lee, M. A. 1991, ApJ, 368, 316

Scholer, M. 1988, in Activity in Cool Stars Envelopes, ed. O. Havnes et al., (Dordrecht : Reidel), 195

Shlesinger, M. F., Zaslavsky, G., & Frisch, U. (ed.) 1995, Levy Flights and Related Topics in Physics, (Berlin : Springer-Verlag)

Tang, C., & Bak, P. 1988, Phys. Rev. Lett., 60, 2347

Trottet, G. 1994, Space Sci. Rev., 68, 149

van den Oord, G. H. J. (ed.) 1994, Fragmented Enegry Release in Sun and Stars, (Dordrecht : Kluwer)

Vilmer, N. 1993, Adv. Space Res., 13, No9, 221

Vilmer, N. 1987, Sol. Phys., 111, 207

Vlahos, L. 1996, in Radio Emission from the Stars and the Sun, eds. A. R. Taylor & J. M. Paredes, ASP Conference Series, 93, 355

Vlahos, L. 1993, Adv. Space Res., 13, No9, 161

Vlahos, L. 1989, Sol. Phys., 121, 431

Vlahos, L., Georgoulis, M., Kluiving, R., & Paschos, P. 1995, A&A, 299, 897

Vlahos, L., et al., 1989, in Energetic Phenomena on the Sun, ed. M. R. Kundu et al., (Dordrecht:Kluwer), 127

This manuscript was prepared with the AAS LaTeX macros v4.0.

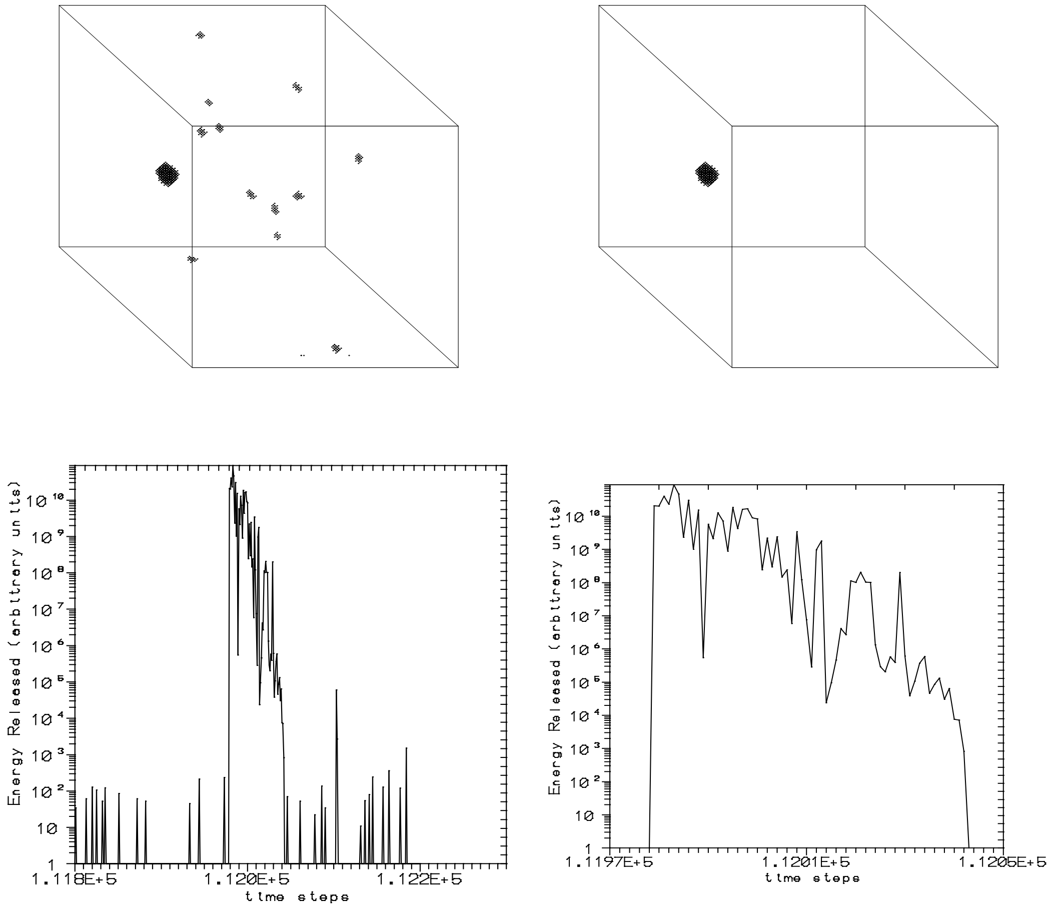

Fig. 1.— **a** Spatial configuration of the thee-dimensional simulation box, illustrating the clustering of unstable grid points. **b** The biggest cluster formed. **c** The energy release time series corresponding to the configuration **a**. **d** The profile of the energy release time series for the cluster presented in **b**

