Theory of Nonlinear Ballooning Modes

C J Ham, S C Cowley, G Brochard and H R Wilson

17th EFTC Athens 2017

Thanks also to: Jarrod Leddy, Guillermo Bustos Ramirez, G S Yun, S Ohdachi, S Saarelma, F Militello …

Research Councils UK Energy For a Low Carbon Future

CCFE is the fusion research arm of the **United Kingdom Atomic Energy Authority.**
This work was funded by the RCUK Energy Programme [grant number EP/P012450/1] .

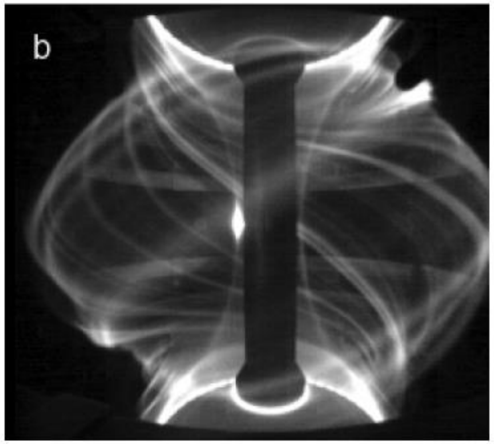
CCFE CULHAM CENTRE FOR FUSION ENERGY

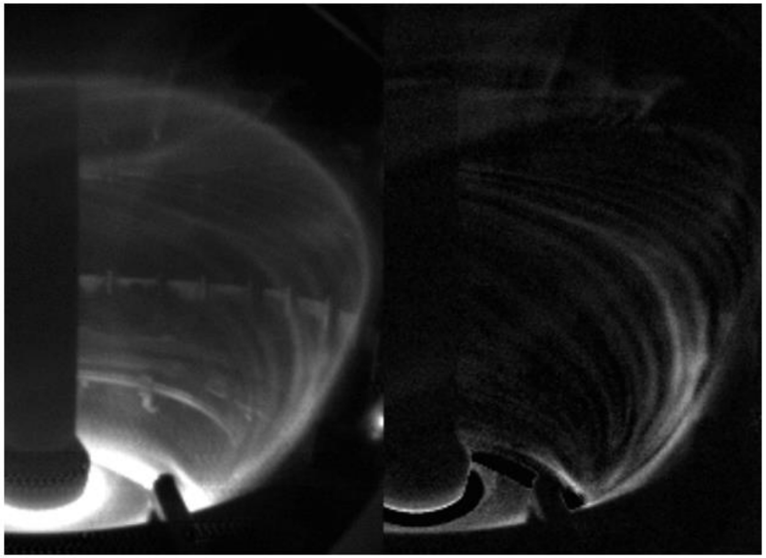
Overview

- Motivation – MAST, KSTAR,TFTR, LHD
- Hard and soft limits
- Review of some previous work
- Box geometry
- Toroidal geometry
- Discussion

Edge Localized Modes - MAST

- Edge Localized Modes (ELMs) are periodic eruptions of plasma from the pedestal/edge region
- Driven by the plasma pressure gradient
- Explosive ballooning mode filaments
- H mode pedestal limited by critical gradient





CCFE
CULHAM CENTRE FOR FUSION ENERGY

KSTAR

- ELMs are observed on KSTAR
- ECEI diagnostic shows nonlinear evolution
- Instability can saturate before crash
- Saturated state shows finger like filaments

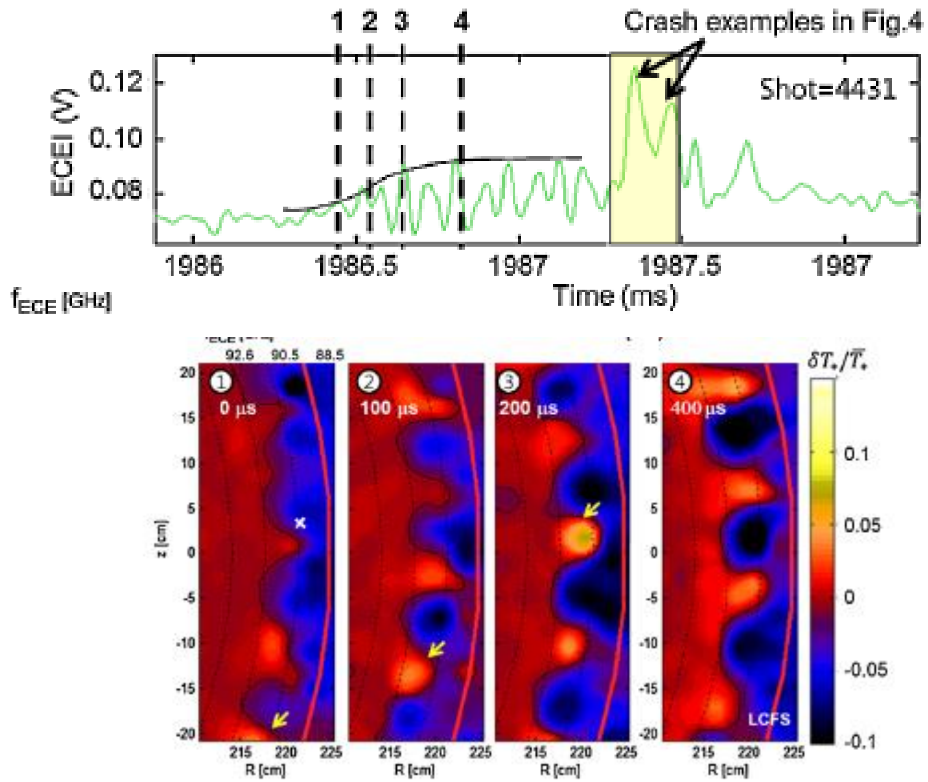


FIG. 3 (color online). Simultaneous emergence and growth of multiple ELM filaments (shot no. 4431). Solid curves are contour lines of the same $\delta T_*/\bar{T}_*$ value representing the approximate boundary of the filaments. The arrows follow the same filament illustrating the counterclockwise rotation.

G S Yun *et al.* Phys Rev Lett **107** 045004 (2012)

ITB

- Internal transport barrier (ITB) discharges have high pressure and confinement
- This would provide higher fusion power for a given magnetic field
- Thus more economic fusion power
- However, they are prone to very fast disruptions (30-100μs) which are difficult to control
- Without disruption control ITBs are unusable therefore understanding the disruption process is vital.
- There may be other problems with ITBs too

TFTR

- TFTR experiments show ballooning modes appearing prior to ITB disruption
- This is on top of $n=1$ mode activity
- ECE diagnostic results shown in figure

