

**Local, up-down asymmetrically shaped, analytical
tokamak-equilibrium model**

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Although magnetic equilibria are at the basis of almost every phenomena in tokamak plasmas, accurate numerical solutions of the Grad-Shafranov equation are not always the best way to gain insight into such complex processes. Often, a simplified description is preferable, either to achieve analytically tractable expressions or to perform parameter scans without the need to recompute a numerical equilibrium at every scan step. Therefore, it is not surprising that several local equilibrium models have seen wide application in tokamak plasmas, ranging from analytical studies on stability (e.g., ballooning modes, Alfvén eigenmodes, zonal flows) and charged-particle orbits, to simplified magnetic-field descriptions on large-scale numerical simulations with gyrokinetic codes.

Most of these local equilibrium models are built by expanding the poloidal-field flux in powers of some radial coordinate around a magnetic surface of prescribed shape, which may range from shifted circles [1] to more sophisticated parametrizations written in terms of the conventional shaping parameters: shift, elongation, and triangularity [2]. Despite the elegant magnetic-surface description achieved by such approaches, they usually result in non-trivial curvilinear coordinates and yield complicated expressions for the magnetic-field components [2,3], which turn analytical work into a difficult task.

To ease these difficulties, a local magnetic-equilibrium model is presented, with finite aspect ratio and up-down asymmetrically shaped cross section, where the poloidal-field flux is expanded as a series of Solovév solutions [4,5] with radially changing coefficients. Here, the focus changes from simple magnetic-surface parametrizations to a more convenient flux description, accurate to fourth-order terms in the inverse aspect ratio. It depends on eight free parameters, one for each of the four independent poloidal-angle harmonics (even and odd), of which three can be related with the conventional shift, elongation, and triangularity. In contrast with other local equilibrium approaches, the proposed model is intentionally built to afford tractable analytical expressions for the magnetic-field components. Therefore, it is particularly suitable for analytical assessments of equilibrium-shaping effects on a variety of tokamak-plasma phenomena. A couple of example applications are provided to illustrate this ability.

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