An analytical model for scrape-off layer plasma dynamics at arbitrary collisionality

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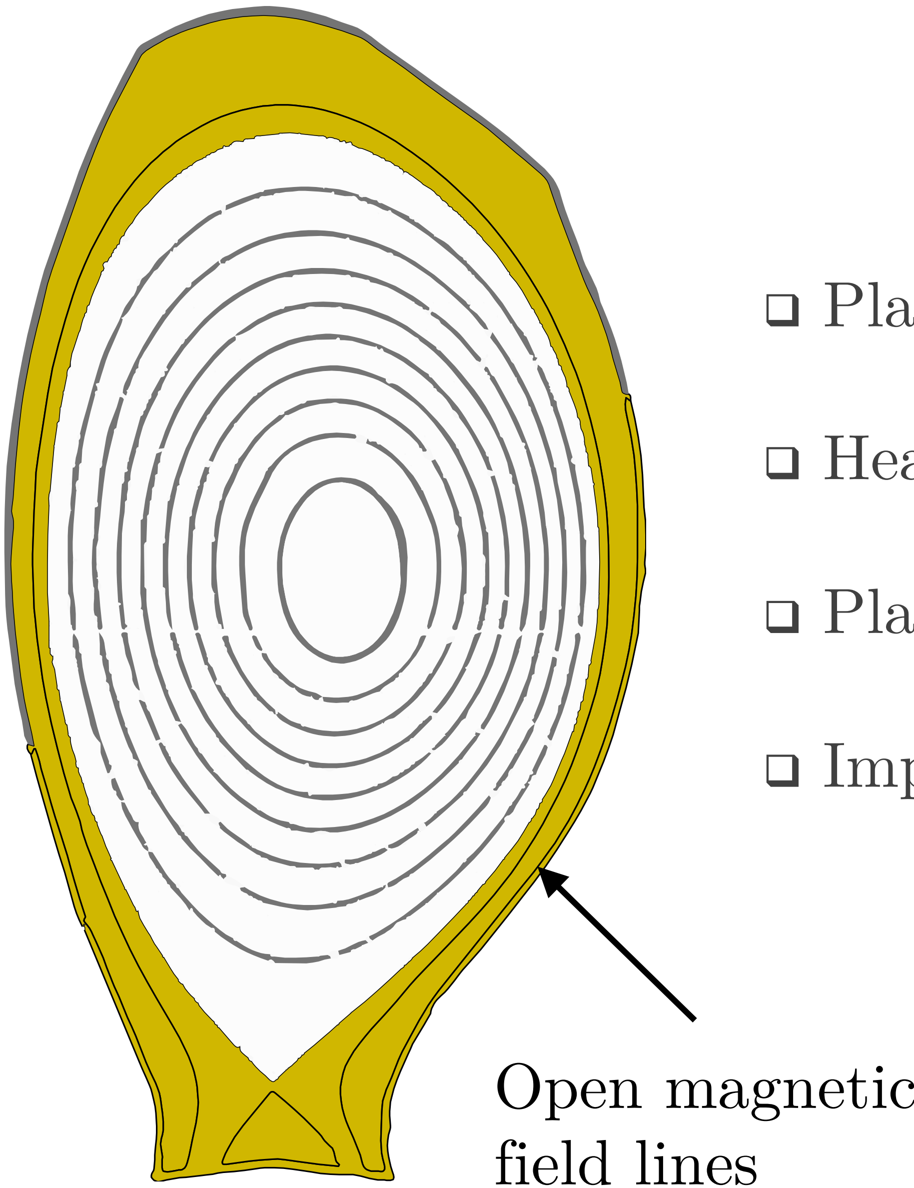
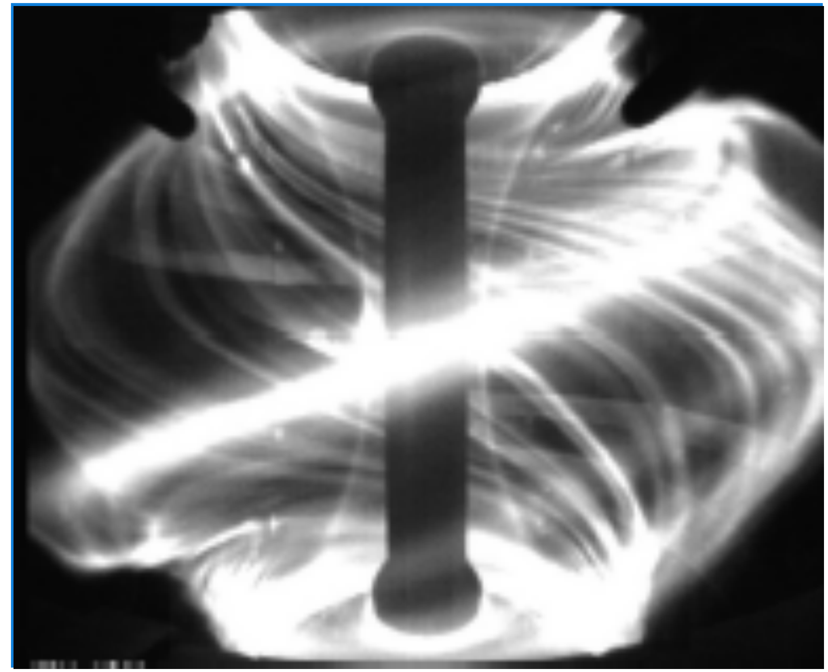
The Scrape-off Layer (SOL)