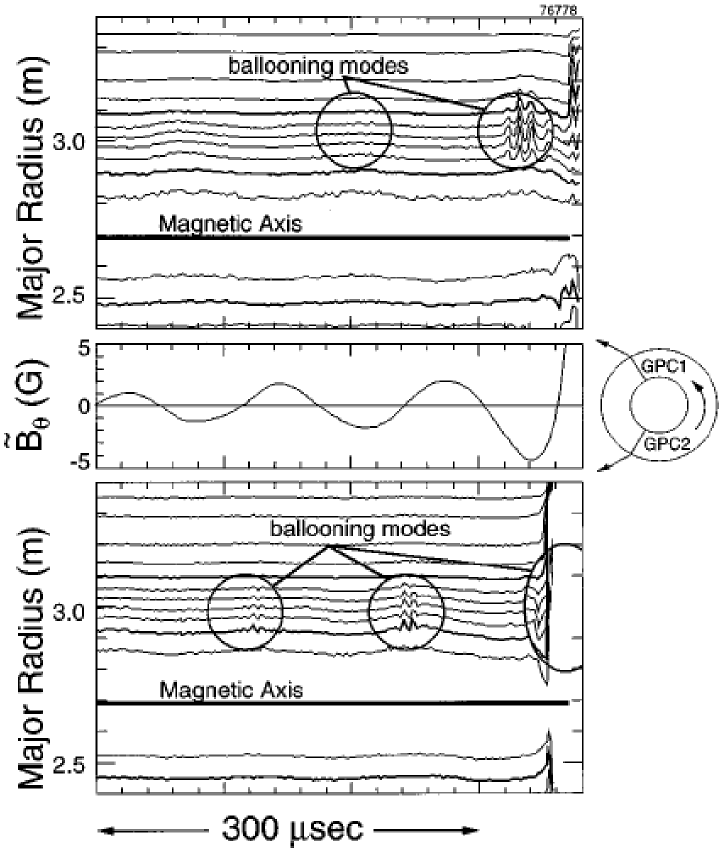
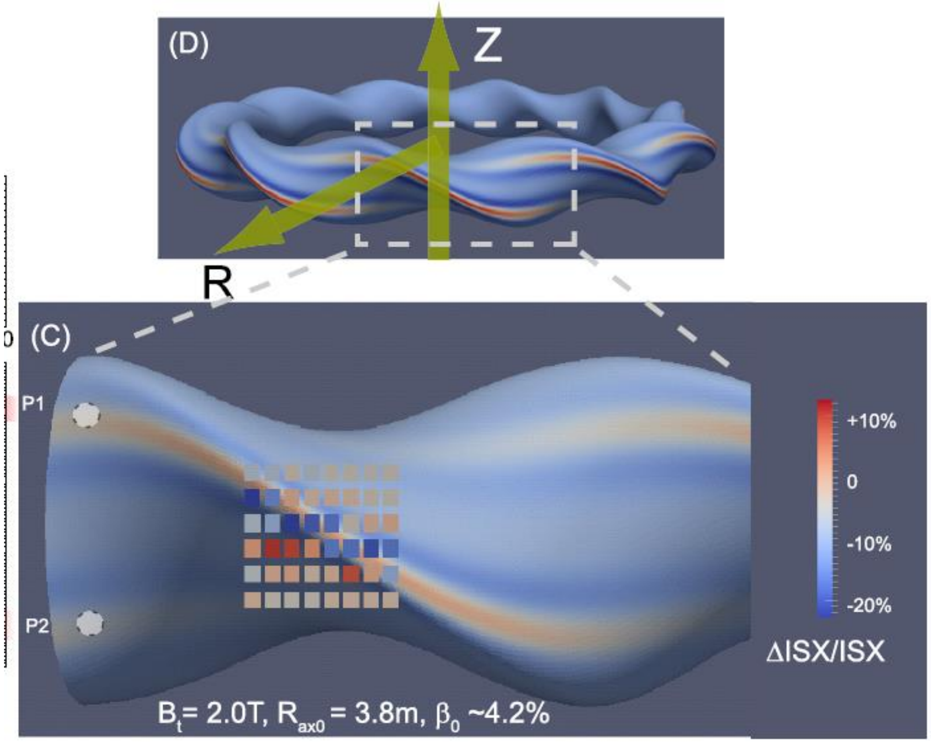


FIG. 1. Contour plot of constant electron temperature across the plasma midplane versus time. Data from the two GPCs, separated by 126° in the toroidal direction is shown. The shaded region indicates the 180° of toroidal angle where the $n=1$ kink pushes outward in the major radius [$I_P=2.5$ MA, $B_T=5.1$ T, $q(a)=4.0$, $\beta_n^{dia}=1.9$].

Fredrickson et al., Phys. Plasmas **3** 2620 (1996)

Large Helical Device

- LHD is a large Heliotron device
- Super-Dense-Core (SDC) experiments can be terminated by a Core Density Collapse (CDC)
- CDC seems to be ballooning in nature
- Stellarator version of TFTR distruption?

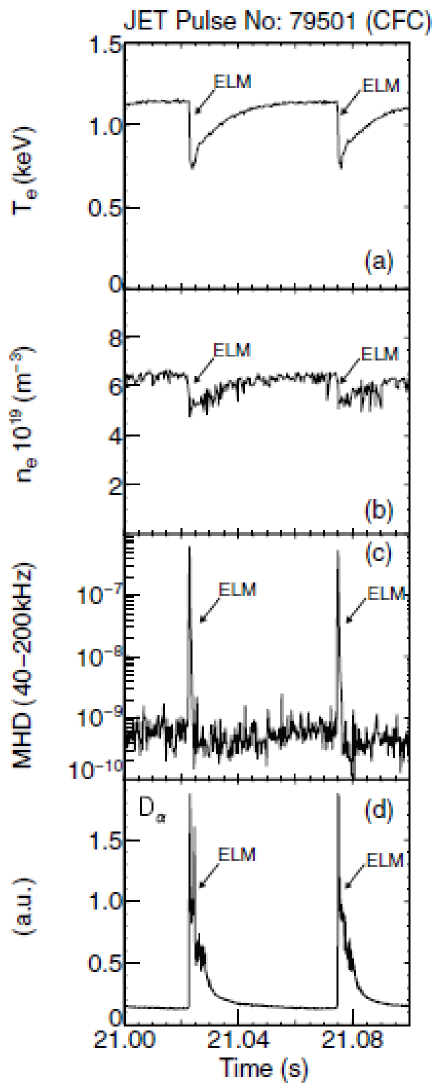


Hypothetical mode structure with SX measurements

Ohdachi et al. Nucl. Fusion 57 (2017) 066042

Soft and Hard limits

- Linear stability can only tell us so much
- Nonlinear phase of fast MHD instabilities determines if there is a *hard* or *soft* limit
- Soft limit usually pins the pressure gradient to some critical value
  - possibly high n ballooning modes in the pedestal
  - JET pulse 79501 shows saturation of T_e at the pedestal long before an ELM crash



Frassinetti et al Nucl. Fusion 55 (2015) 023007

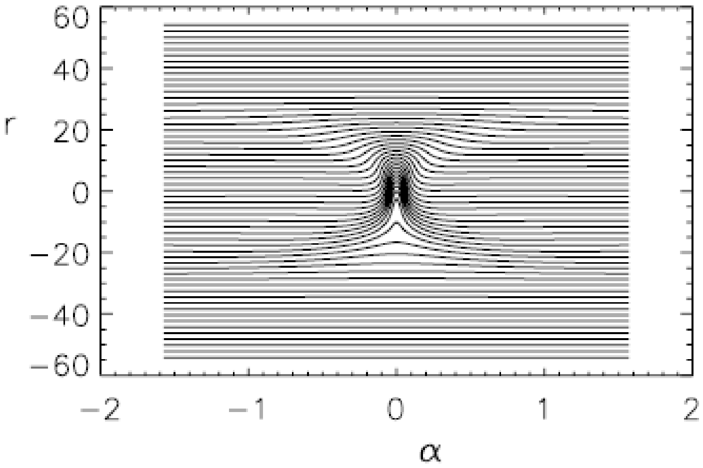
CCFE CULHAM CENTRE FOR FUSION ENERGY

Soft and Hard limits

- Hard limit rapidly destroys confinement
  - e.g. edge localized mode (ELM)
  - disruption
- ELMs show that ballooning modes may produce a hard limit as well as a soft limit
- Want to understand when we have hard or soft limits
- Most importantly how to remove hard limits

