Measuring the Neutron-Star EOS with Gravitational Waves from Binary Inspiral

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Measuring the Neutron-Star EOS with Gravitational Waves from Binary Inspiral

I. Imprint of the EOS on the inspiral waveform
   A. EOS and deformability
   B. Double-neutron-star (DNS) inspiral
   C. BH-NS inspiral

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   A. EOS parametrization
   B. Measure only λ

III. With what accuracy?

IV. Comments on GW observations
   A. Model independence
   B. Waveform accuracy; NS spin

Early papers on EOS from inspiral waveforms:

Kochanek ’92

Lai, Rasio, Shapiro ’94

Vallisneri ’00

Faber, Grandclement, Rasio, Taniguchi ’02

Mora, Will ’04

Berti, Iyer, Will ’08 Ferrari, Gualtieri, Pannarale ‘10

“We find that $f_{\text{tidal disruption}}$ depends strongly on the NS radius R and estimate that LIGO-II (ca. 2006–2008) might measure R to 15% precision at 140 Mpc”

Vallisneri ’00

NS-NS:

Read, Markakis, Shibata, Uryu, Creighton, JF ’09

F. Pannarale, L. Rezzolla, F. Ohme, and J. S. Read, ’11

Read, Baiotti, Giacomazzo, Rezzolla, Shibata, Brady, JF ’13

Baiotti, Damour, Giacomazzo, Nagar, Rezzolla

Damour, Nagar, Vilain ’12

Bauswein, Janka, Hebeler, Schwenk ’12 (post-merger)

Bernuzzi, Nagar, Thierfelder, Bruegmann ’12

Del Pozzo, Li, Agathos, Van Den Broeck, Vitale ’13

+ Meidam, Tompitak, Veitch ’15

Wade, Creighton,Ochsner, Lackey,Farr,Littenberg,Raymond ‘14

Lackey, Wade ’14

Early inspiral

Flanagan, Hinderer ’08; Hinderer ’08

Hinderer, Lackey, Lang, Read ’10

Postnikov, Prakash, Lattimer ’10

Vines, Hinderer, Flanagan ‘11

BH-NS:

Vallisneri ’00

Shibata, Koutarou, Yamamoto, Taniguchi ’09

Kyutoku, Shibata, Taniguchi ’10

Pannarale, Rezzolla, Ohme, Read ’11

Ferrari, Gualtieri, Pannarale ’09, ‘10

Duez, Foucart, Kidder, Ott, Teukolsky ’10

Foucart, Deaton, Duez, Kidder, MacDonald, Ott,
Pfeiffer, Scheel, Szilagyi,Teukolsky ’13

Lackey, Kyutoku, Shibata, Brady, JF ’13, ’14

A. EOS and deformability

Neutron stars are cold:

kT << Fermi energy per nucleon

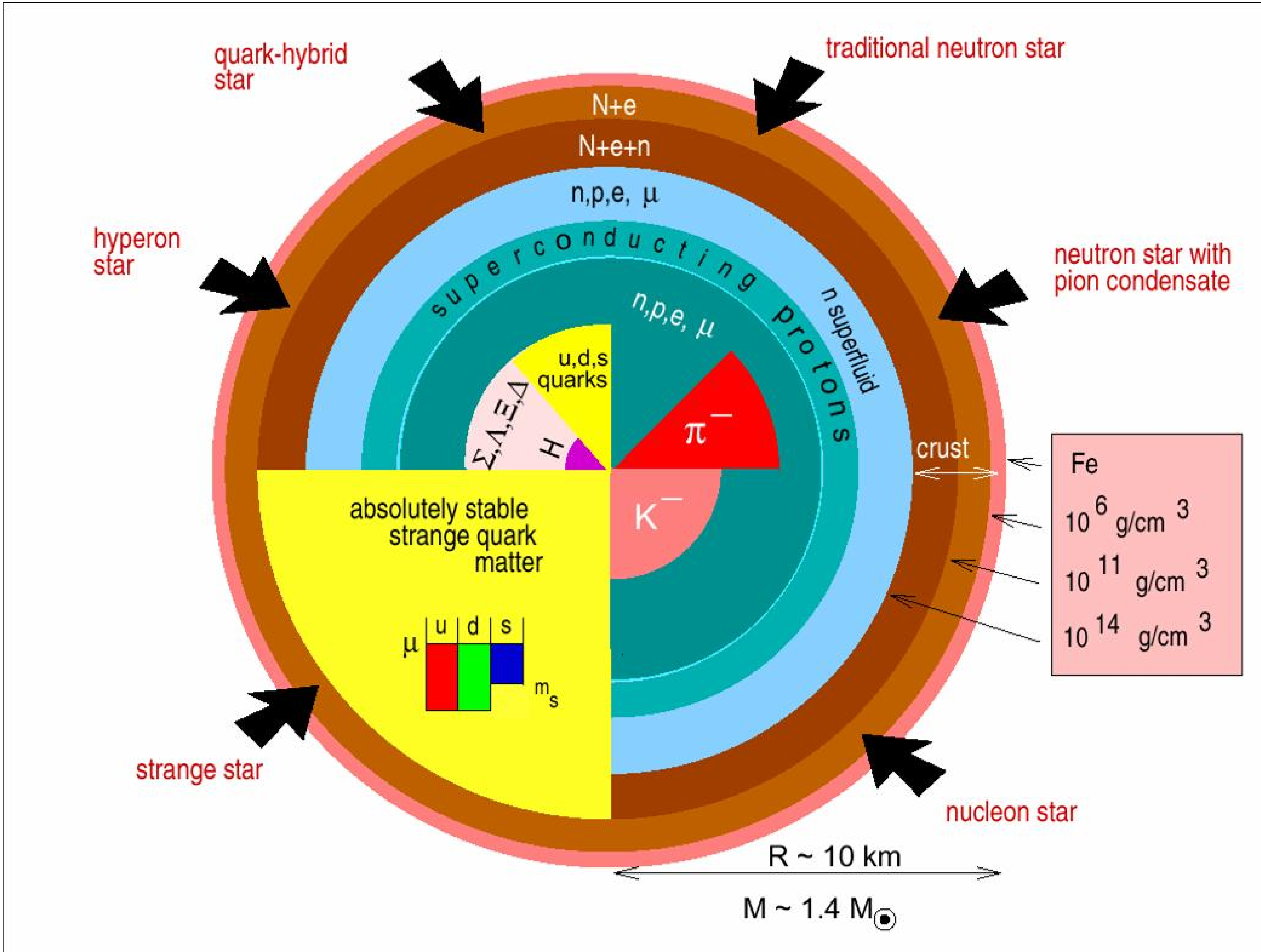
The EOS is essentially the zero-temperature EOS, depending on only one parameter:

$$p = p(\rho)$$
$$\varepsilon=\varepsilon(\rho)$$

ρ = rest mass density

ε = energy density

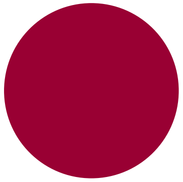
p = pressure



quark-hybrid star
traditional neutron star
hyperon star
neutron star with pion condensate
strange star
nucleon star
N+e
N+e+n
n,p,e, μ
superconducting protons
n superfluid
n,p,e, μ
u,d,s quarks
Σ,Λ,Ξ,Δ
H
π⁻
K⁻
absolutely stable strange quark matter
μ
u
d
s
m_s
crust
Fe
10^6 g/cm^3
10^11 g/cm^3
10^14 g/cm^3
R ~ 10 km
M ~ 1.4 M_⊙

If the EOS of cold matter above nuclear density is stiff the radius of a $1.4\ M_{\odot}$ neutron star will be large

Soft EOS: p small for $\rho \sim 2\rho_{\text{nuclear}}$ implies
star more centrally condensed,
R small



If the EOS of cold matter above nuclear density is stiff
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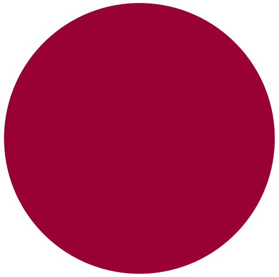
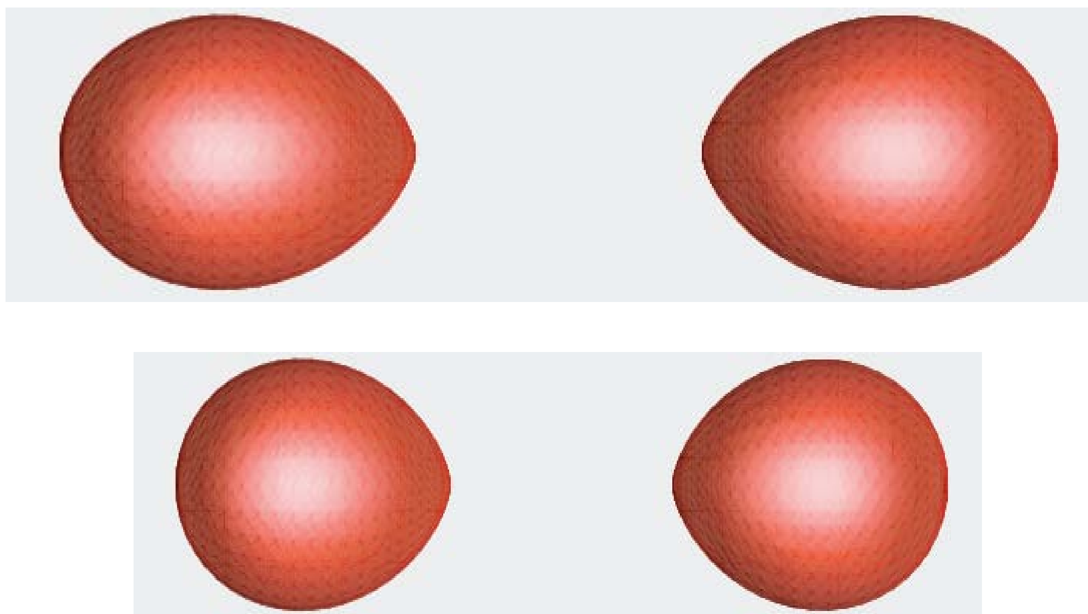


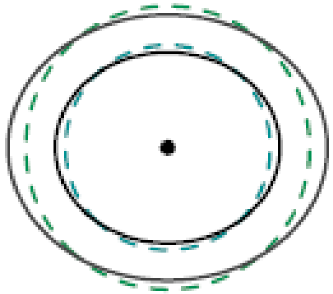
Imprint of EOS on inspiral waveform

In a binary system, the tides raised on each star depend on the deformability of that star: Because the tides are larger for large radii



a stiff NS EOS will yield higher tides.

Formally, define deformability λ

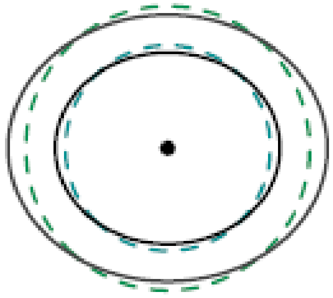


For an imposed asymptotic quadrupole field E_{ij}, a star acquires a quadrupole moment Q_{ij}

$$E_{ij} = \lambda Q_{ij}$$

The deformability λ is larger for larger radii and hence for stiffer EOSs.

Deformability, λ



In fact, λ is roughly proportional to R^5

$$\lambda = \frac{2}{3} k_2 R^5$$

and measuring λ is roughly equivalent to measuring R.

B. DNS inspiral: Imprint of EOS on waveform

As the orbit shrinks,

orbital energy → gravitational waves
orbital energy → stellar deformation

With the added loss of energy to deformation, the orbit of a star with large λ shrinks faster: Frequency f increases more quickly.

And tidal disruption ends the inspiral sooner – the cutoff frequency is lower.

$$E = \text{energy of system}$$

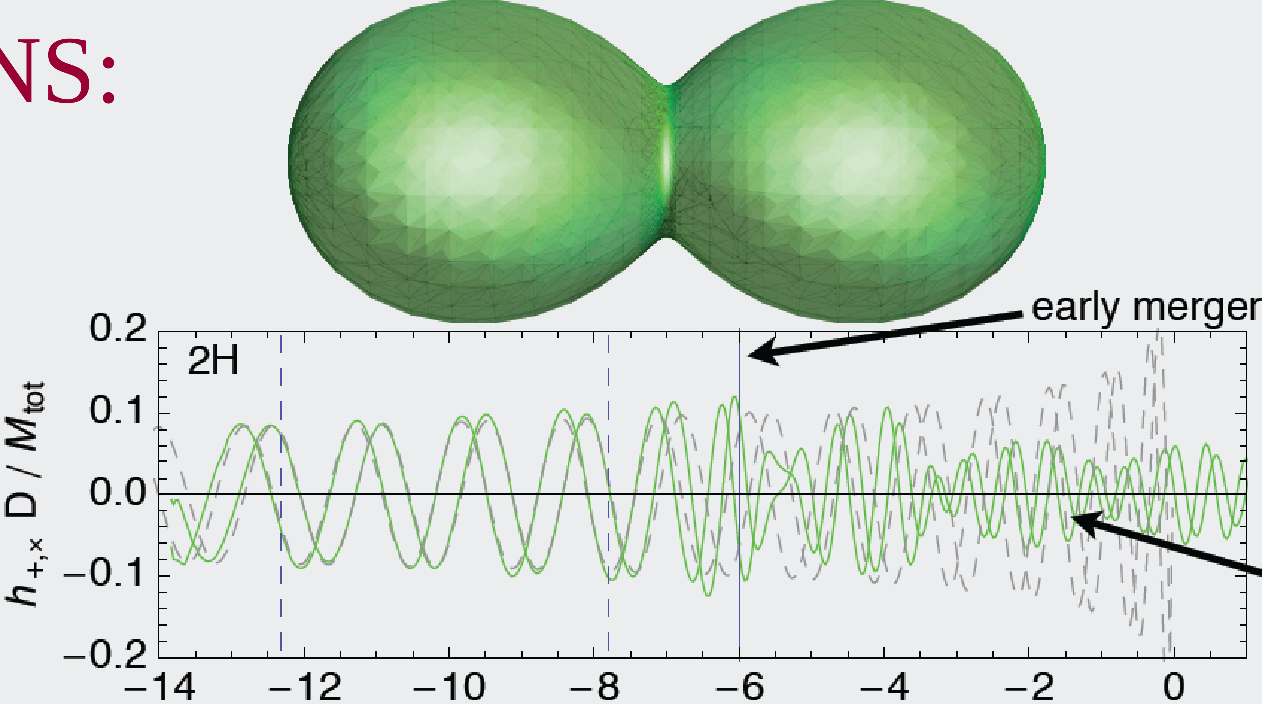
$$= -E_{\text{point-particle}}\left(1 - \frac{1}{2}Q_{ij}^{A}E_{ij}^{B} - \frac{1}{2}Q_{ij}^{B}E_{ij}^{A}\right)$$

$$= -E_{\text{point-particle}}\left(1 - \lambda Q_{ij}Q_{ij}\right),$$

for equal-mass neutron stars.