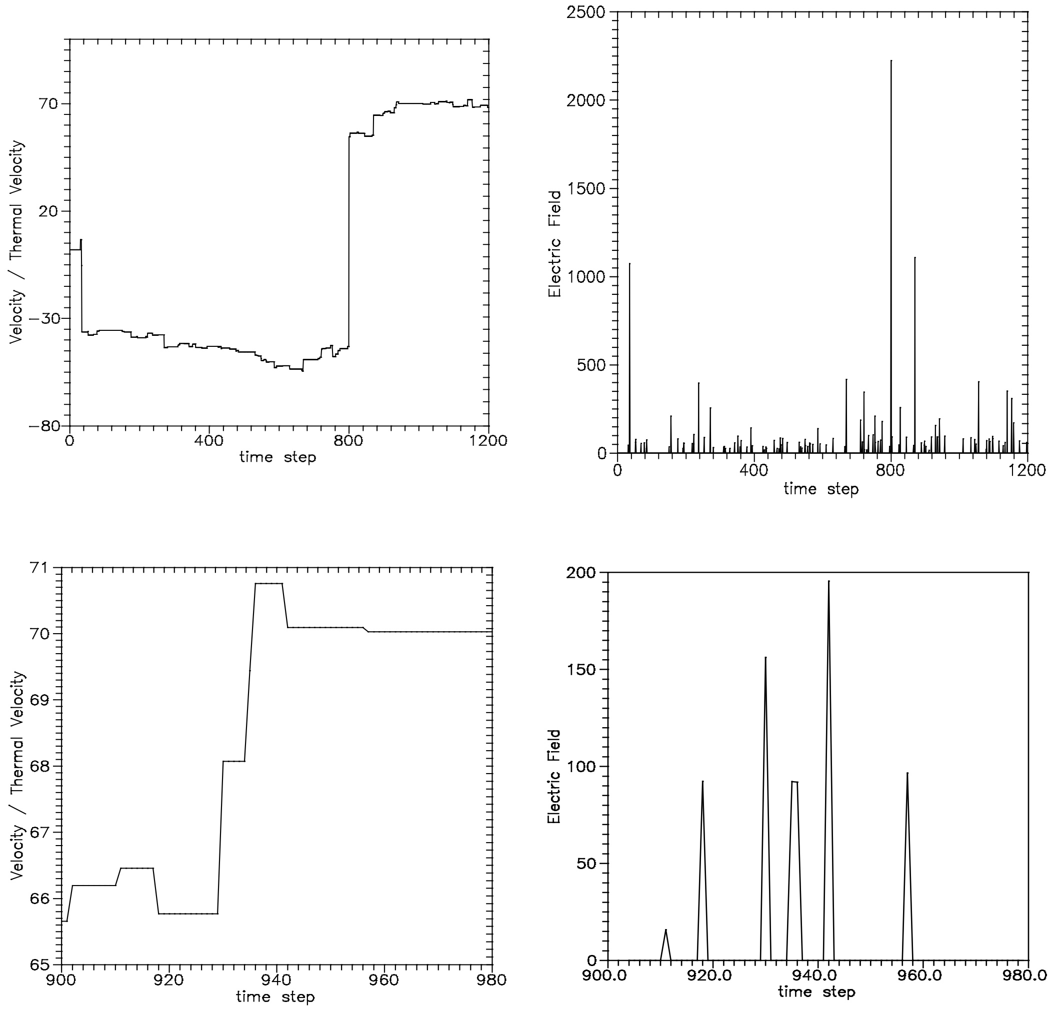

Fig. 2.— **a** The temporal evolution of the velocity (normalised to the thermal one) of a single electron. The sign of the velocity corresponds to the direction of motion. **b** The electric field time series which the electron experiences. **c** A rescaled detail of **a**. **d** A rescaled detail of **b**