- Plasma boundary conditions
- Heat exhaust
- Plasma fueling and ashes removal
- Impurity control

Properties of SOL Turbulence



- Large structures
- Field aligned

$$\rho_i \ll L_\perp \ll L_\parallel$$

- No separation between equilibrium and fluctuations

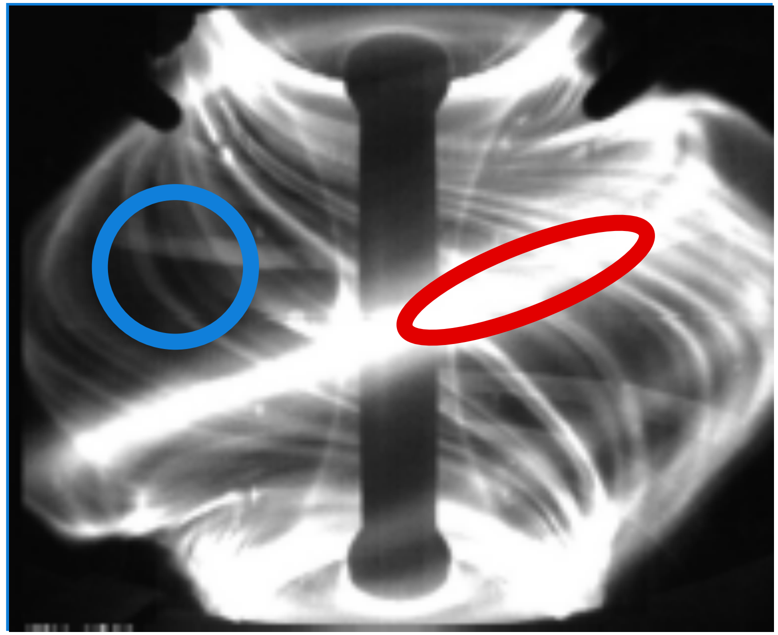
$$\langle n \rangle_t \sim \tilde{n} \qquad L_{\langle n \rangle_t} \sim L_{\tilde{n}} \qquad \frac{e\phi}{T_e} \sim 1$$

- $T \sim 5 - 200 \text{ eV}$

Is the SOL Collisional?