Nonlinear ballooning review

- Previous work on nonlinear ballooning modes has focussed on the early nonlinear state
- This shows that the mode is expected to form narrow finger-like structure
- Mode gets narrower as it grows
- Explosive growth expected
- Finite time singularity



Wilson & Cowley Phys Rev Lett 92 175006 (2004)

CCFE CULHAM CENTRE FOR FUSION ENERGY

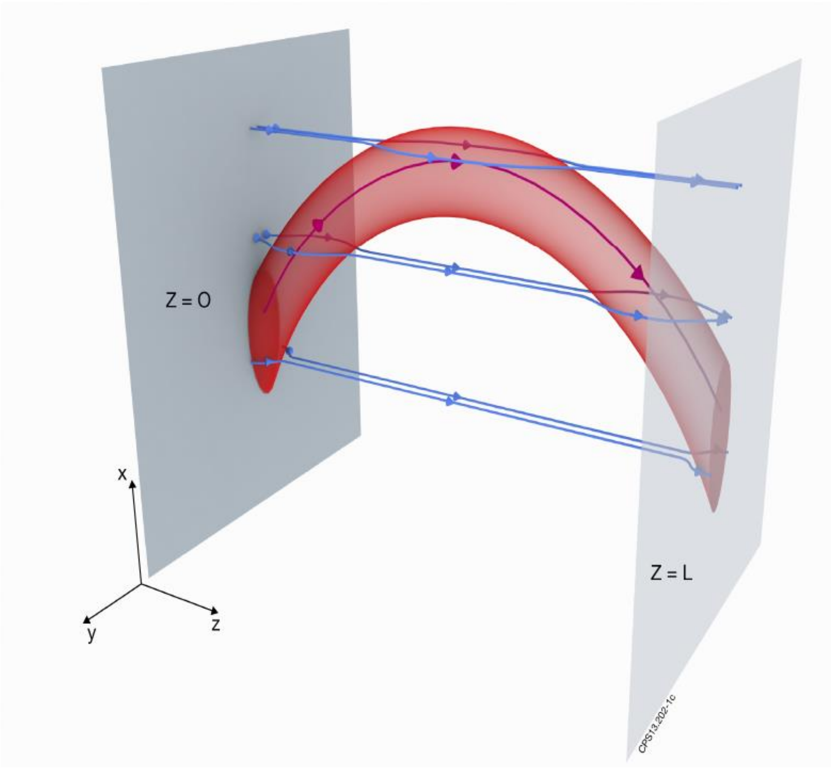
Slab geometry

- Myers *et al.* studied a line-tied slab with gravity and a density gradient numerically using ideal MHD – no reconnection
- They found a number of phases
  - Initial transient
  - Linear
  - Quasilinear
  - Explosive growth
- Explosive growth phase not fully resolved, extra physics required

Myers *et al.* Plasma Phys. Control. Fusion **55** (2013) 125016

Box model

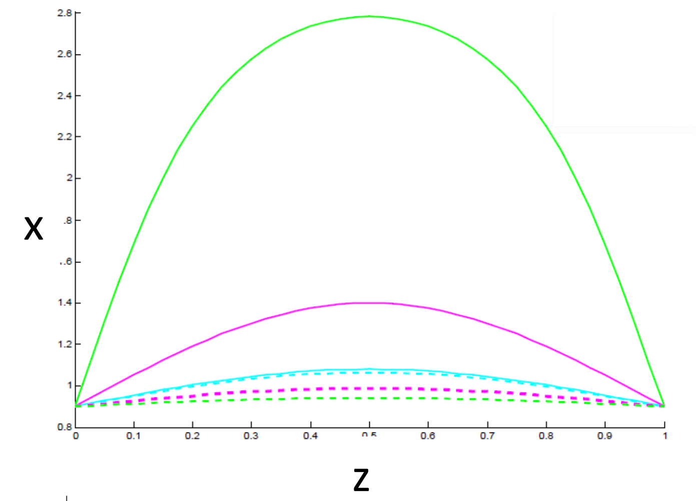
- Cowley *et al.* investigated a box model again with ideal MHD and line tied boundary conditions
- Assume filaments are narrow to reduce field line bending
- Buoyancy balanced against magnetic curvature

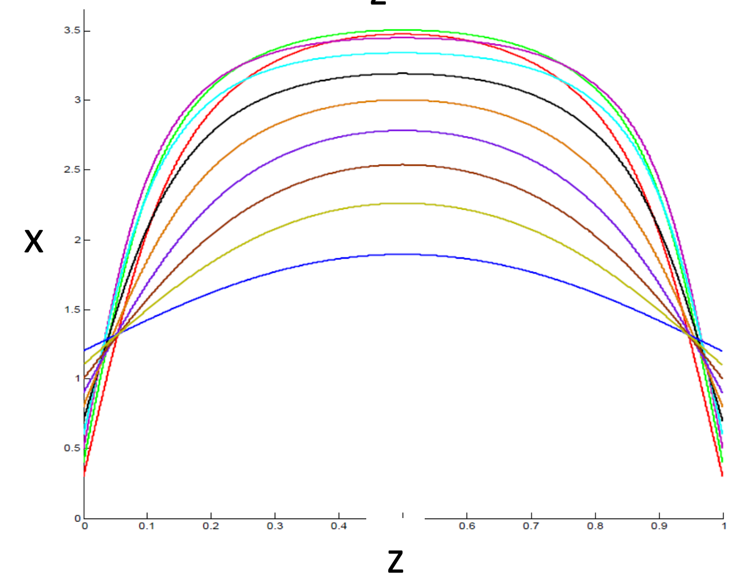


Cowley *et al.* Proc. R. Soc A **471** 20140913 (2015)

Box model

- Saturated states were found
- Linearly stable yet nonlinearly unstable flux tubes
- Flux tubes lower in the atmosphere erupted further than those above