DNS:

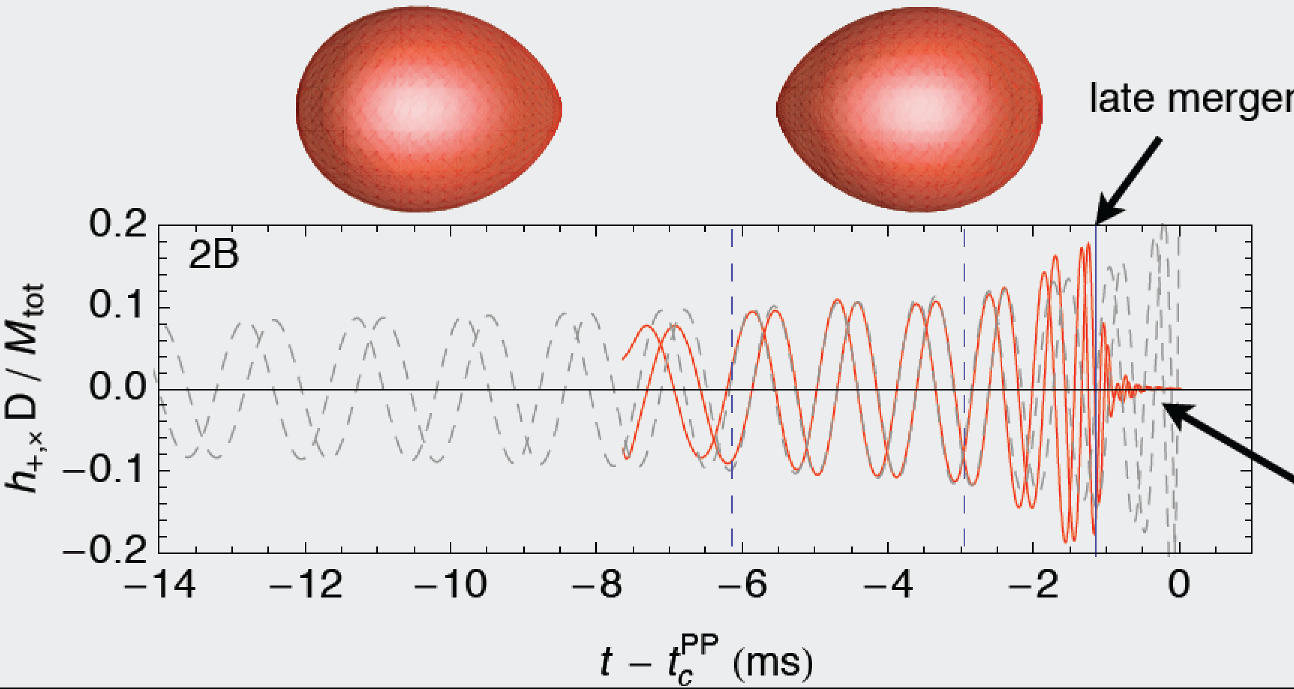
Stiff EOS



Post-merger governed by hot EOS

Post merger oscillations of hypermassive NS

Soft EOS



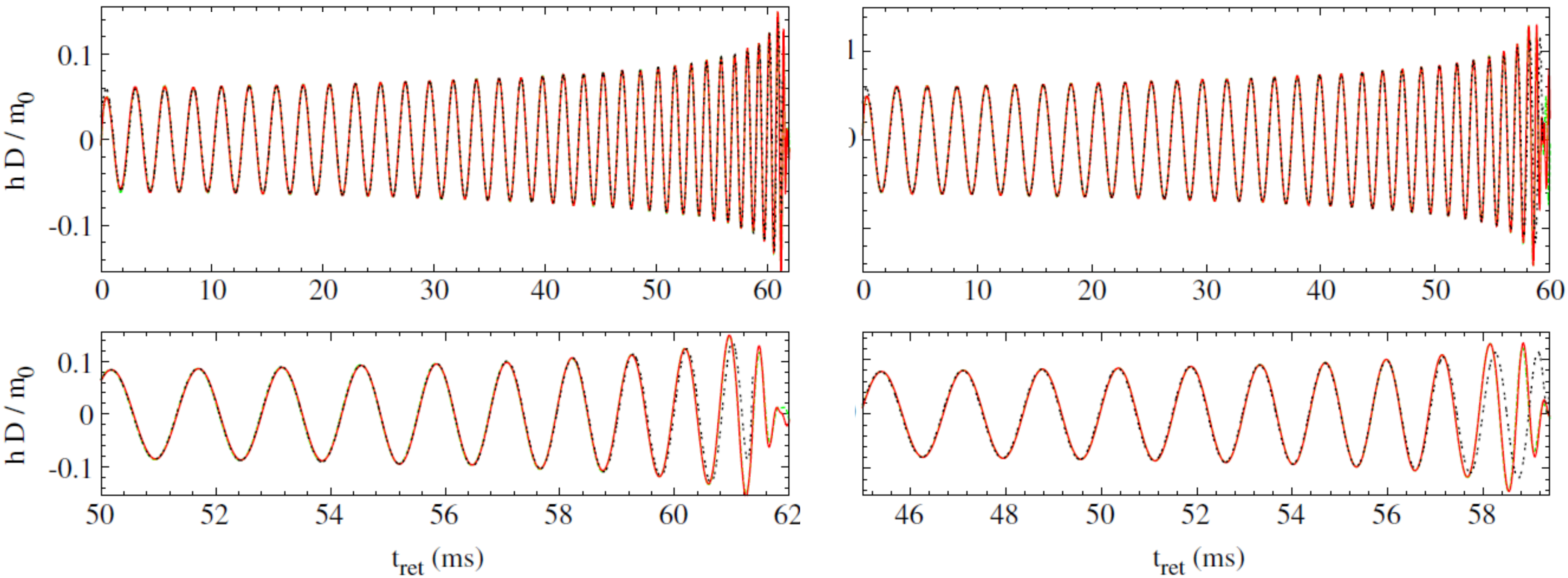
Higher mass gives:

Prompt collapse to BH and quasinormal ringdown

DNS

Equal mass ($1.35M_\odot$) from f = 370 Hz

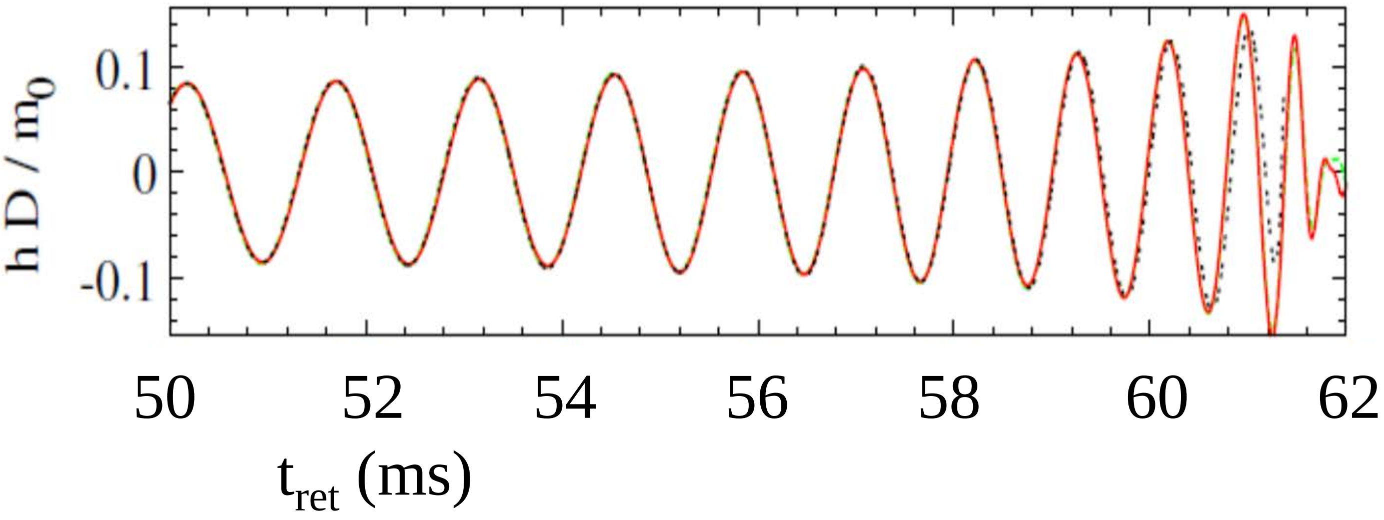
APR4: R = 11.1 km H4: R = 13.6 km



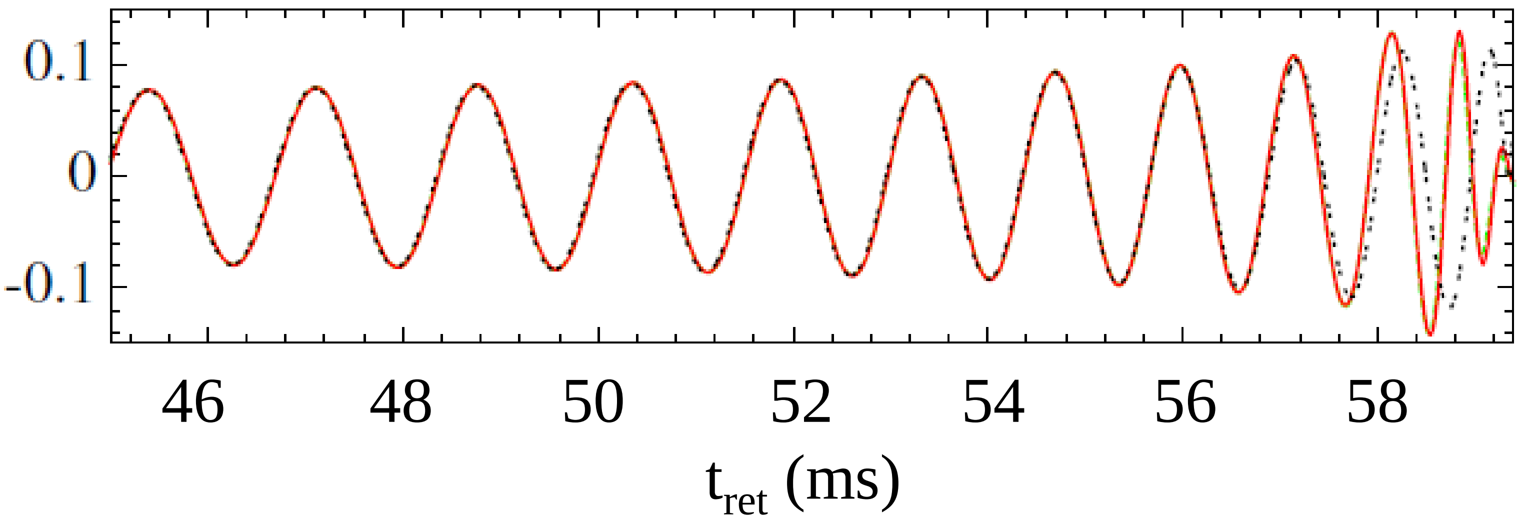
(Hotokezaka, Kyutoku, Okawa, Shibata, '15)

APR4 (R = 11.1 km)

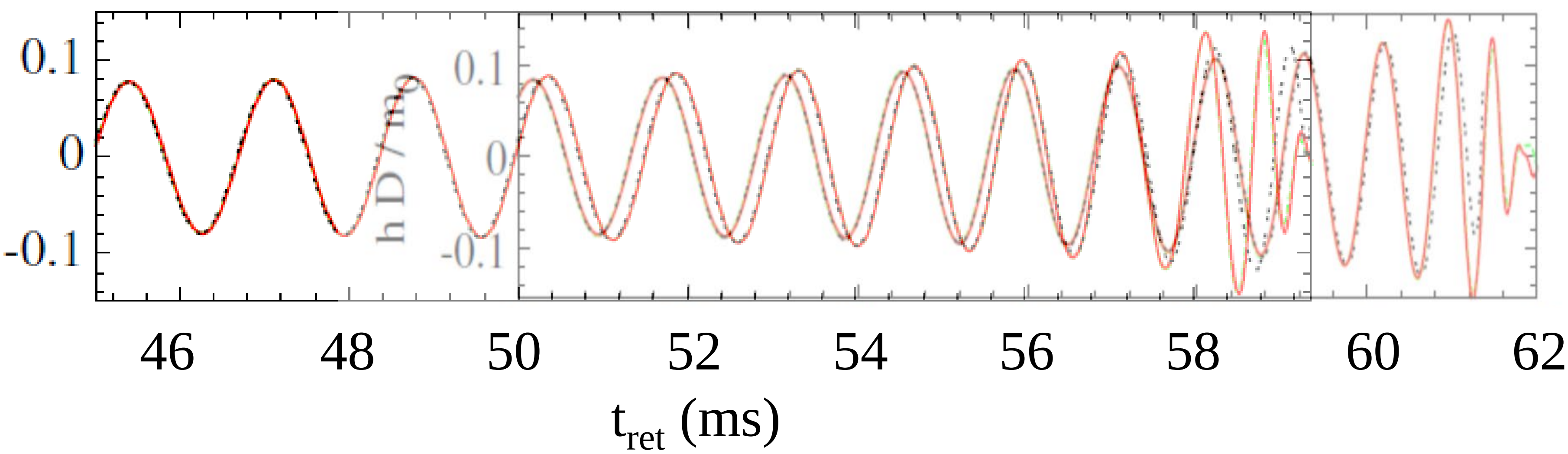
Last 5 orbits



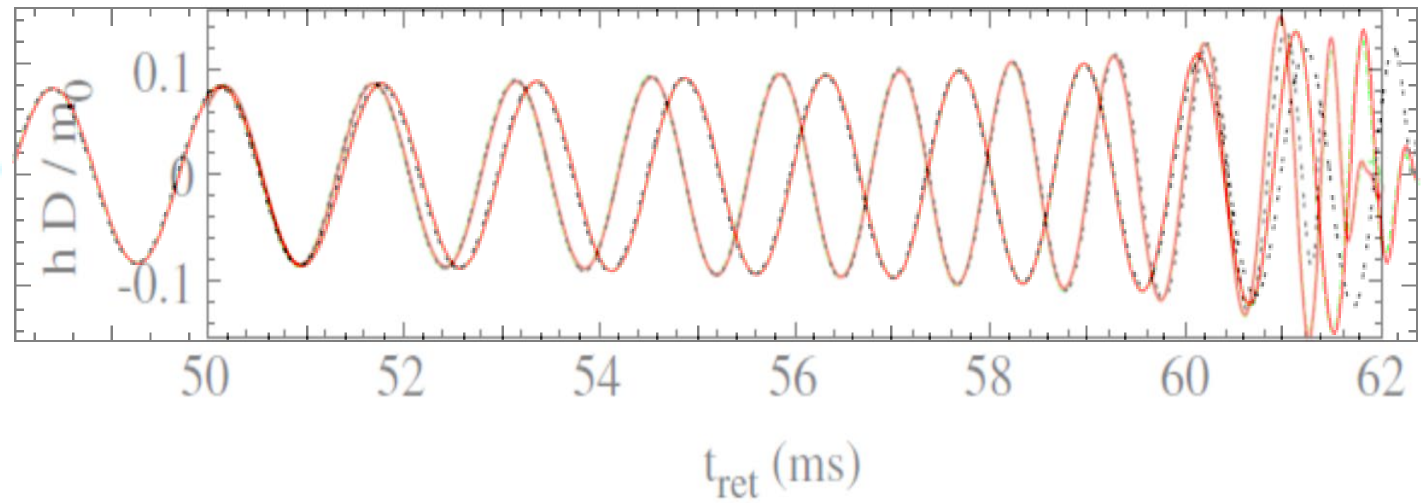
H4 (R = 13.6 km)
Last 5 orbits



Phase aligned at t=0 (start of run), showing earlier coalescence for stiffer EOS



Phase aligned at left of figure shows frequency difference in final orbits



C. Black-hole –neutron-star inspiral

Again obtain surface in EOS space by measuring departure of NS-BH waveform from point-particle inspiral.