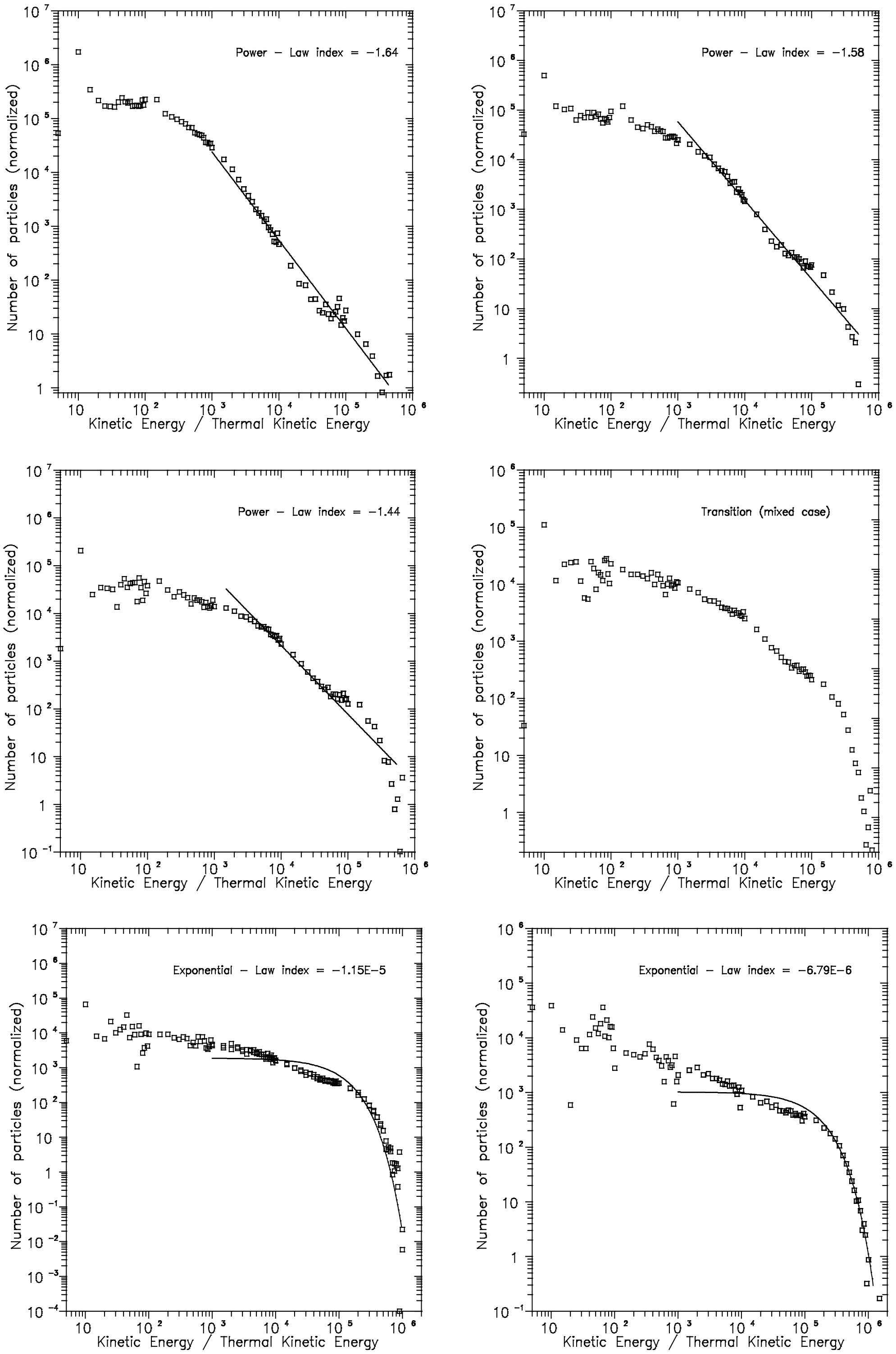

Fig. 3.— **a-f.** The numerically evaluated kinetic energy distribution of accelerated electrons (average of 10 sample runs) as a function of the maximum trapping time. **a** $N_{max} = 500$, **b** $N_{max} = 2000$, **c** $N_{max} = 5000$, **d** $N_{max} = 10000$, **e** $N_{max} = 20000$ and **f** $N_{max} = 40000$

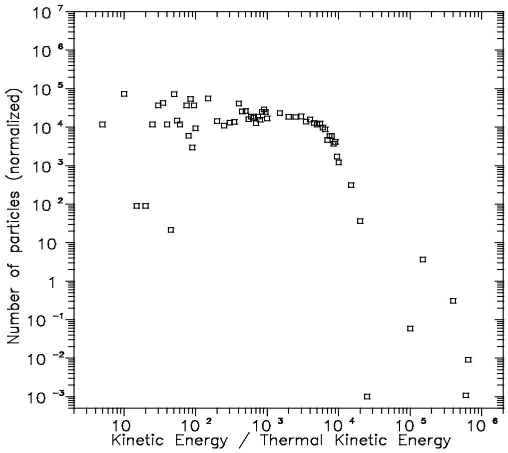

Fig. 4.— Same as Fig. 3, but for random type time series and $N_{max} = 500$