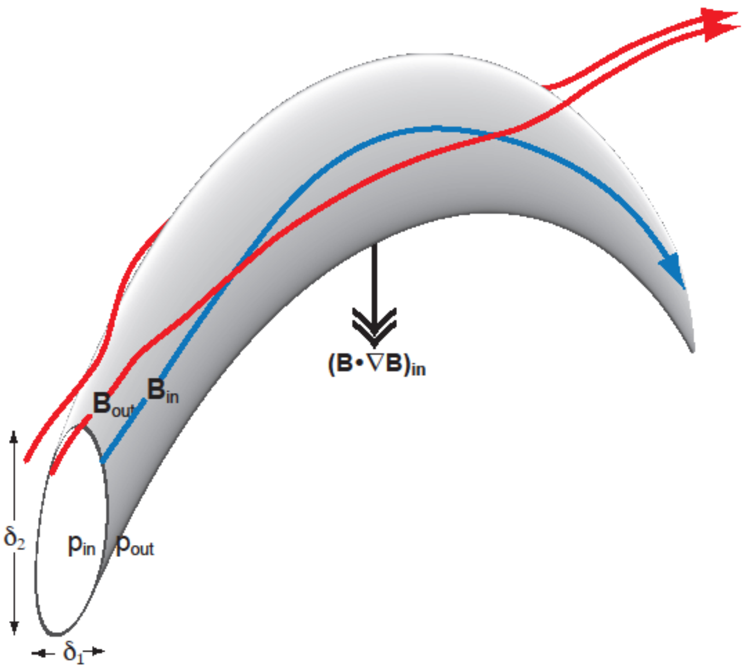




Cowley *et al.* Proc. R. Soc A **471** 20140913 (2015)

Toroidal geometry

- Develop box geometry to a torus to be able to study ELMs and disruptions quantitatively
- Consider a highly elliptical flux tube, $\delta_1 << \delta_2$
- Tube sufficiently narrow that the field and pressure outside are unperturbed



Ham *et al.* Phys Rev Lett **116** 235001 (2016)

Forces on flux tube

- Denote field inside the tube as
  - $\mathbf{B}_{in} = \mathbf{B}_{in}(\theta, r_0, t)$
  - θ, distance along field line
  - r_0, starting flux surface
  - t, time
- The motion is assumed to be slow compared to the (sound) time to equalize pressure along the tube
  - $p_{in}(\theta, r_0, t) = p(r_0)$

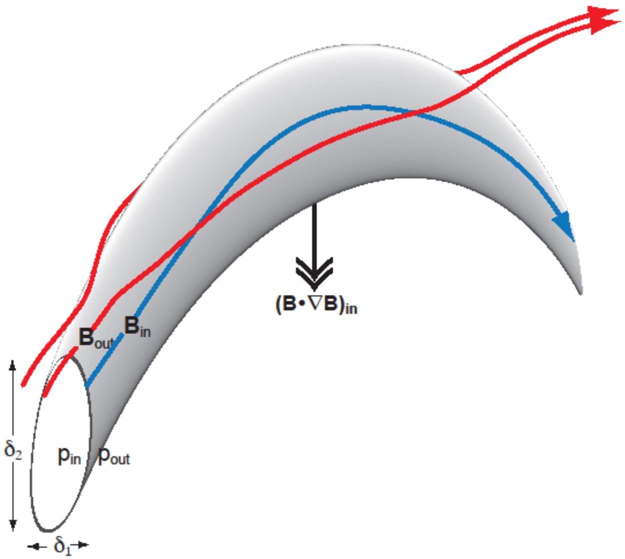
Force on flux tube

- We calculate the nonlinear behaviour of erupting flux tubes by consideration of the two components of the MHD force equation perpendicular to the magnetic field

$$\mathbf{F} = \frac{1}{\mu_0}\left[\mathbf{B}\cdot\nabla\mathbf{B} - \nabla\left(\frac{B^2}{2} + \mu_0 p\right)\right]$$

Across the tube

- Force in the narrow direction is formally large and so must cancel to this order - $O(p/\delta_1)$
- This implies total pressure inside must equal total pressure outside



$$\left[\frac{B_{in}{}^2}{2} + \mu_0 p_0(r)\right]_{in} = \left[\frac{B_0{}^2}{2} + \mu_0 p_0(r_0)\right]_{out}$$

$$B_{in}^2(\theta, r_0, t) = B_0^2(\theta, r) + 2\mu_0\left[p_0(r) - p_0(r_0)\right]$$

Forces on flux tube

- In the radial direction we have

$$F_r = \frac{1}{\mu_0}\left[B_{in}\cdot\nabla B_{in} - \nabla\left(\frac{B^2}{2}+\mu_0 p\right)\right]\cdot e_r$$

- Using the results on the previous slide a generalized *Archimedes' principle* can be derived

$$F_r = \frac{1}{\mu_0}\left[B_{in}\cdot\nabla B_{in} - B_0\cdot\nabla B_0\right]\cdot e_r$$

- Net force is the curvature force of the tube minus the curvature force of the tube it has displaced

Model

- The physics of this model is represented by the following equations

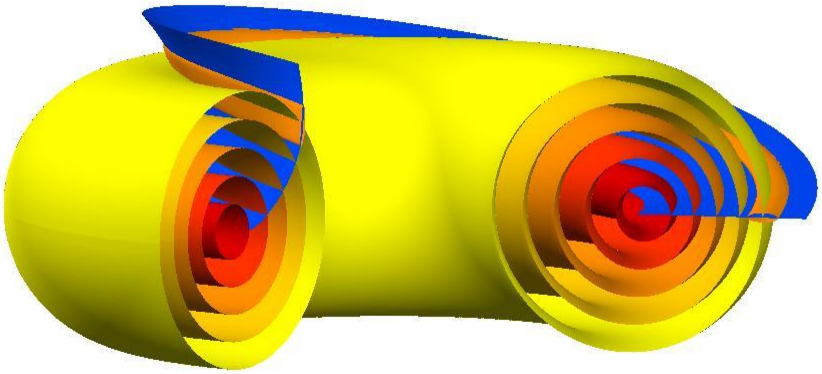
$$B_{in}^2(\theta, r_0, t) = B_0^2(\theta, r) + 2\mu_0\left[p_0(r) - p_0(r_0)\right]$$

$$F_r = \frac{1}{\mu_0}\left[B_{in} \cdot \nabla B_{in} - B_0 \cdot \nabla B_0\right] \cdot e_r$$

- The rest is geometry!

Toroidal geometry

- Flux tube must follow the surface S which is tangent to both the surrounding field lines and the flux tube
- Consider displacement along the $S=0$ surface
- Field line shape will be given by $r=r(\theta,r_0,t)$