For black hole, with circumferential radius used to define the $1/r^3$ part of metric,

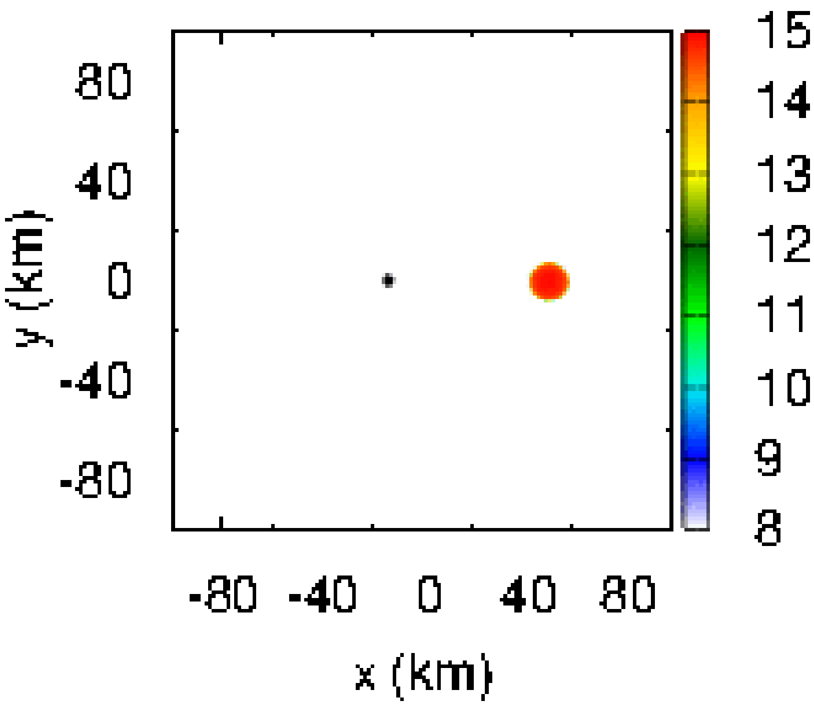
$\lambda=0$,

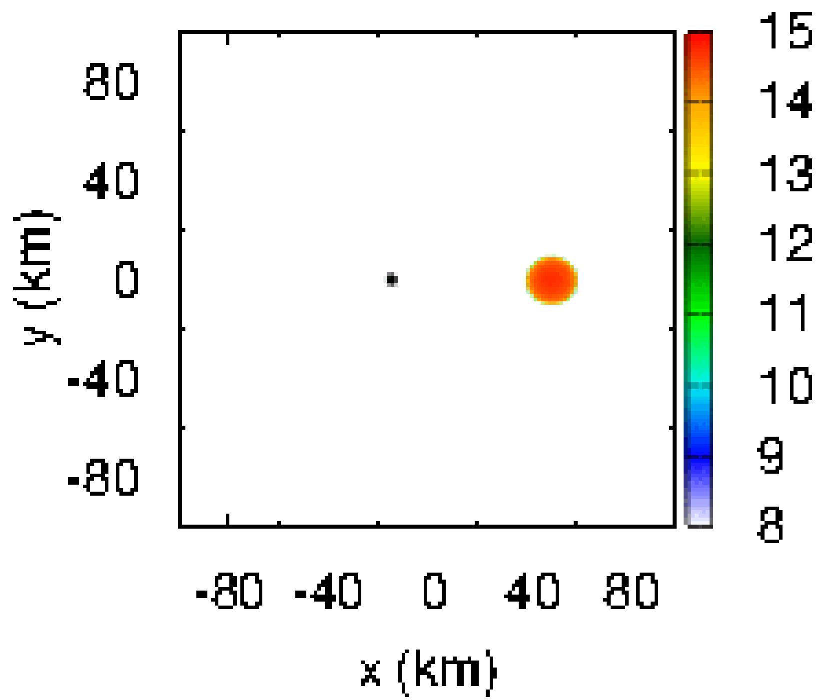
- related to the no-hair theorem forbidding asymptotically flat stationary perturbations with nonzero quadrupole moment. (Poisson)

BH-NS simulation

$M_{NS} = 1.35 M_{\odot}$ $M_{BH}/M_{NS} = 3$

APR 4 EOS: 11.1 km H4 EOS: 13.6 km

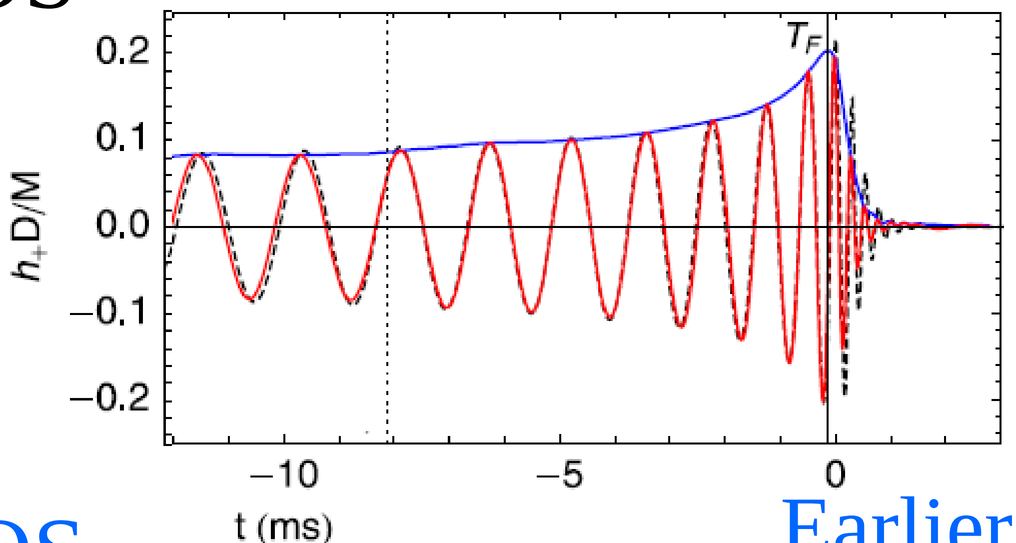




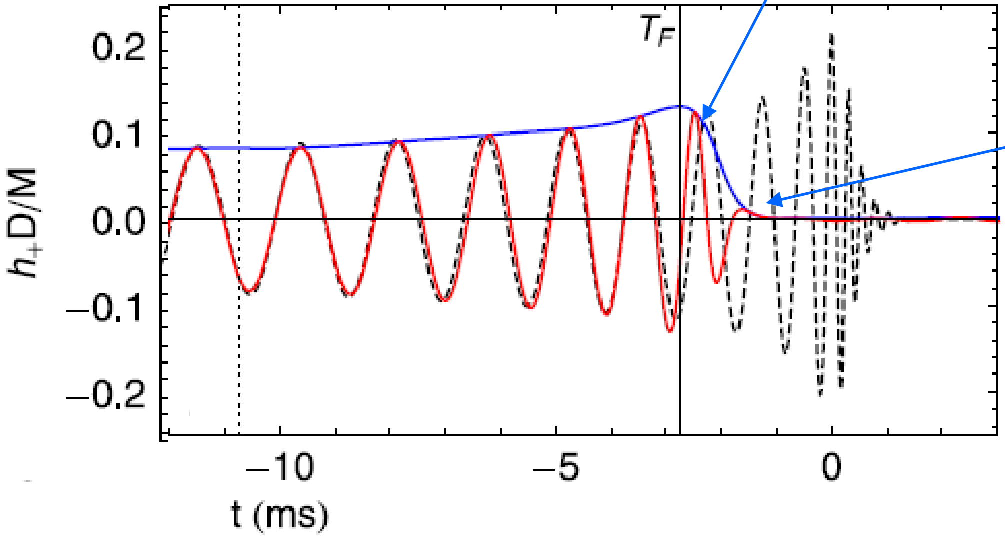
movies by Koutarou Kyutoku

Imprint of EOS on waveform: BH-NS

Softer EOS



Stiffer EOS



Earlier merger

Ringdown suppressed

BH-NS and NS-NS comparison

Long-term phase difference important for both, but cutoff frequency as important as cumulative phase difference for BH-NS.

Smaller tides in BH-NS and fewer expected events for a given mass ratio mean less likely to gain EOS information, despite lower frequency of final orbits.

What NS property can be measured?

Systematizing Constraints on EOS

To answer this and to systematize the constraints on the EOS, one parameterizes the space of possible EOSs, of functions $p(\rho)$.

Noting that any function can be approximated by a piecewise linear function, we can approximate

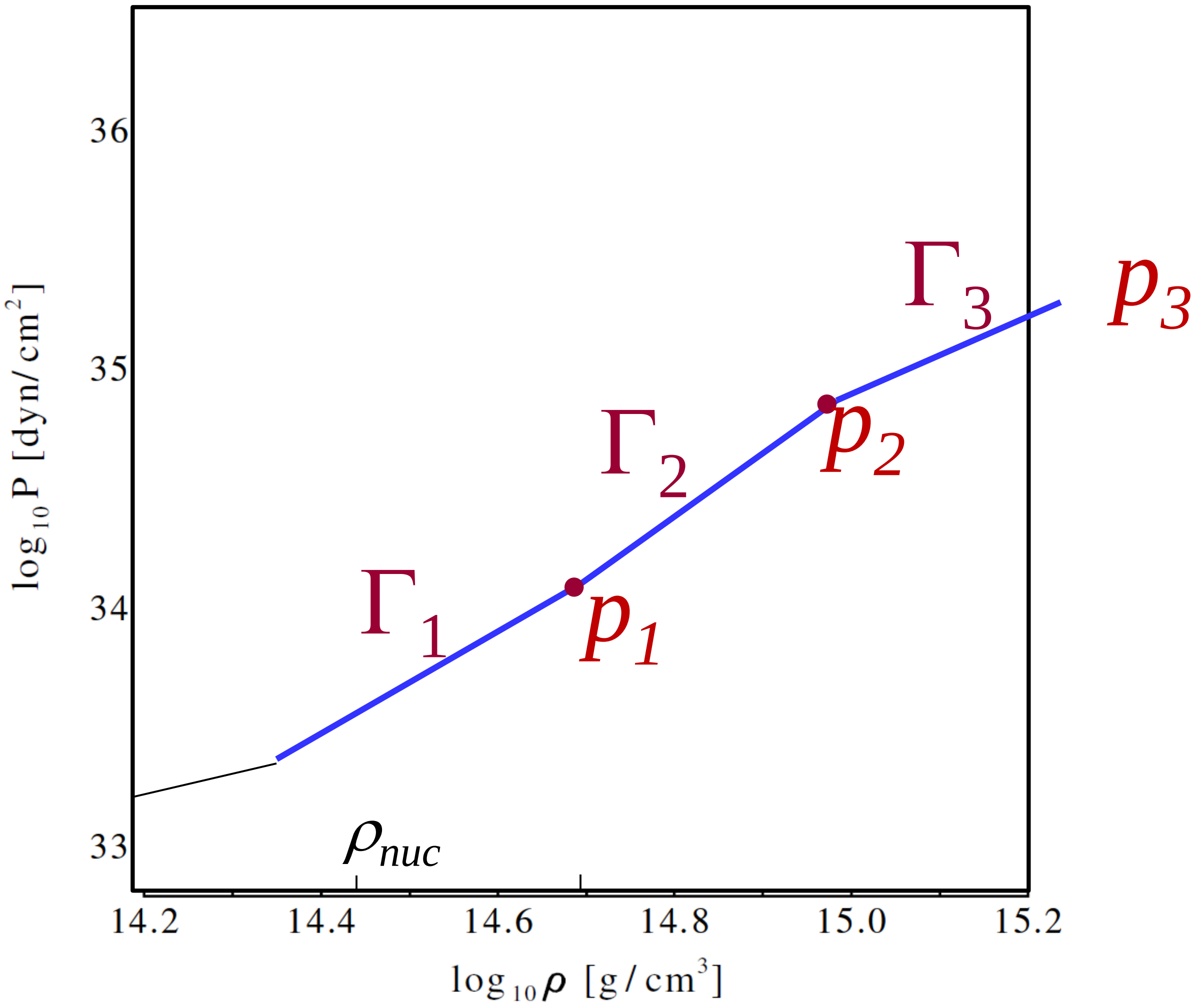
$$\log p\,(\log \rho)$$

in this way.