$$T_e = 5 \text{ eV}$$

$$\frac{\lambda_{\text{mfp}}}{L_{\|}} \simeq 0.002$$

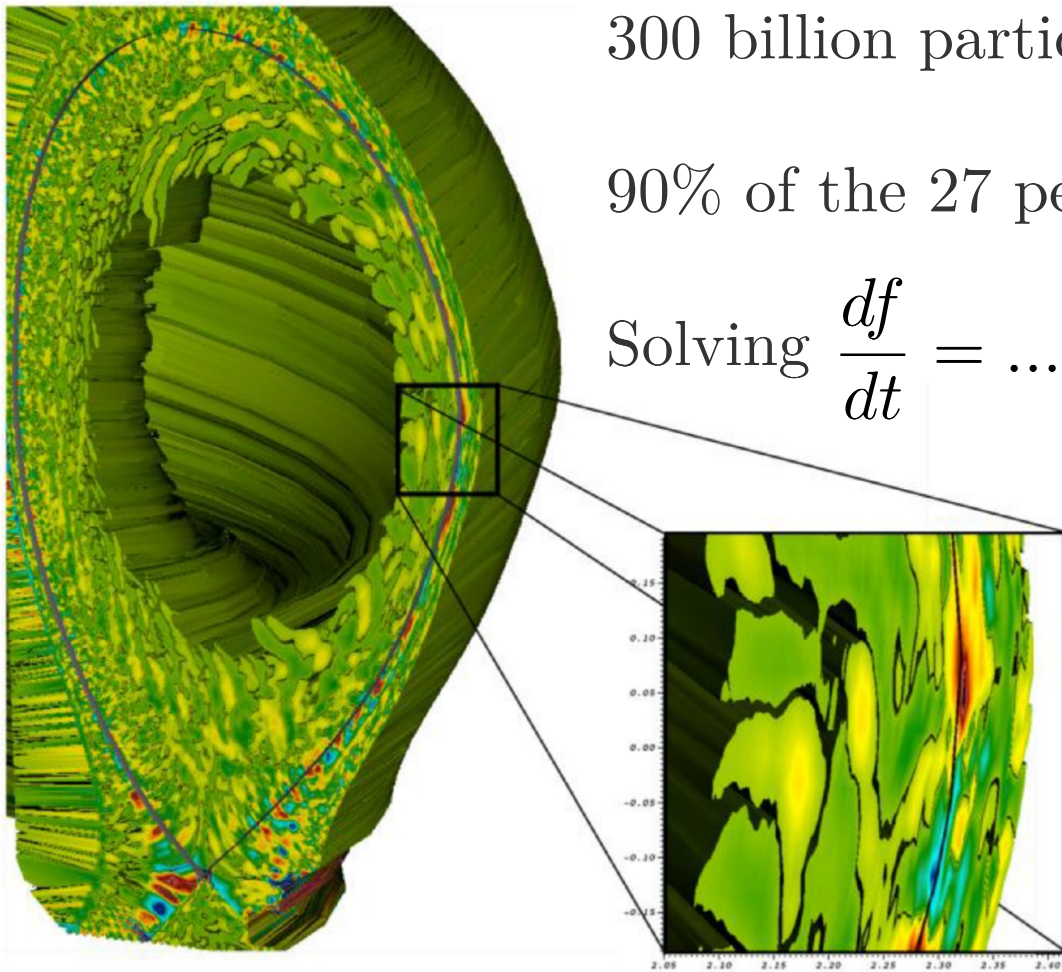


$$T_e = 200 \text{ eV}$$

$$\frac{\lambda_{\text{mfp}}}{L_{\|}} \simeq 3$$

Extremely different collisionality regimes!

Kinetic Simulations (collisionless + collisional)