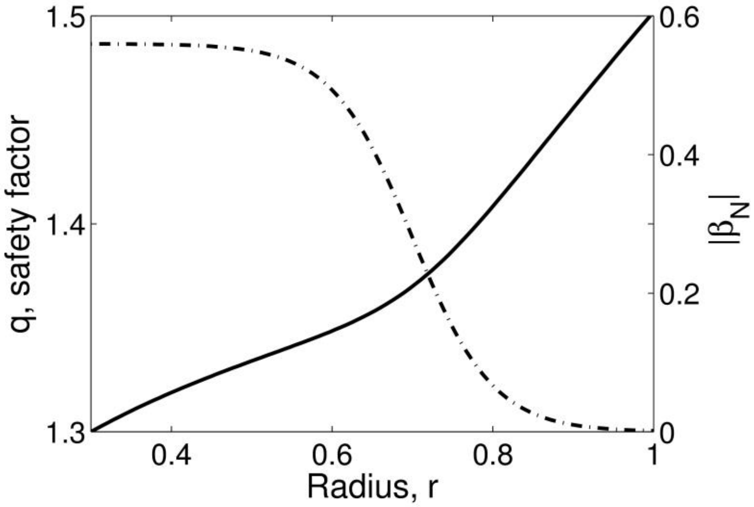


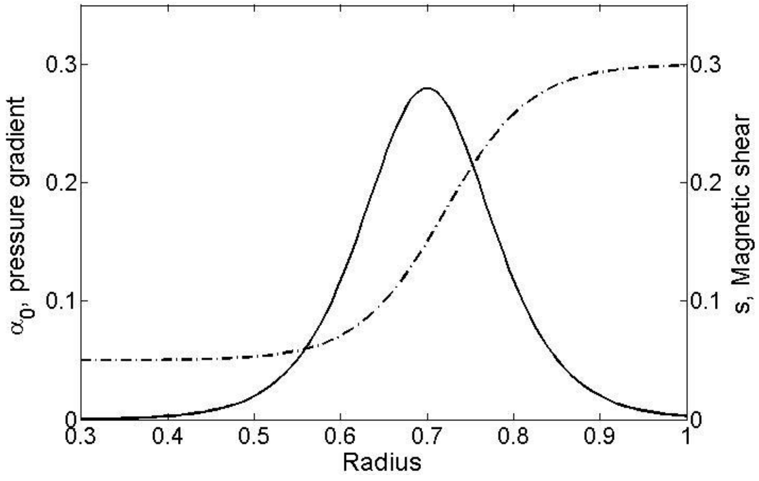
$$B_0 = -\overline{B}_0 R_0 \left\{ f(r) \nabla r \times \nabla S \right\}$$

$$S = \phi - q(r)(\theta - \theta_0(r))$$

Equilibrium

- Large aspect ratio, circular cross section torus
- Region of steep pressure gradient
- Region of change of magnetic shear
- Model of some internal transport barriers





Force equation

- We now use an ‘s-α’ model to produce the required geometry

$$F_r = \frac{1}{\mu_0}\left[B_{in}\cdot\nabla B_{in} - B_0\cdot\nabla B_0\right]\cdot e_r$$

- Becomes

$$F_\perp = \left(\beta_N(r_0) - \beta_N(r)\right)\left[\cos\theta + \sin\theta\left(\alpha\sin\theta - s\theta\right)\right]$$
$$+\left(\frac{\partial}{\partial\theta}\right)_{r_0}\left(\left[1+\left(\alpha\sin\theta - s\theta\right)^2\right]\left(\frac{\partial r}{\partial\theta}\right)_{r_0}\right) - \frac{1}{2}\left(\frac{\partial r}{\partial\theta}\right)^2_{r_0}\left(\frac{\partial}{\partial r}\right)_\theta\left(\alpha\sin\theta - s\theta\right)^2$$

- Saturated states when force is zero.
- Nonlinear generalization of `s- α’ model

where: $s=rq'(r)/q(r)$ – magnetic shear
$\alpha(r)=-d\beta_N/dr$ - pressure gradient

Evolution equation

- An evolution equation can be derived using the previous equation, assuming drag evolution

$$\nu\left(\frac{\partial r}{\partial t}\right)\left[1+(\alpha\sin\theta-s\theta)^2\right]=\left(\beta_N(r_0)-\beta_N(r)\right)\left[\cos\theta+\sin\theta(\alpha\sin\theta-s\theta)\right]$$

$$+\left(\frac{\partial}{\partial\theta}\right)_{r_0}\left(\left[1+(\alpha\sin\theta-s\theta)^2\right]\left(\frac{\partial r}{\partial\theta}\right)_{r_0}\right)-\frac{1}{2}\left(\frac{\partial r}{\partial\theta}\right)_{r_0}^2\left(\frac{\partial}{\partial r}\right)_{\theta}(\alpha\sin\theta-s\theta)^2$$

where: $s=rq'(r)/q(r)$ – magnetic shear

$\alpha(r)=-d\beta_N/dr$ - pressure gradient

Linearized evolution equation

- We can linearize to find usual \`s- α' model

=α

$$F_\perp = \left(\beta_N(r_0) - \beta_N(r)\right)\left[\cos\theta + \sin\theta\left(\alpha\sin\theta - s\theta\right)\right]$$

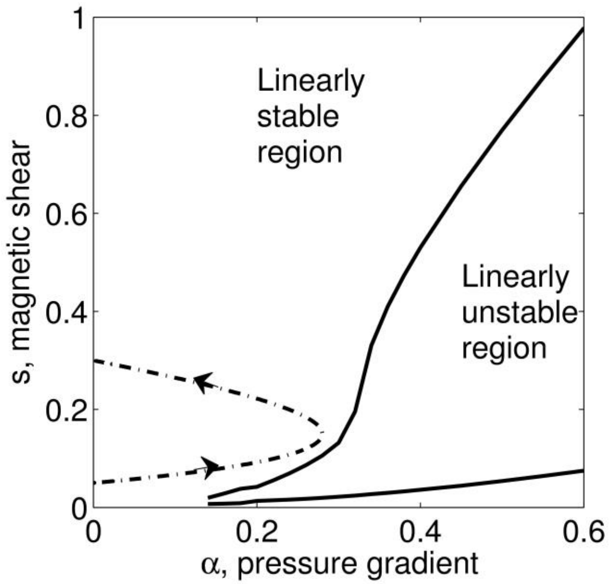
$$+\left(\frac{\partial}{\partial\theta}\right)_{r_0}\left(\left[1+\left(\alpha\sin\theta - s\theta\right)^2\right]\left(\frac{\partial r}{\partial\theta}\right)_{r_0}\right) - \frac{1}{2}\left(\frac{\partial r}{\partial\theta}\right)_{r_0}^2\left(\frac{\partial}{\partial r}\right)_{\theta}\left(\alpha\sin\theta - s\theta\right)^2$$

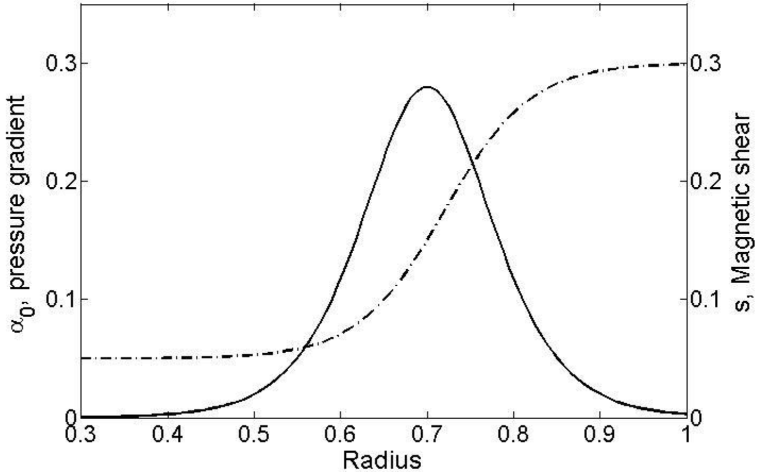
=0

where: $s=rq'(r)/q(r)$ – magnetic shear
$\alpha(r)=-d\beta_N/dr$ - pressure gradient

Linearly stable case

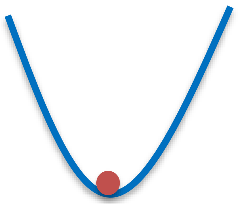
- We investigate a case which is linearly stable across the whole profile but is nonlinearly unstable
- Physical motivation: Pedestal will be held at critical gradient by soft limit i.e. marginal stability