These are piecewise polytropes

Parameters are then pressures at specified densities:

p_0, p_1, p_2, p_3



or

first pressure and 3 slopes: $p_1, \Gamma_1, \Gamma_2, \Gamma_3$

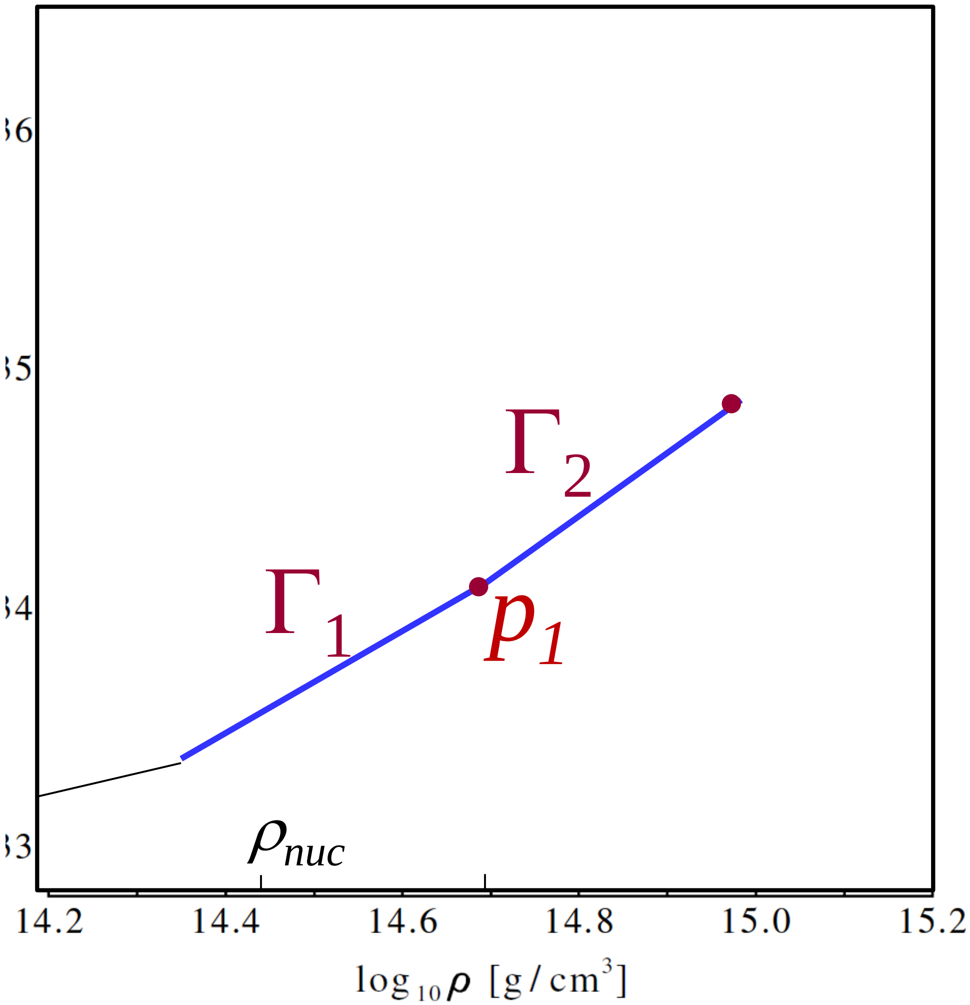
For observed DNS systems,

$$m < 1.6 M_\odot$$

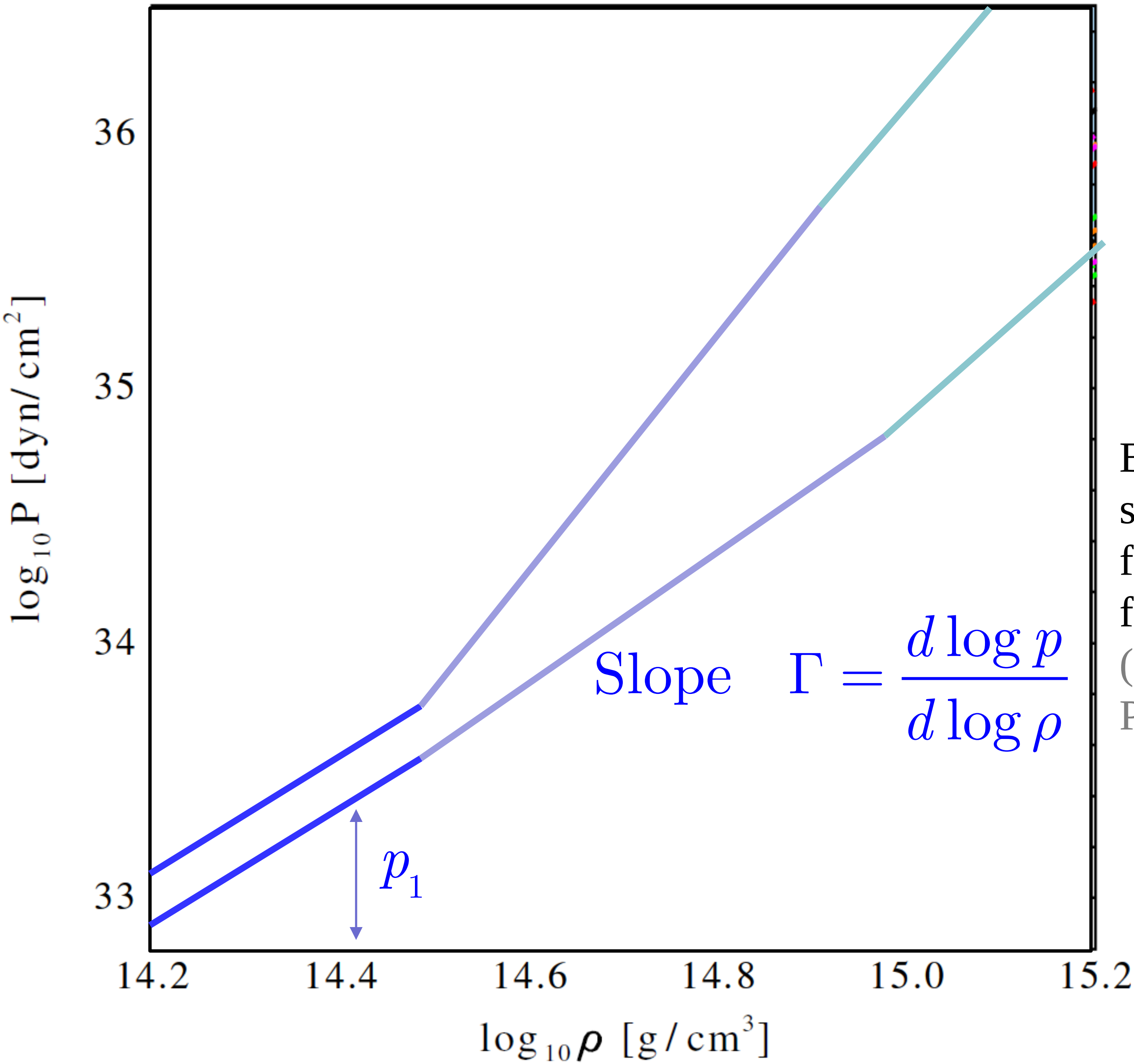
EOSs that allow the largest observed neutron star ($2M_\odot$) have

$$\rho < 10^{15} \mathrm{g/cm^3}$$

when $M < 1.6 M_\odot$. Only this part of the EOS is likely to be sampled.



Example: Constraints imposed by nuclear theory



Band between lines satisfies constraints from chiral effective field theory.
(Hebeler, Lattimer, Pethick, Schwenk. '10)

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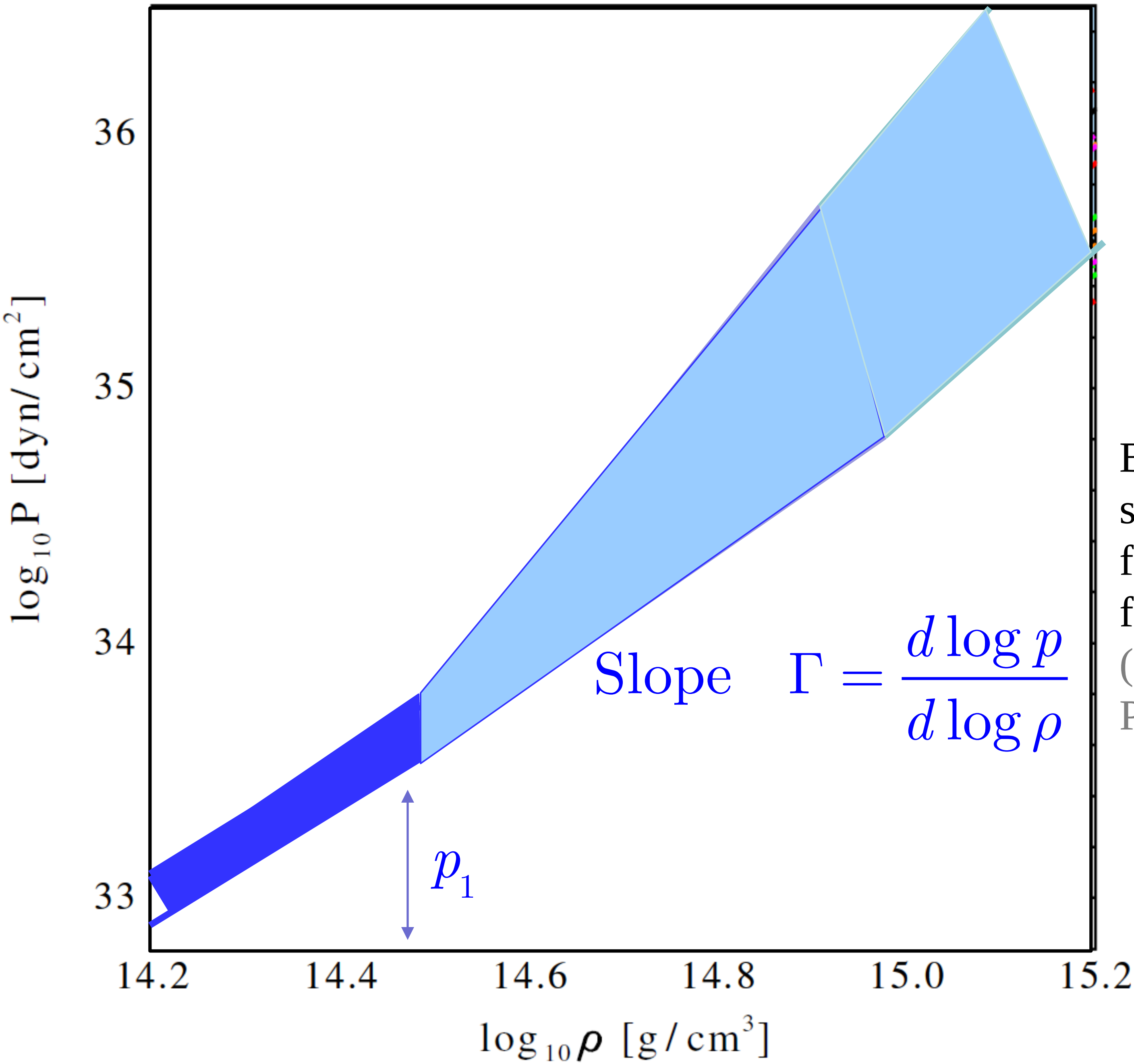

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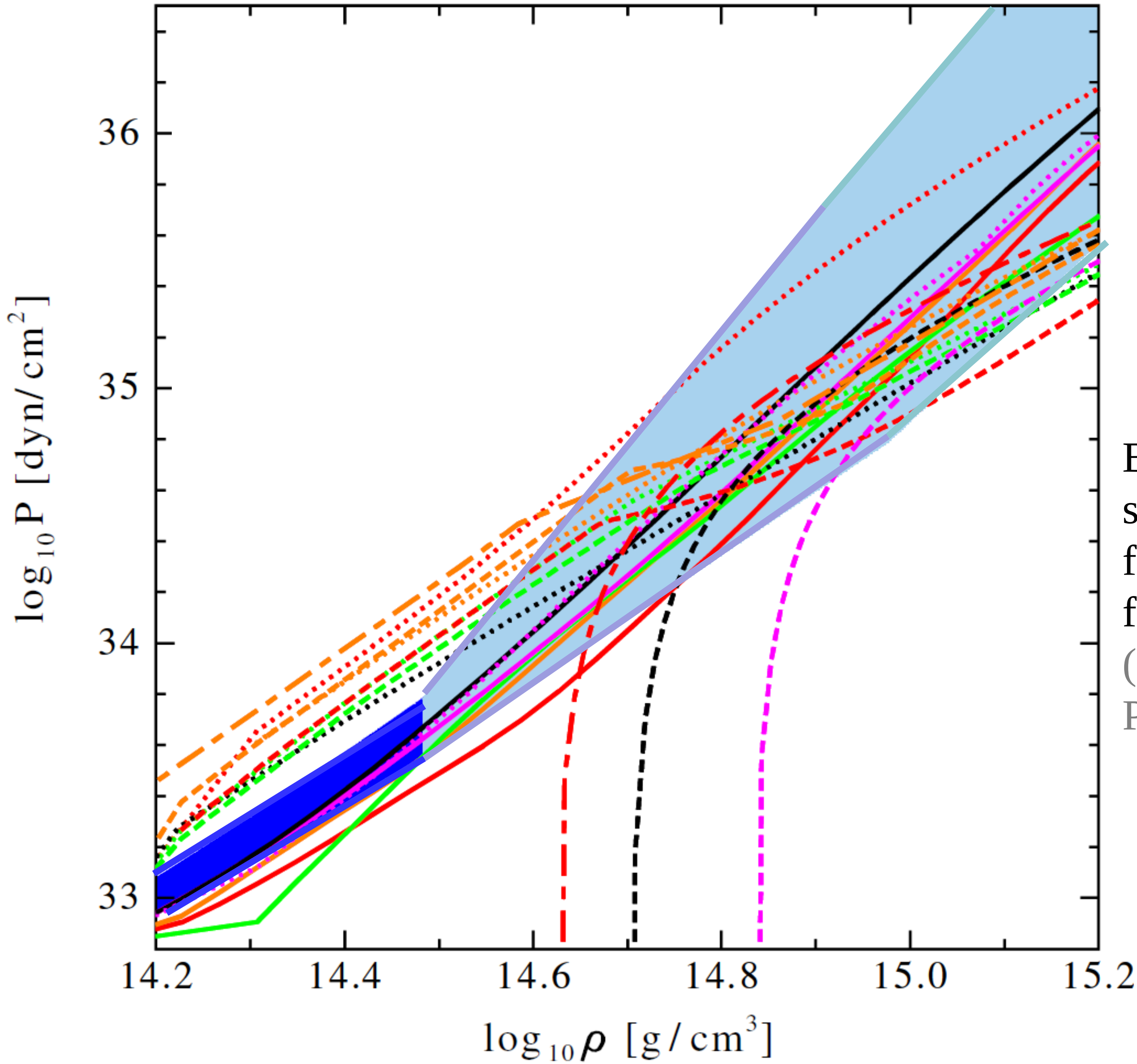