300 billion particles

90% of the 27 petaflop Titan supercomputer

Solving $\frac{df}{dt} = \ldots$

Extremely Expensive

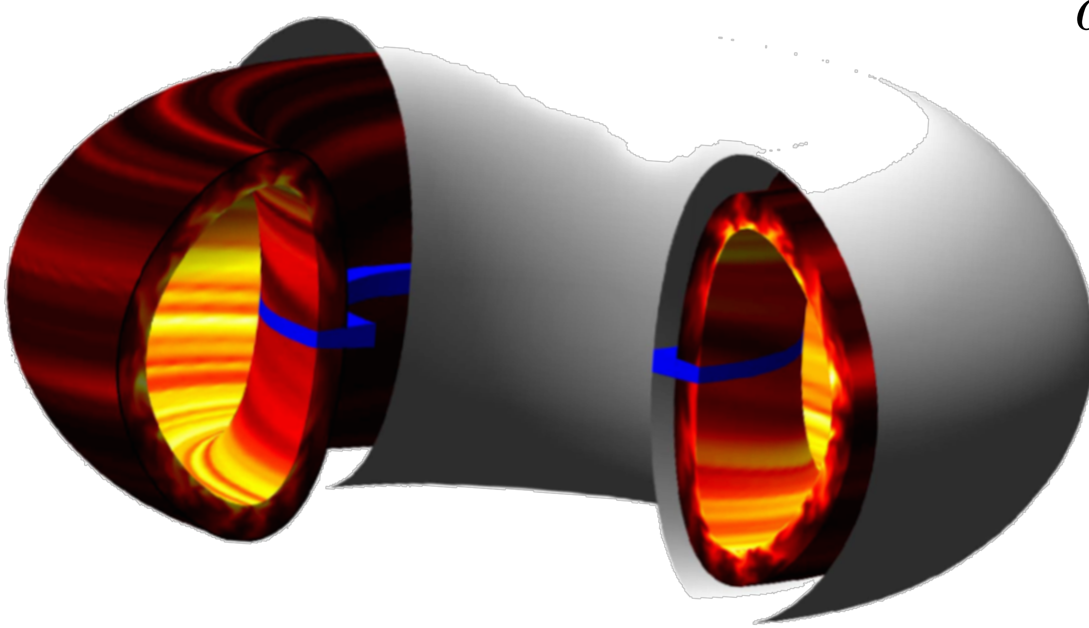
XGC1 code
Chang et. al., Nuclear Fusion **57** (2017)

Fluid Simulations (collisional)

Considerably less expensive

SOL Confinement time scales

Solving $\frac{dn}{dt} = ...$



Assume High Collisionality

GBS code
Ricci et. al., Plasma Phys. Controlled Fusion **54** (2012)

Our Goal – Develop a Model

- Retain necessary kinetic effects (and no more)
- Remain numerically tractable

↓

Hierarchy of Fluid Equations

Outline

Kinetic → Drift-Kinetic → Moment Hierarchy → Closure

while retaining

- Full-F, $\langle n \rangle_t \sim \tilde{n}$
- Full Coulomb collisions
- Simple Maxwellian (collisional) limit

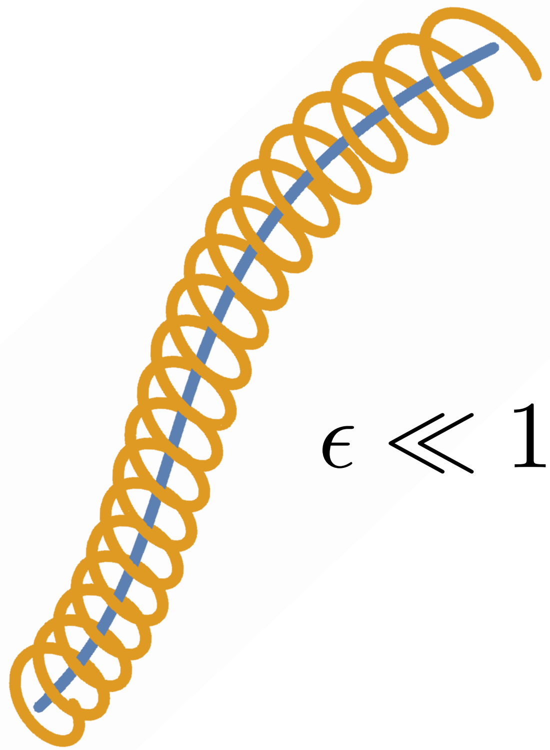
Kinetic Model – Our Ordering Assumptions