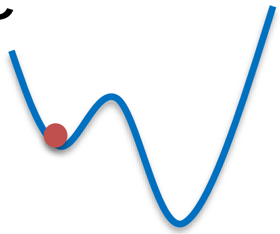


Equilibrium

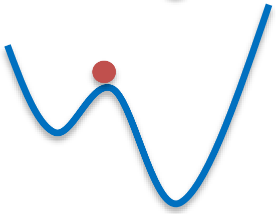
- Quartic potential energy function
- Either trivial equilibrium



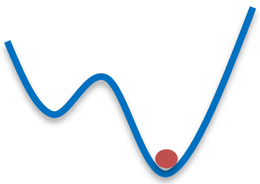
- Or three equilibria available
  – Unperturbed field line



  – A critical field line



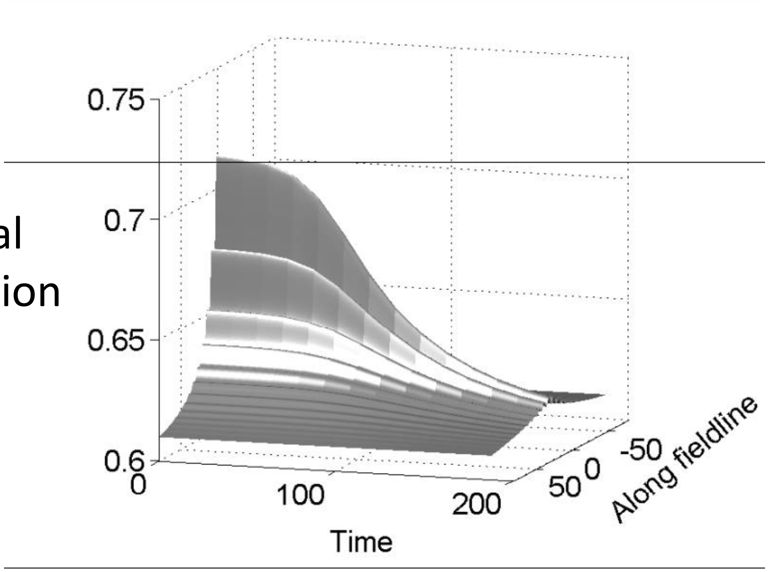
  – A nonlinear saturated state

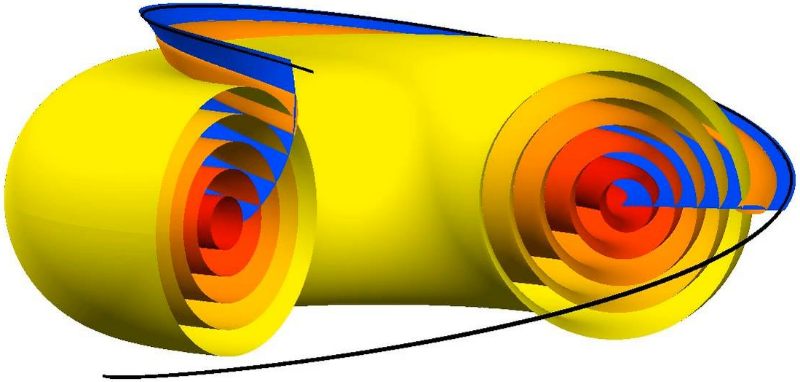


Downwards evolution

- If the field line is initially perturbed just below the critical amplitude the field line drops back to the initial location

Radial position

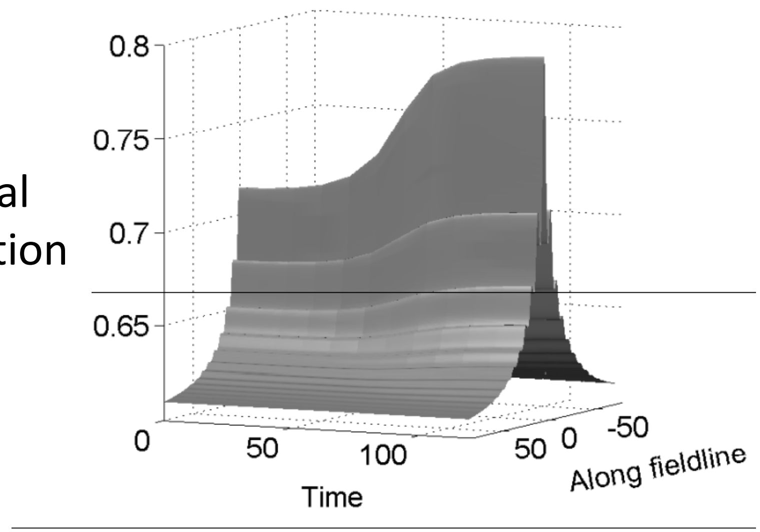


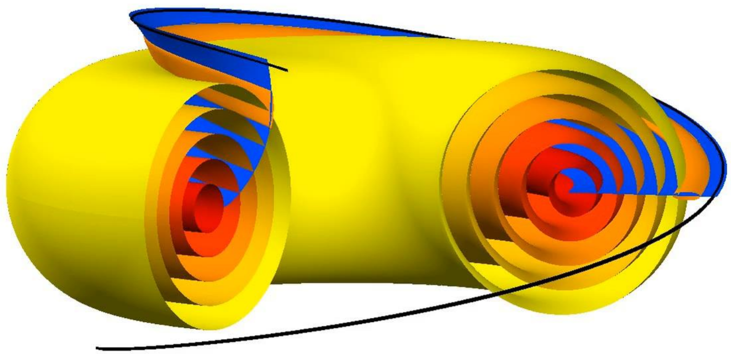


Movie

Upwards evolution

- If the field line is initially perturbed just above the critical amplitude the field line evolves to the saturated state