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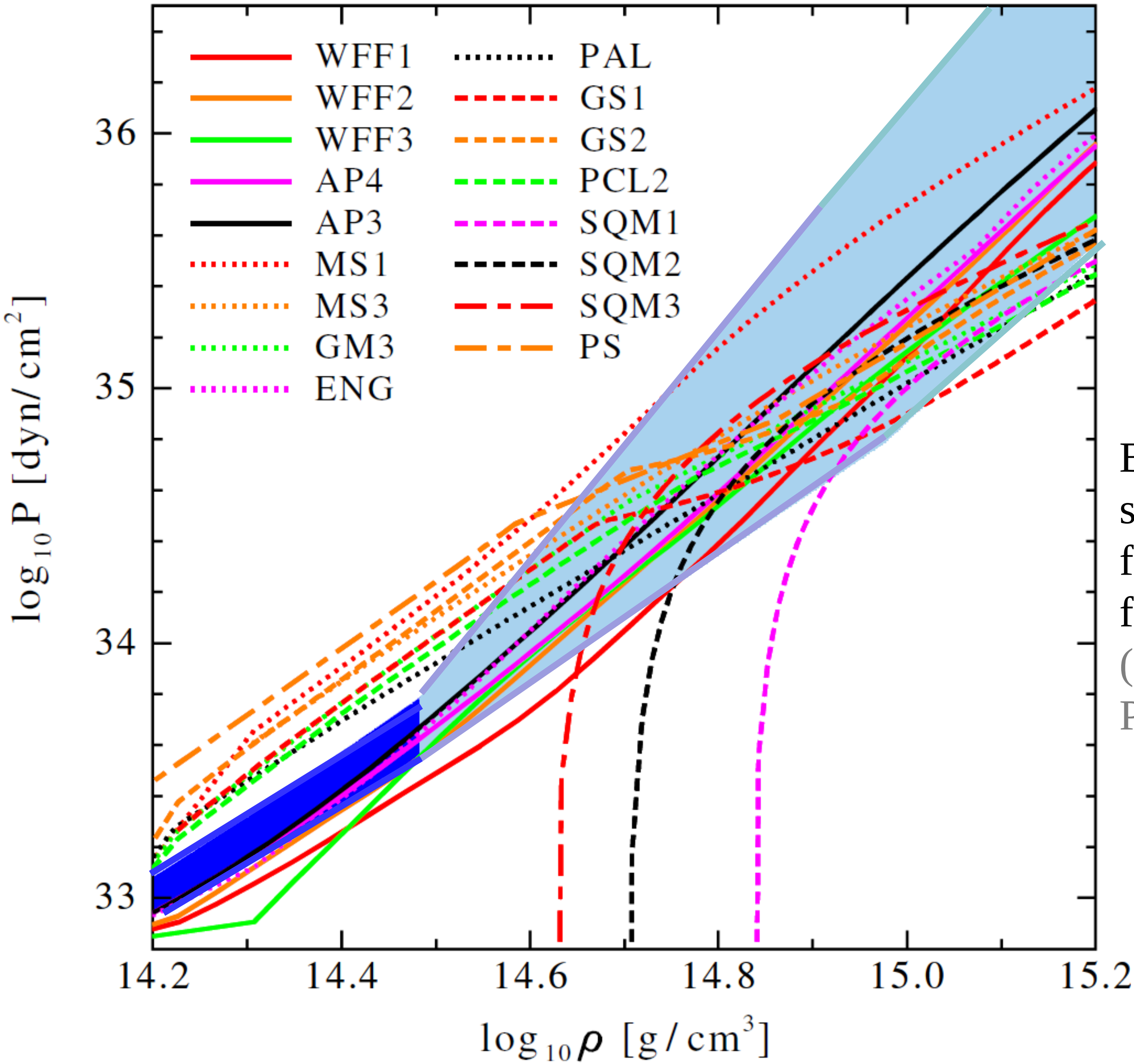

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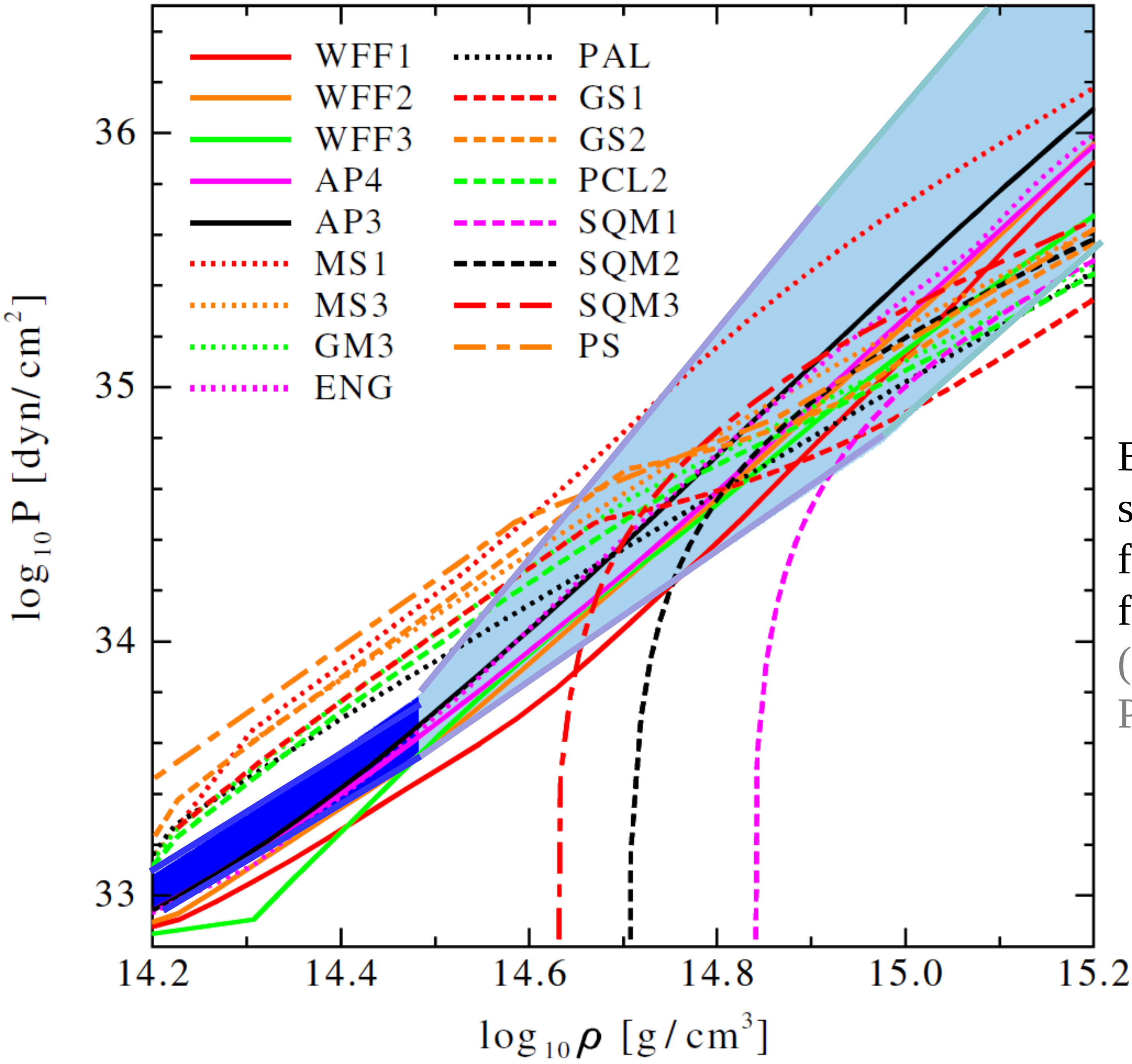

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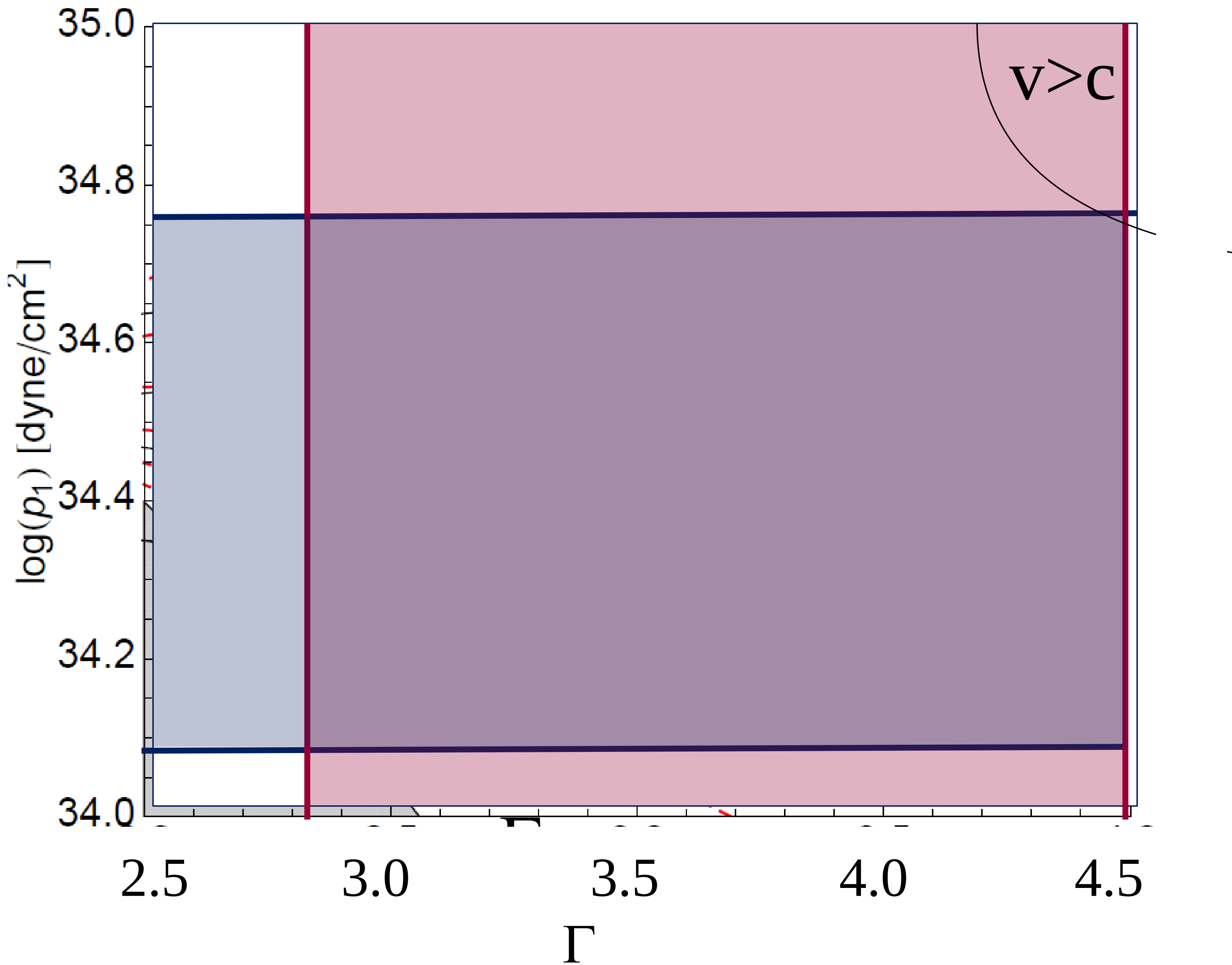
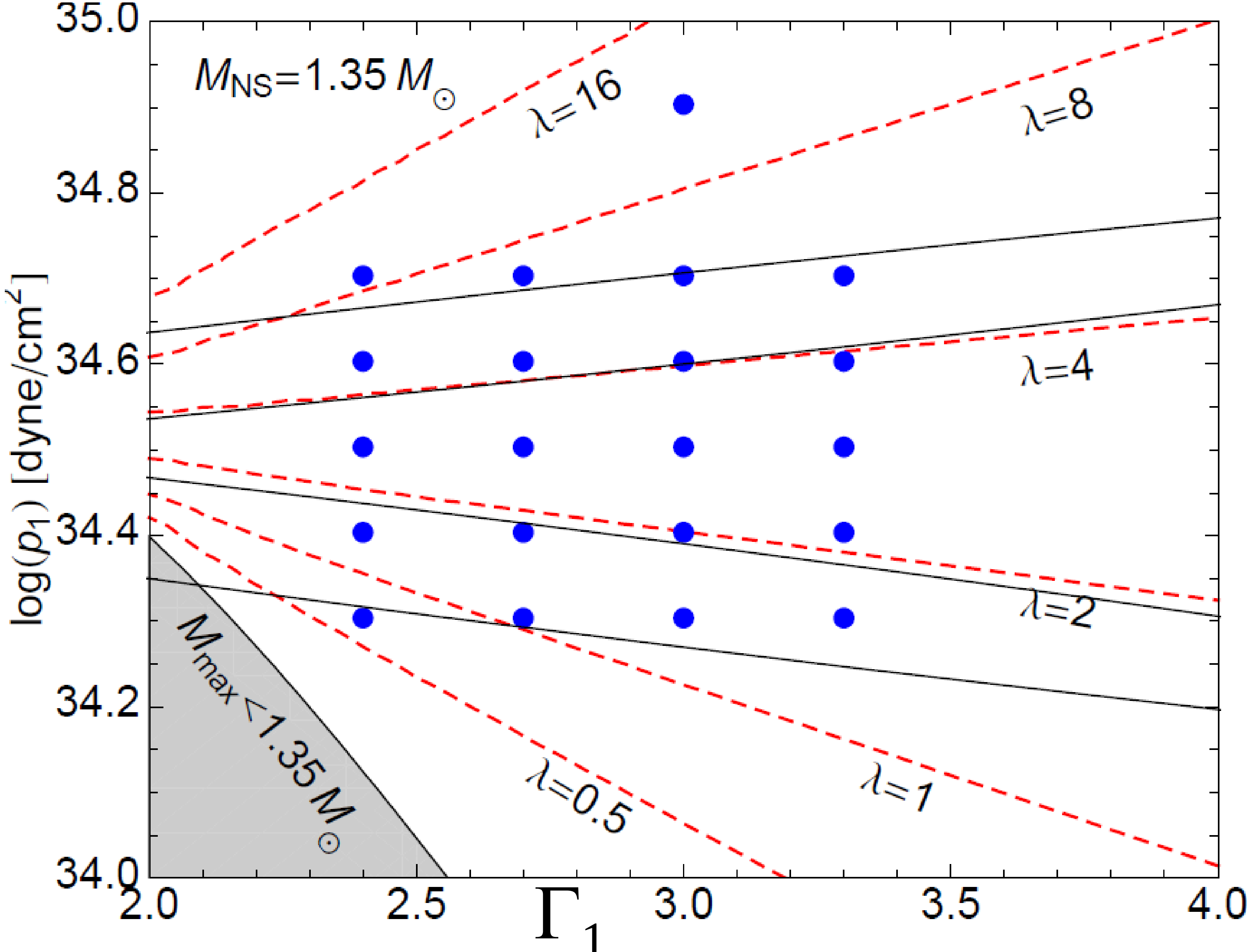


v>c
35.0
34.8
34.6
34.4
34.2
34.0
log(p_1) [dyne/cm^2]
2.5
3.0
3.5
4.0
4.5
Γ

What NS property can be measured?