Spatial Scale
(Drift-Kinetic)

$$k_\perp \rho_i \sim \epsilon$$

Temporal Scale
(low frequency)

$$\frac{\omega}{\Omega_i} \sim \epsilon$$

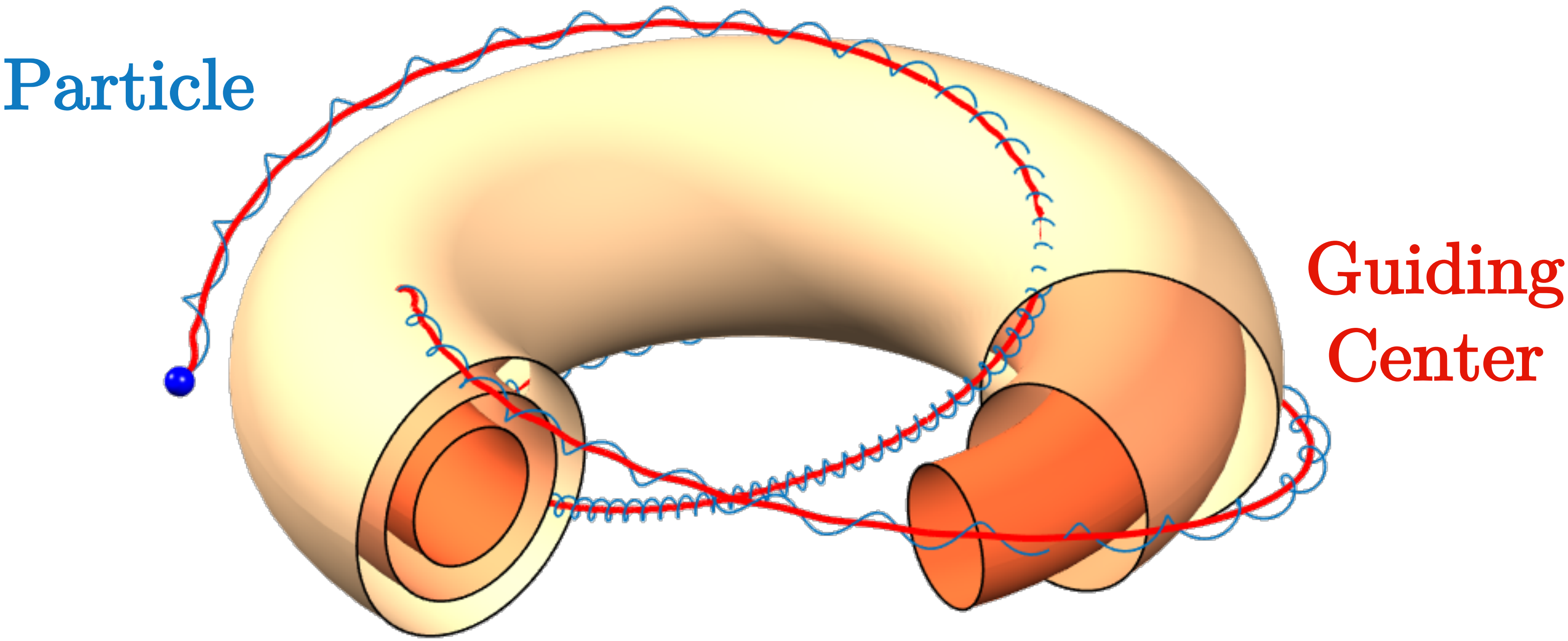


$$\epsilon \ll 1$$

From collisionless to
collisional (still magnetized)