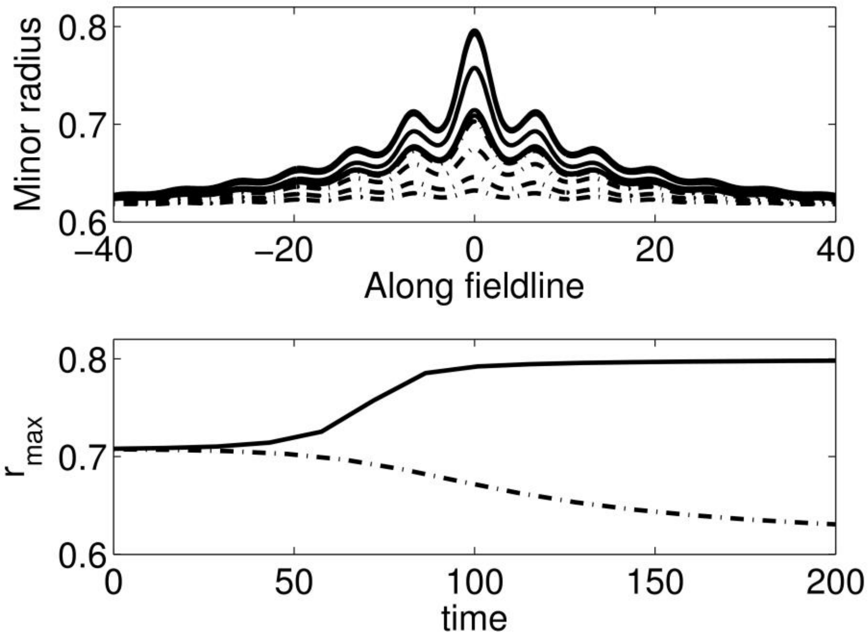




Movie

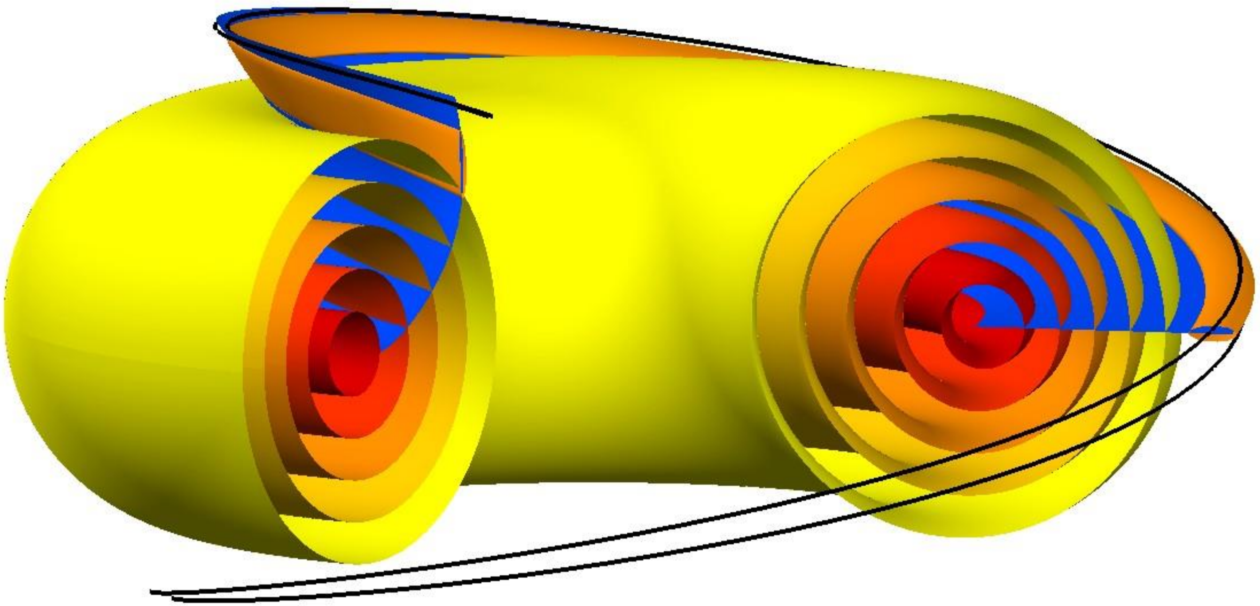
Evolution

- The evolution to the nonlinear saturated state shows explosive behaviour
- No resistivity is included in this model so the field lines remain frozen in