Can now use the parameter space to find the parameter that is measured by the departure of inspiral from spinless BH-BH inspiral.

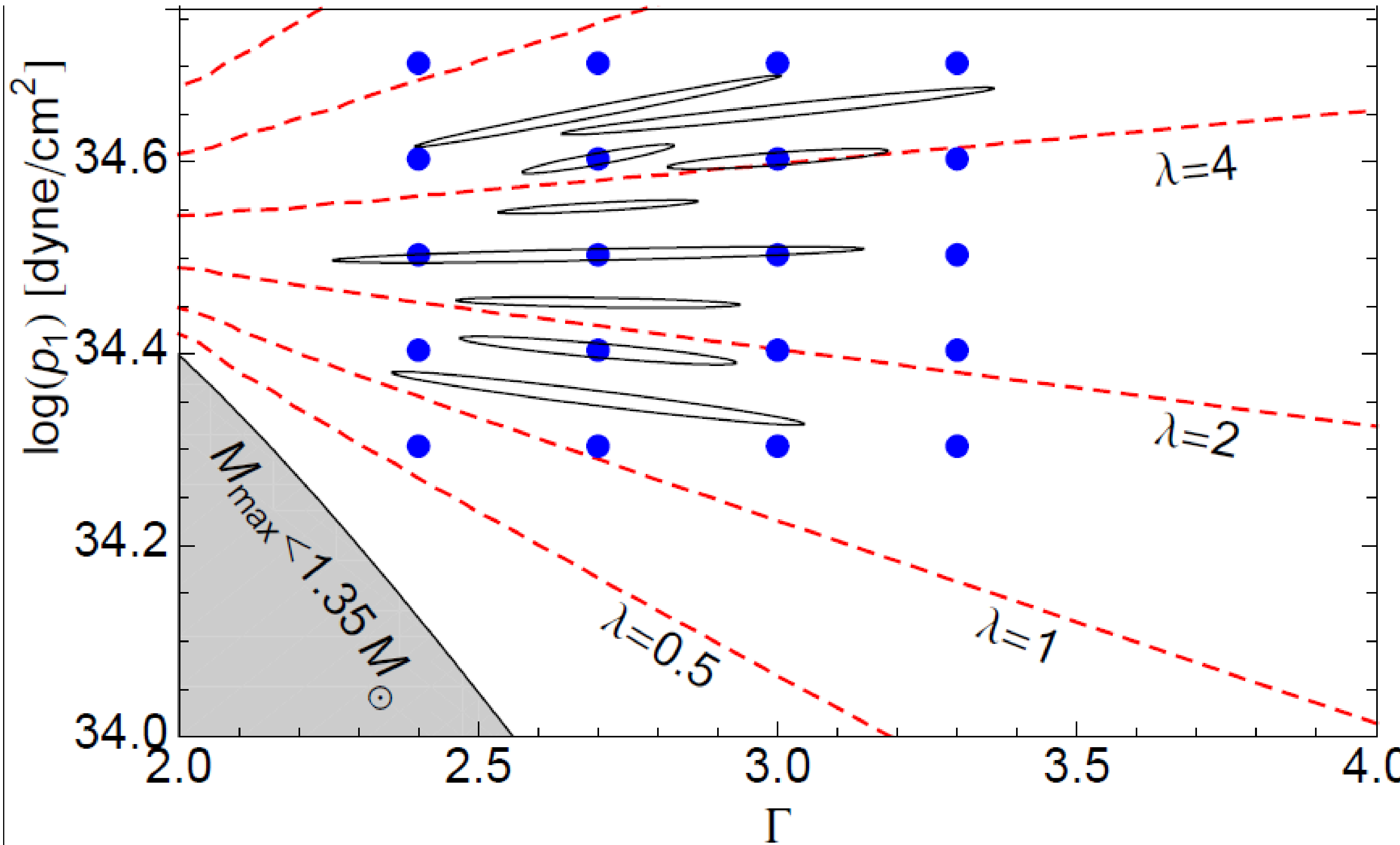
134 NS-BH simulations sample the EOS parameter space



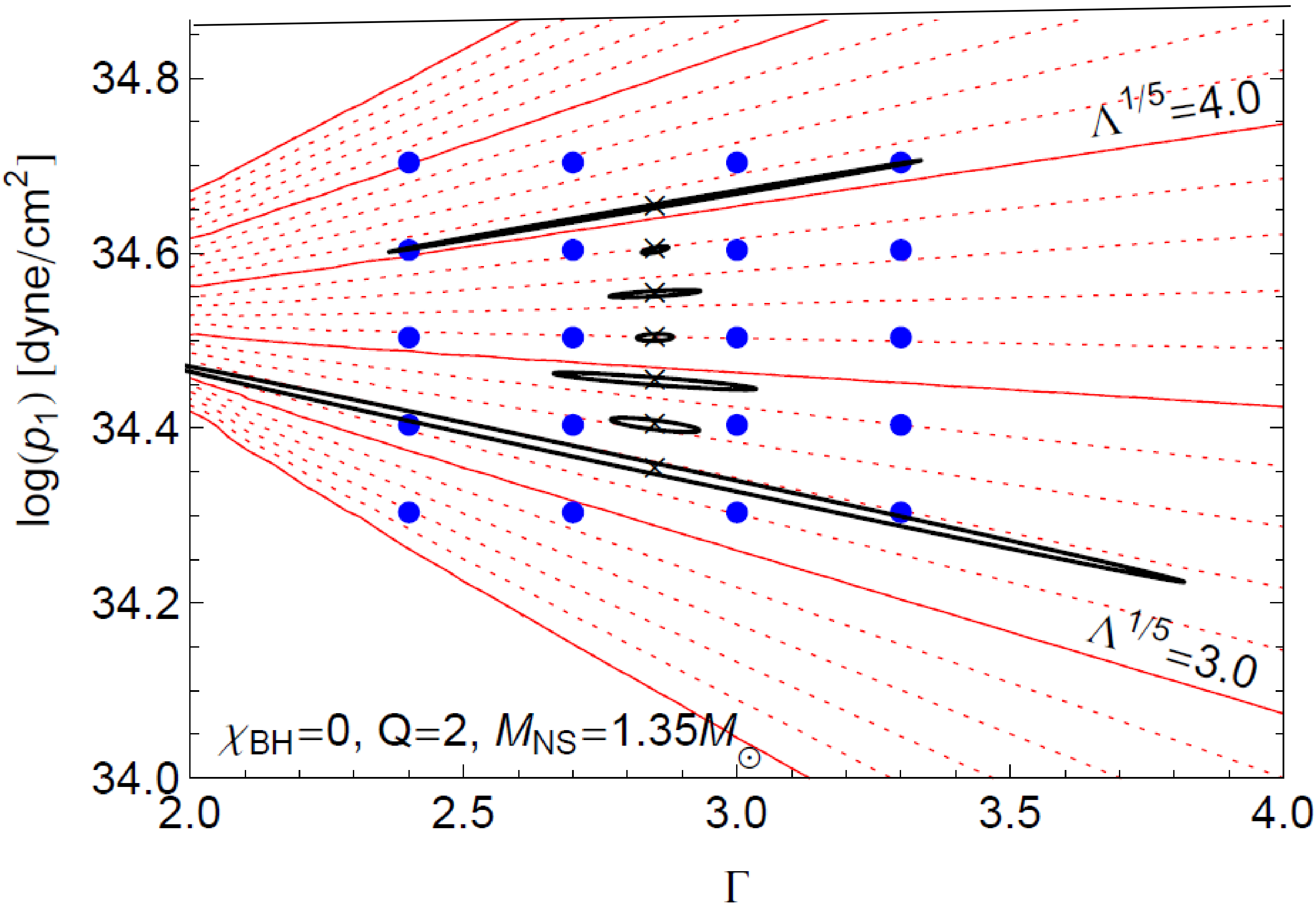
Lackey, Kyutoku, Shibata, Brady, JF '13, '14

Ellipses show 1-σ error for estimated noise of Einstein Telescope (ET-D)