Saturated state

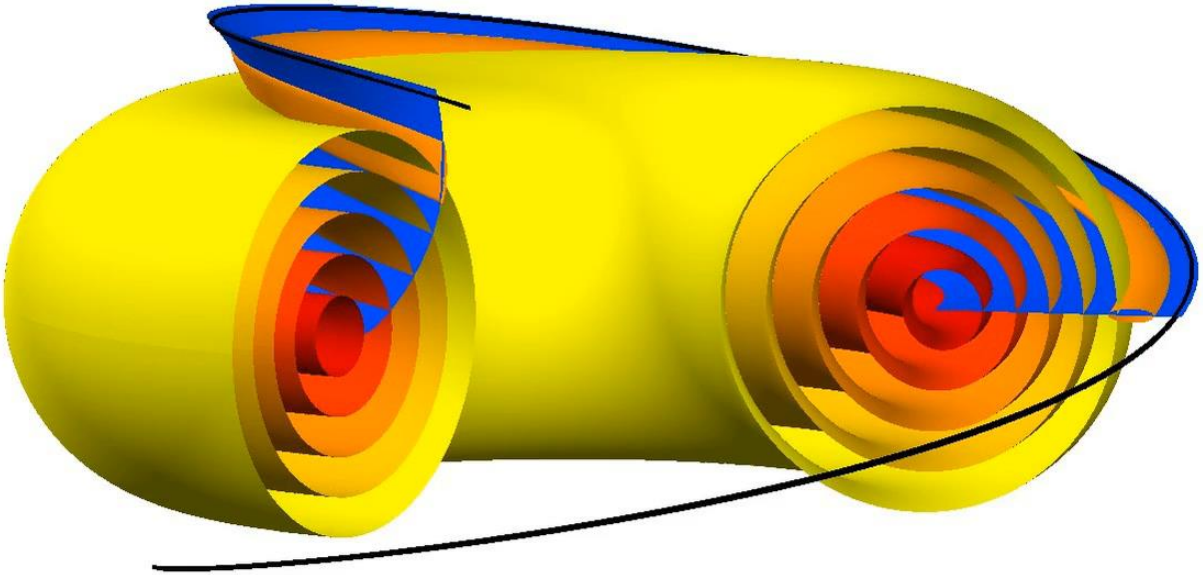
- Saturated state



Colour just for illustration

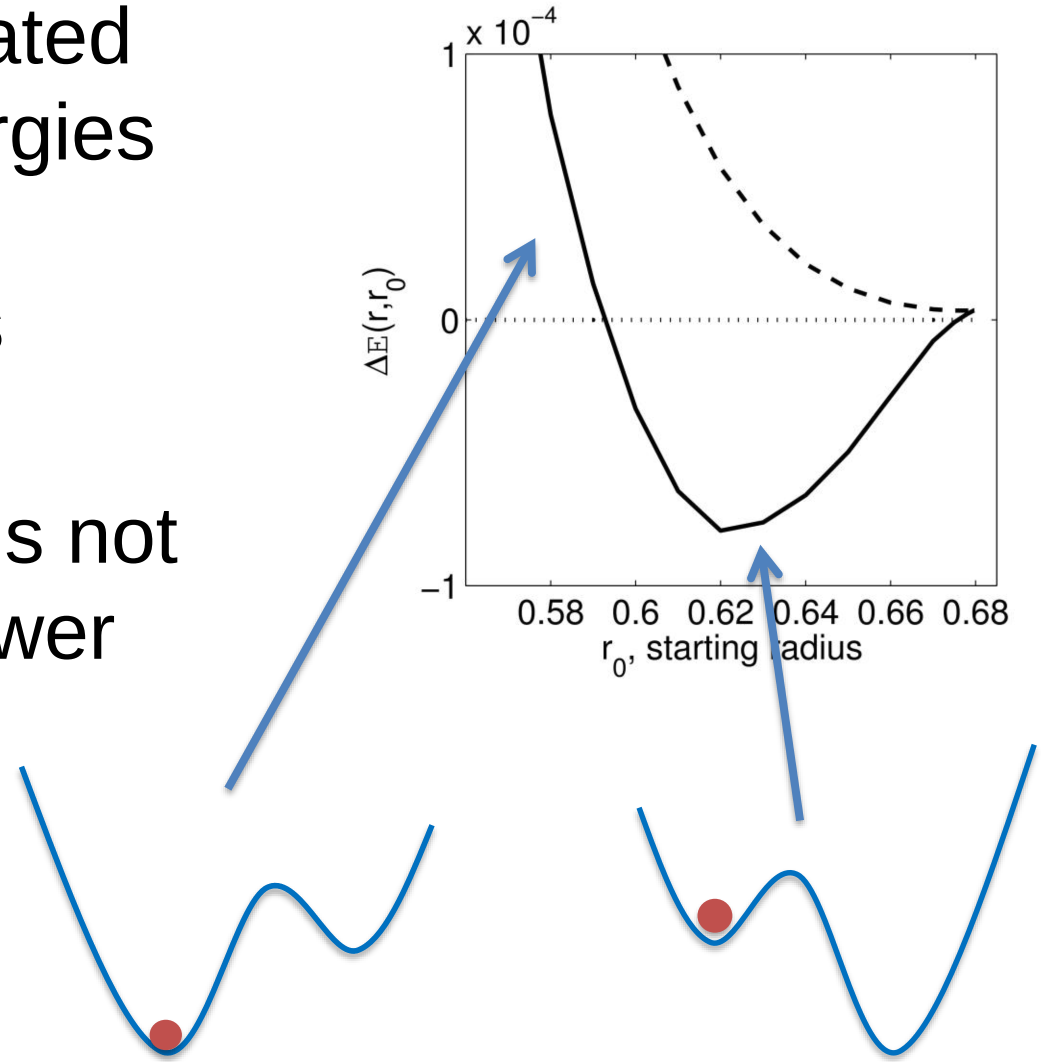
Flux tube dynamics

- Movies of flux tube dynamics



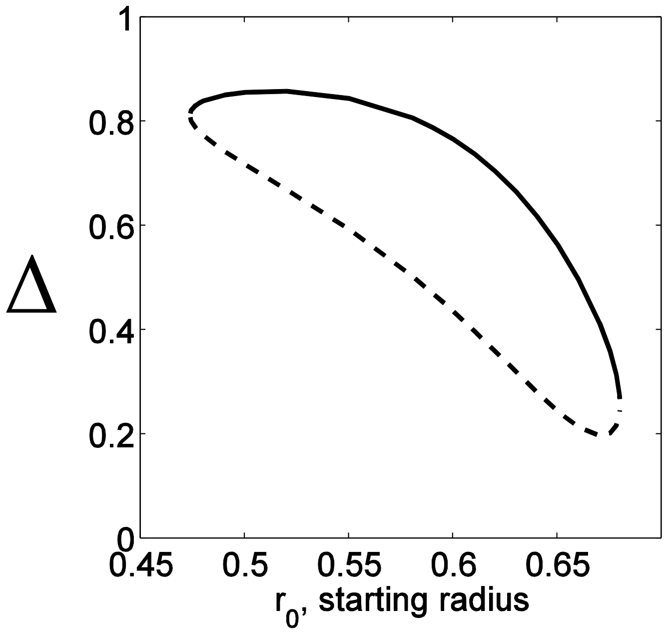
Energy

- We have calculated the relative energies of the initial and saturated states
- The nonlinear saturated state is not necessarily a lower energy state



Ballooning displacement

- The ballooned field lines stretch across most of the pressure step
- Some field lines start closer to the core and end up closer to the edge i.e. they overtake.



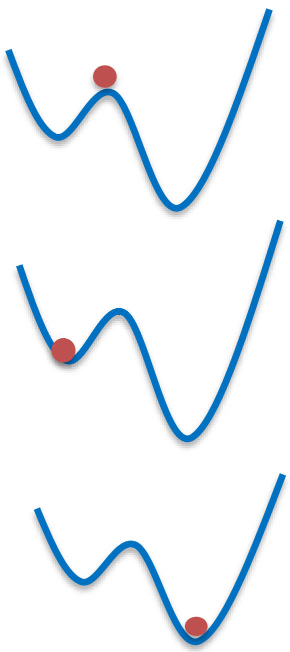
- saturated state

-- critical state

$$\Delta = \frac{\beta_N(r_0) - \beta_N(r_{max})}{2\varepsilon_p \alpha_0}$$

Linearly unstable case

- We could start with a linearly unstable case
- Three equilibria available
  - Unperturbed field line
  - Inward saturated state
  - Outward saturated state



- Field lines balloon inwards and outwards

Discussion

- Drag evolution is an approximation of the real dynamics. However, it is likely to capture key features such as:
  - – explosive dynamics
  - – equilibrium states
- Tube is assumed to have elliptical shape:
  - – mildly nonlinear flux tube results
  - – physical intuition
  - – But, overtaking may change shape

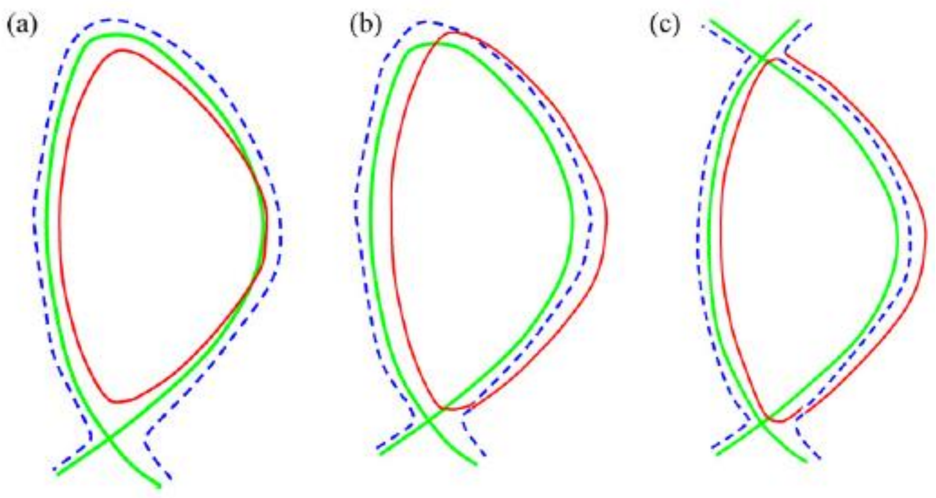
Transport

- Model assumes perfectly conducting plasma i.e. no reconnection
- Good for fast eruption but other processes will take over once in saturated state:
  - Disconnection
  - Cross field transport (`leaky hosepipe')
  - Secondary instability (K-H, ITG etc.)
- Where does reconnection occur (if it occurs)?
- Future work required

Wilson *et al.* Plasma Physics Control. Fusion **48** (2006) A71

Realistic geometry

- Work in progress to look at real tokamak equilibria
- Numerical circular cross section equilibrium has been successfully modelled and saturated states found
- Next step to look at the effect of the separatrix on ideal MHD saturated states



Wilson *et al.* Plasma Physics Control. Fusion **48** (2006) A71

Numerical work

- Potentially difficult to capture these filaments in nonlinear MHD codes
- Narrowing of the ballooning mode structure is important and would require very high toroidal resolution

Conclusion

- We have produced a nonlinear version of the ballooning `s-α' model
- We have shown that a linearly stable profile may still allow nonlinear instabilities
- The resulting flux tubes can have displacements large enough to connect the two sides of the transport barrier
- This may be a mechanism to explain elements of fast disruptions of TFTR ITB shots and ELMs