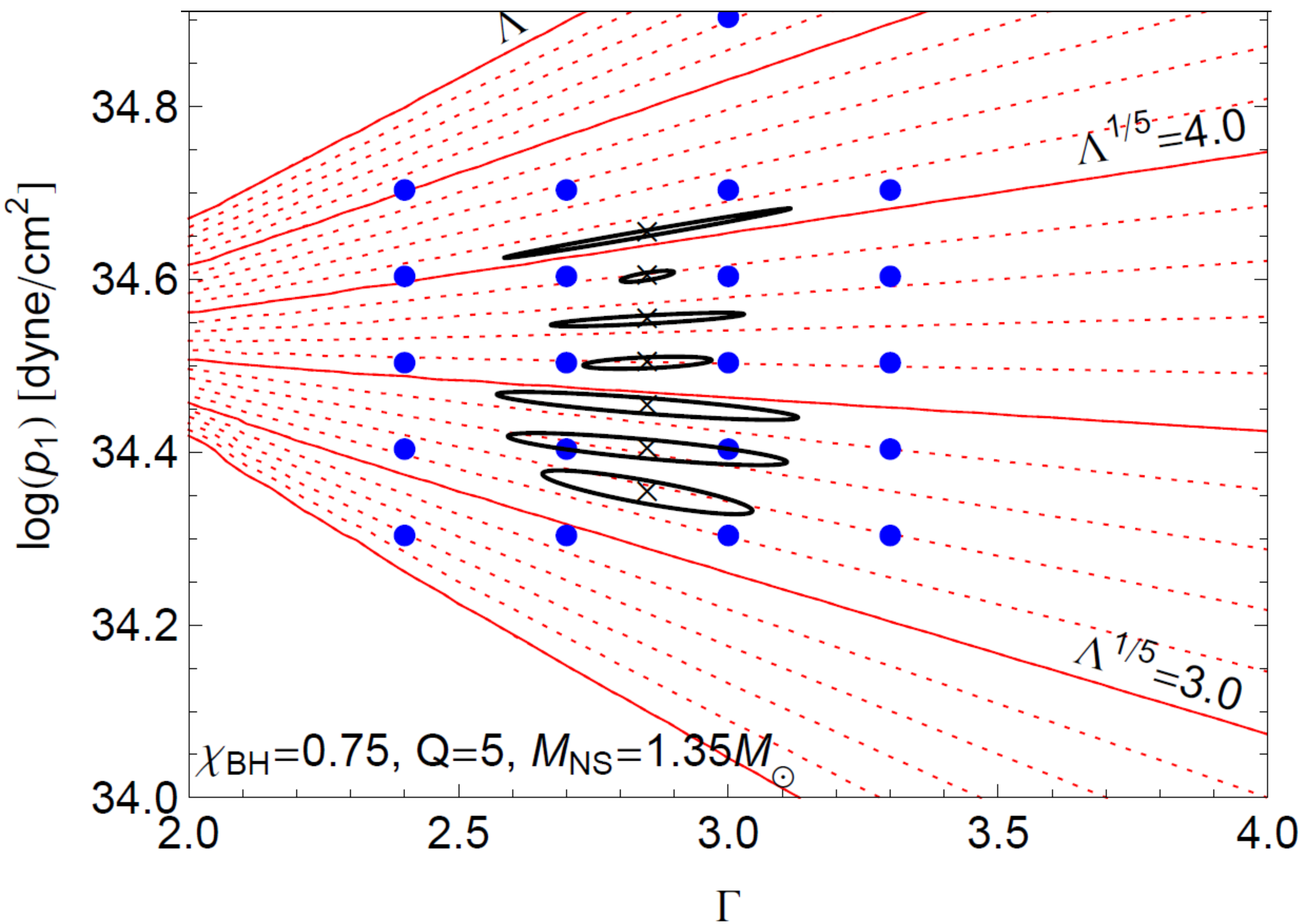
Inspiral determines λ



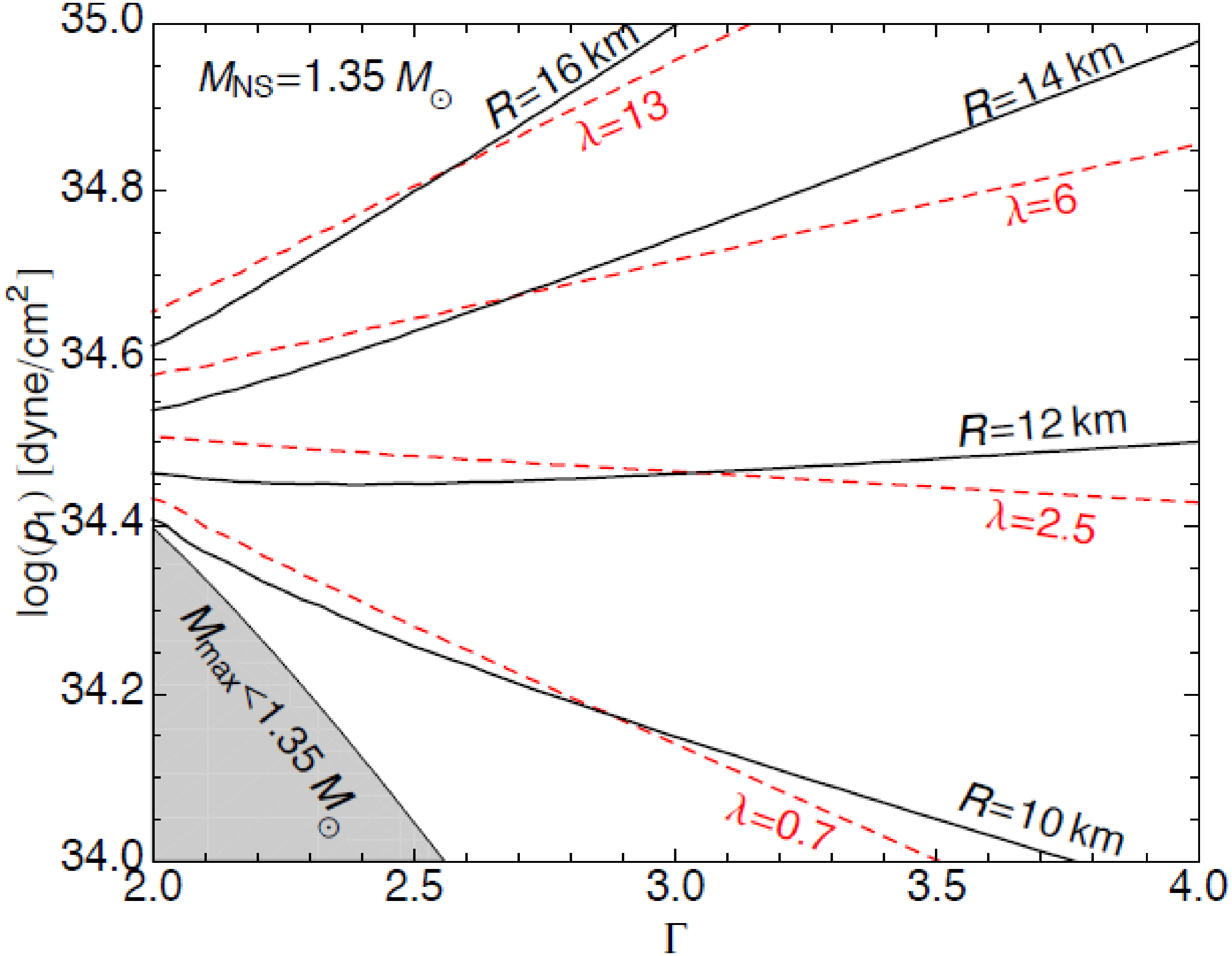
Changing the mass ratio does not break the degeneracy of waveforms from models with the same deformability λ. $M_{BH}/M_{NS} = 2$:



And BH spin does not break the degeneracy: a/M=.75



Lines of constant λ and *R* are nearly aligned in the EOS parameter space



plot from B. Lackey

- For aLIGO/VIRGO/KAGRA, waveforms from EOS with the same λ are degenerate.

- Because the inspiral measures only λ, one needs accurate templates only for models based on a 1-parameter set of EOS – for example, waveforms for a family of relativistic polytropes $p = K \rho^{\Gamma}$, matched to a low-density EOS.

Inverting observations to give EOS: Spectral parametrization

Lindblom ('10), writing a family of EOS in the form

$$\Gamma(p) = e^{\sum \gamma_k p^k},$$

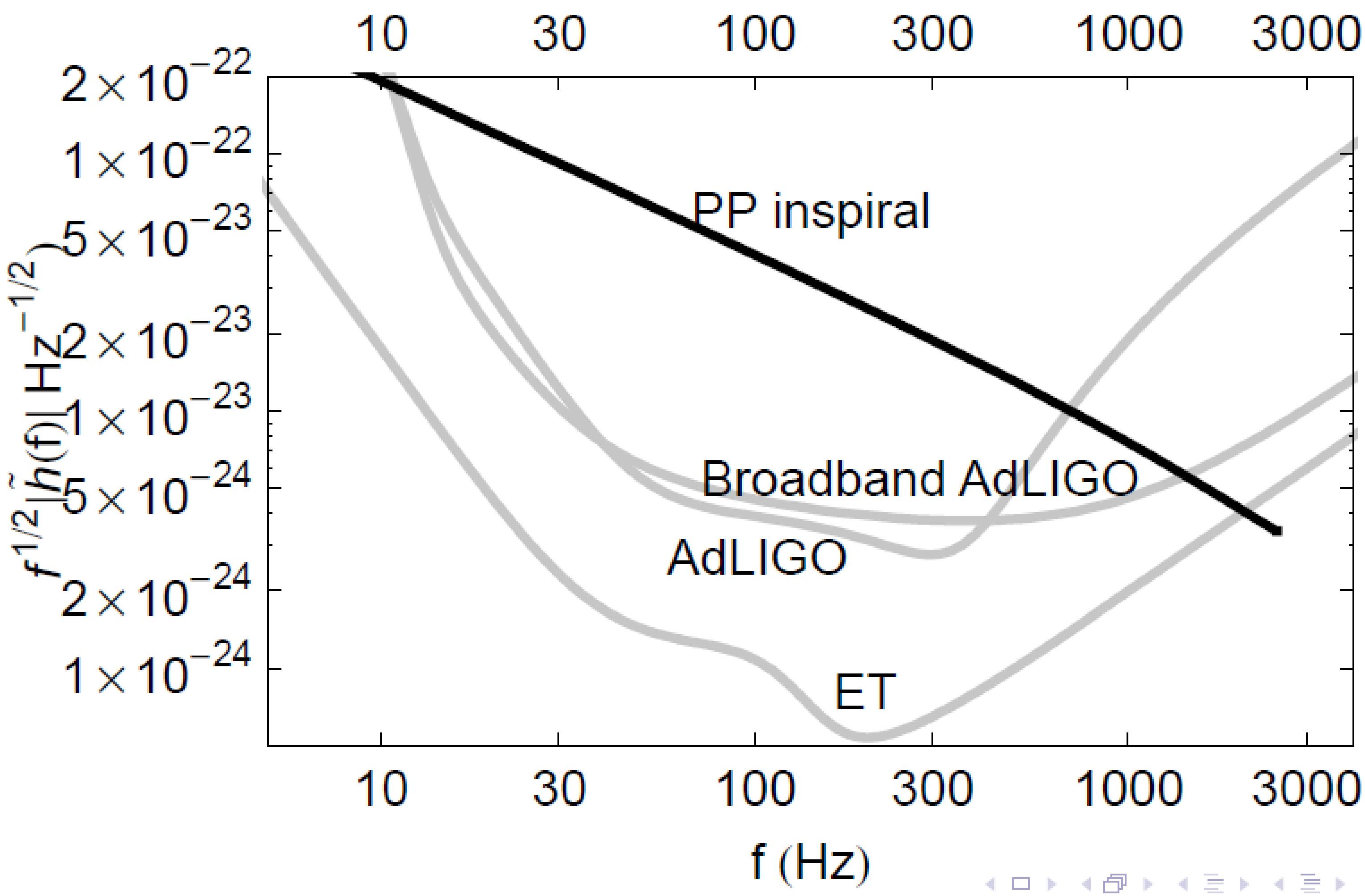
finds a more accurate match to candidate EOS for the same number of parameters. This gives a current best phenomenological way to invert a set of observations to obtain an EOS. Complementary to assuming a form of the nuclear Lagrangian and constraining its parameters.

III. With what accuracy?

See also talks Friday by
Sukanta Bose and Jeroen Meidam

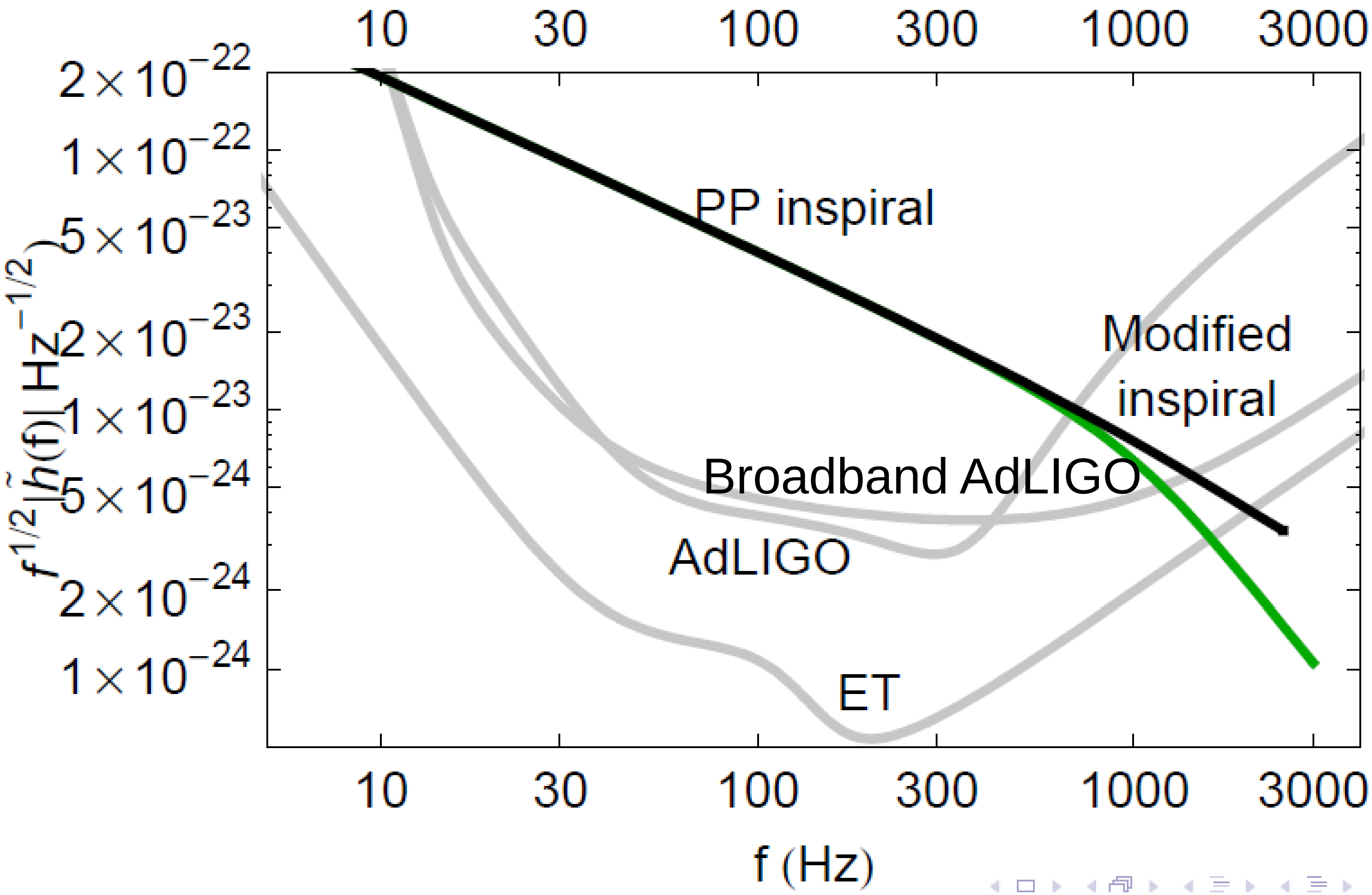
Signal from DNS inspiral

at 100 Mpc



Signal from DNS inspiral

at 100 Mpc:



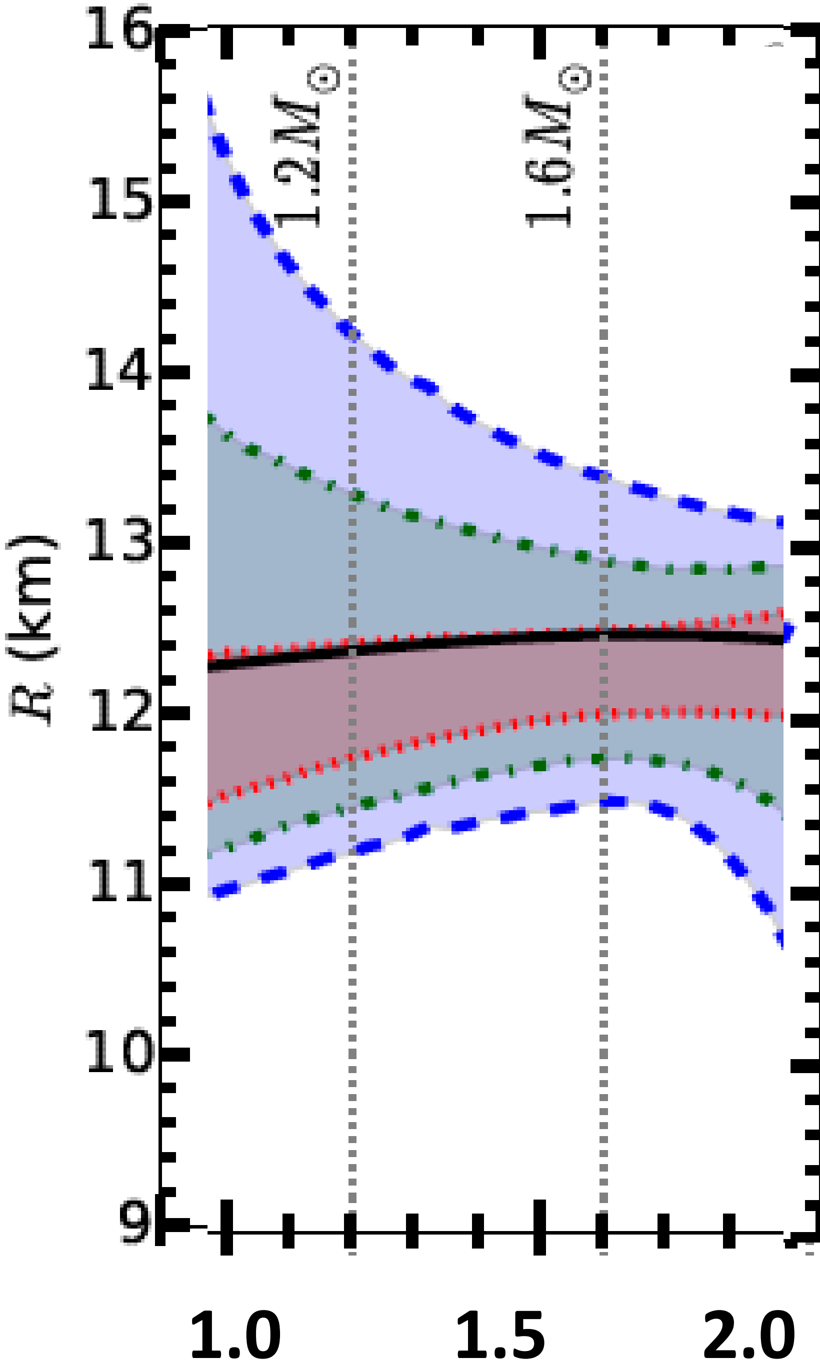
Estimated accuracy with which radius can be extracted from inspiral waveform for two $1.4\ M_\odot$ stars at 100 Mpc

R (km)	Broadband	Narrow band 1150 Hz
R=10.3	$\pm .61$	$\pm .57$
R=11.25	$\pm .78$	$\pm .78$
R=13.4	± 1.75	± 2.13

Read et al.

- Including correlations between λ and other parameters increases uncertainty in R by a factor of about 3.
- Uncertainty decreases by $\sqrt{\text{number of sources}}$

These opposing effects are incorporated in recent Markov-Chain Monte-Carlo analyses:

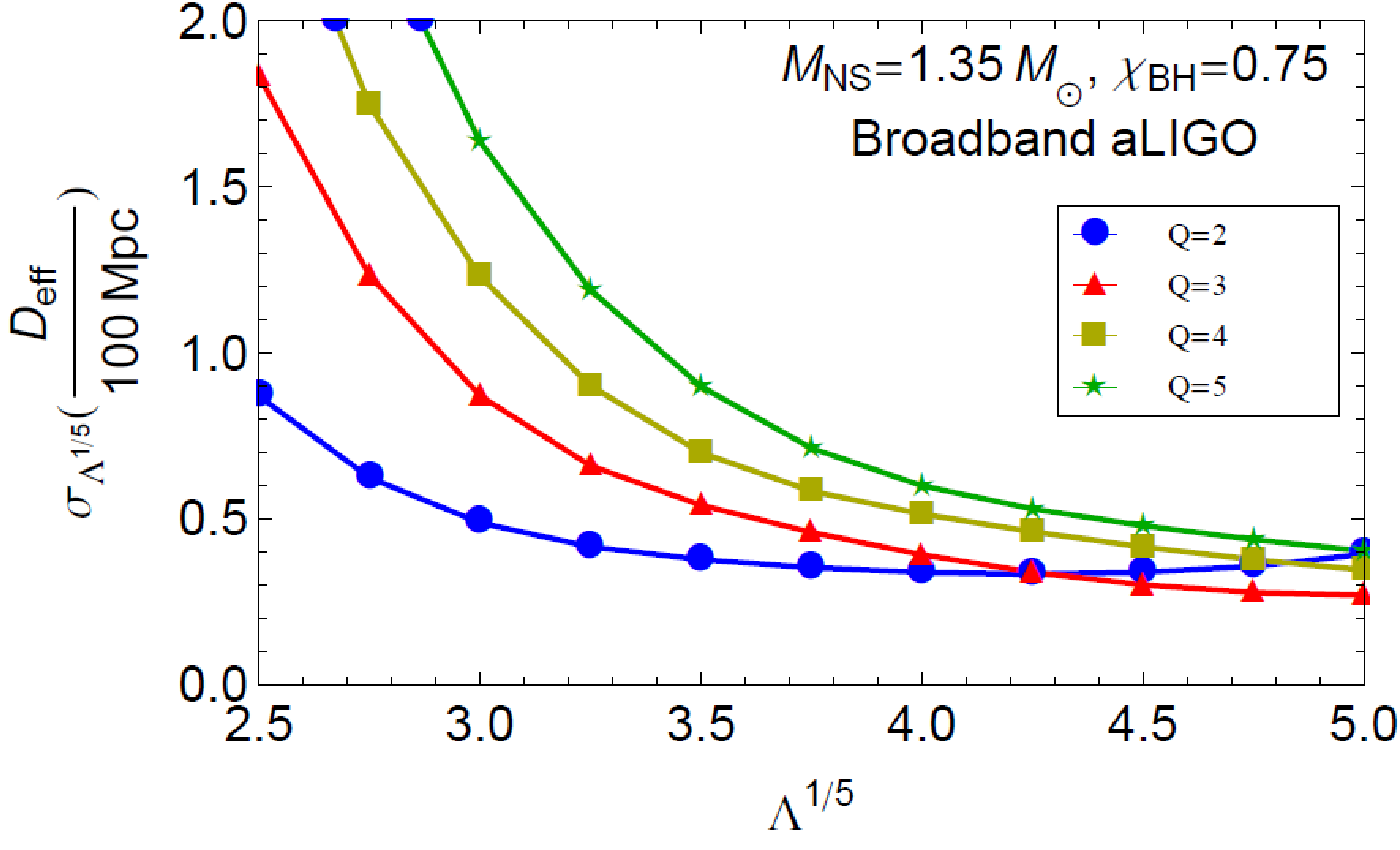


1σ, 2σ and 3σ contours for R, from loudest 20 of 40 events.

Uniformly distributed masses, 1.2 to 1.6 $M_\odot$, give same error bars as narrowly peaked range of masses.

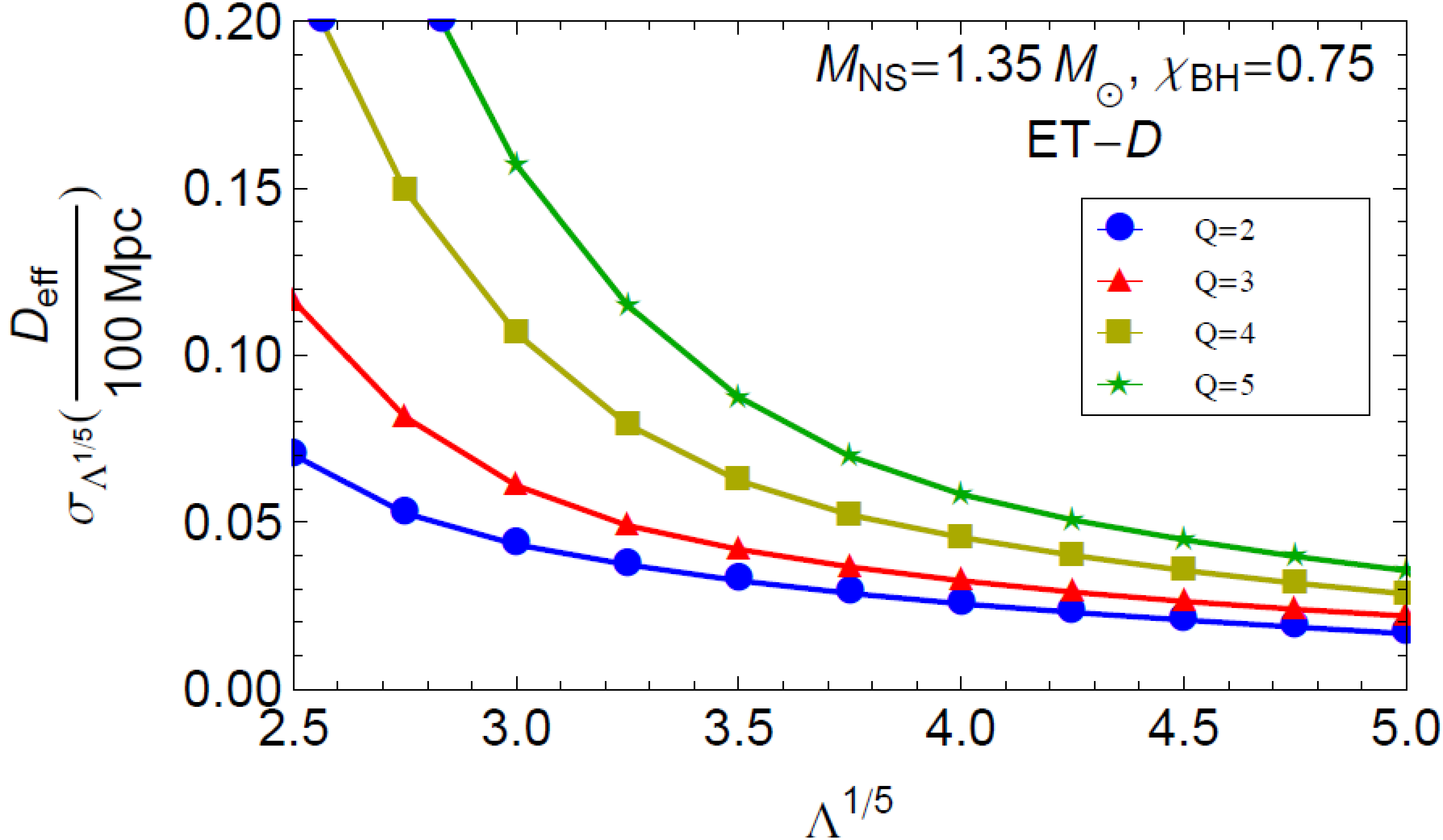
Lackey, Wade ‘14

BH-NS inspiral



Lackey, Kyutoku, Shibata, Brady, JF ’13, ’14

For the proposed ET telescope, high accuracy is possible:1-σ error for 100 pc optimal orientation



Only a small fraction of the Advanced LIGO events in this parameter range have gravitational-wave signals which could offer constraints on the equation of state of the neutron star (at best ~ 3% of the events for a single detector at design sensitivity)

Foucart, Deaton, Duez, Kidder, MacDonald, Ott, Pfeiffer, Scheel, Szilagyi, Teukolsky

IV. Comments on GW observations

A. Model independence
GW observations are likely to have statistical accuracy no better than the present electromagnetic observations.

But electromagnetic determinations of are model dependent.

Electromagnetic measurements of NS radii for X-ray binaries with measurable mass:

Thermonuclear bursts

Emission at the tail end of the burst after atmosphere has settled back down

Transiently accreting neutron stars in quiescence

Evidence supports isotropic emission from core heated by prior accretion

Model dependence of radii from X-ray observations of quiescent and burst sources:

Systematic errors from, e.g.,
Opacity of model NS atmosphere:
H or He?

Fraction of light absorbed by the interstellar medium

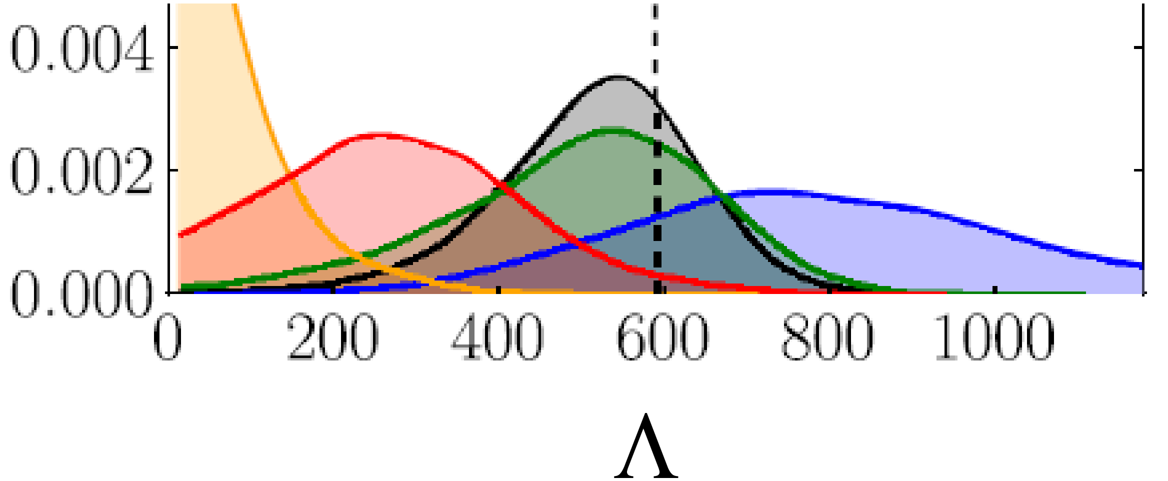
(and for bursts only)
Does the photosphere remain above the NS surface in the cooling tail of the burst?

How isotropic is the emission ?

"A recent analysis (Guillot et al. 2013) of the thermal spectra of 5 quiescent low-mass X-ray binaries. . . determined the radius to be $R = 9.1^{+1.3}_{-1.5}$ km to 90% confidence. However, the masses of the sources were found to range from $0.86 M_\odot$ to $2.4 M_\odot$, and a significant amount of the predicted *M*-*R* region violates causality and the existence of a 2 solar mass neutron star." (Lattimer, Steiner '13)

Demanding causality and the existence of a $2 M_\odot$ neutron star, they find an allowed range of radii between 10.9 and 12.7 km

Waveform accuracy needed:



If have 10 events at SNR 30 or more, for systematic error in waveform to be less than 1σ statistical error from detector noise, need less than 2 radian accumulated phase error by merger. Merger at 1600 Hz Implies less than 0.5 radian phase error at 400 Hz.

NS spin

The spin of pulsars in observed NS-NS systems is small compared to pulsars in low-mass X-ray binaries:

Pulsar	Period (ms)
J0737-3039A	22.699
J0737-3039B	2773.461
B1534+12	37.904
J1756-2251	28.461
J1906+0746	144.071
B1913+16	59.031
B2127+11C	30.529
J1829+2456	41.009
J1518+4904	40.935
J1811-1736	104.1
J0453+1559	45.782

(from Martinez ‘15)

Why?

Each progenitor star was massive enough to end as a neutron star. After the first NS forms, its high-mass companion evolves more quickly than the companion star in an LMXB and the accretion phase is shorter:

Less mass and less angular momentum transferred to the first neutron star.

No accretion onto second NS, so no spin up.
In only one system is the companion seen as a pulsar, and its period is 2.8 s.
For first formed stars:

$$\bar{f} = 24 \pm 11 \text{ Hz}$$

With $$\chi := \frac{c}{G}\frac{J}{M^2},$$

$$\bar{\chi} = 0.01\frac{I}{65 M_\odot \text{km}^2}, \quad \sigma_\chi = .005$$

$$\chi_{\max} = 0.022\frac{I}{65 M_\odot \text{km}^2}$$

This still overstates the spin:
By the time these systems reach the LIGO band they will have spun down for more than 10^8 yr. Using their times to merger,

$$f_{\rm max} = 18\,{\rm Hz},$$
$$\chi_{\rm max} = 0.01$$
$$\bar{\chi} = 0.004$$

(Selection effects in opposite directions here:
We see faster spinning pulsars;
but we see pulsars with larger magnetic fields and so faster spin-down.)

This is too small to affect accuracy of parameter extraction –
but need knowledge beyond GR to avoid a flat prior of $|\chi| = 0.1$, as in Agathos et al.