$$\frac{\nu_{ei}}{\Omega_i} \sim \epsilon_\nu \lesssim \epsilon$$

From Full Particle Dynamics to Guiding Center

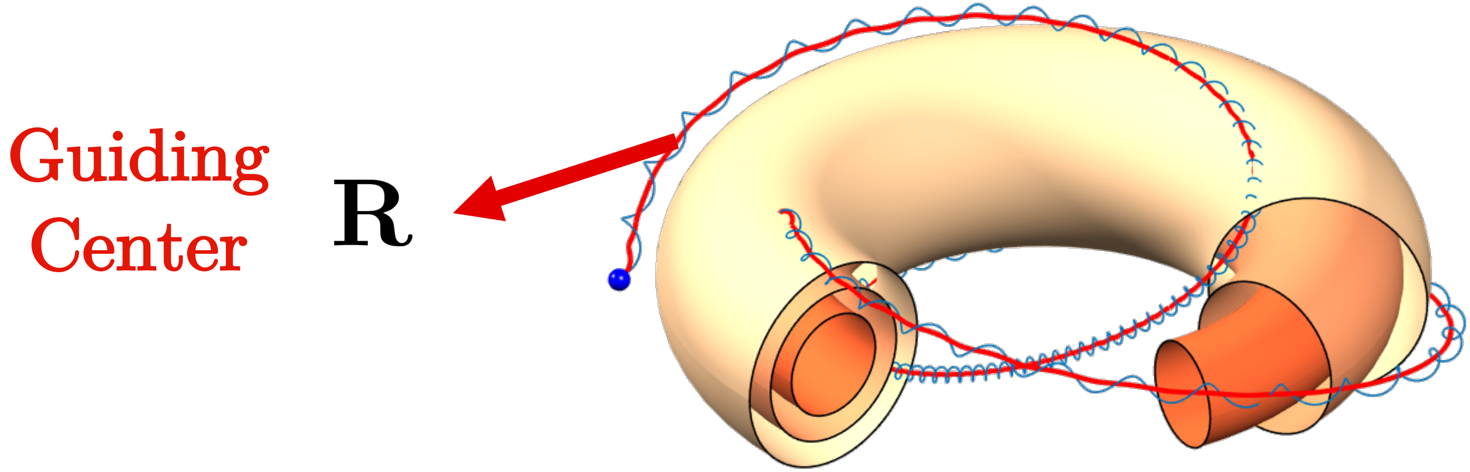


$$L = (q\mathbf{A} + m\mathbf{v}) \cdot \dot{\mathbf{x}} - q\phi - \frac{m_a v^2}{2}$$

↓

Average out fast gyromotion

Guiding Center Equations of Motion



$$\dot{\mathbf{R}} = v_{\|}\mathbf{b} + \mathbf{v}_{\mathbf{E}\times\mathbf{B}} + \text{Other Drifts}$$

$$\dot{v}_{\|} = qE_{\|} + \mu\nabla_{\|}B + \text{Non-Linear Forces}$$

$$\dot{\mu} = 0$$

From Single-Particle to Particle Distribution

Drift-Kinetic Equation

$$\frac{\partial F}{\partial t} + \dot{\mathbf{R}} \cdot \nabla F + \dot{v}_{\|} \frac{\partial F}{\partial v_{\|}} = \langle C(F) \rangle$$

Challenges

F- Full gyroaveraged distribution function

- 5-D + time
- Full Coulomb Collisions

These challenges can be successfully approached by using a moment hierarchy

From DK Equation to Moment Hierarchy

$$\int \boxed{\text{DK Eq.}} (v_{\|})^p (\mu)^j \, dv_{\|} \, d\mu$$

$$(p, j) = (0, 0) \qquad \frac{\partial n}{\partial t} = \ldots$$

$$(p, j) = (1, 0) \qquad \frac{\partial u_{\|}}{\partial t} = \ldots$$

$$\vdots$$

Orthogonal Basis for the velocity space

$$F = F_M \sum_{p,j} N^{pj}(\boldsymbol{R}) H_p(v_{||}) L_j(\mu)$$

Projection coefficients

Orthogonal Basis

Simple expression for

$$N^{pj}(\mathbf{R}) = \int F \; H_p(v_{||}) L_j(\mu) dv_{||} d\mu$$

$=$ moments of F

The choice of basis

Orthogonal with Maxwellian
as weighting function

Hermite Polynomials

$$\int H_p(v_{\|})H_l(v_{\|})e^{-v_{\|}^2}dv_{\|} = \delta_{p,l}$$

Laguerre Polynomials

$$\int L_j(\mu)L_k(\mu)e^{-\mu}d\mu = \delta_{j,k}$$

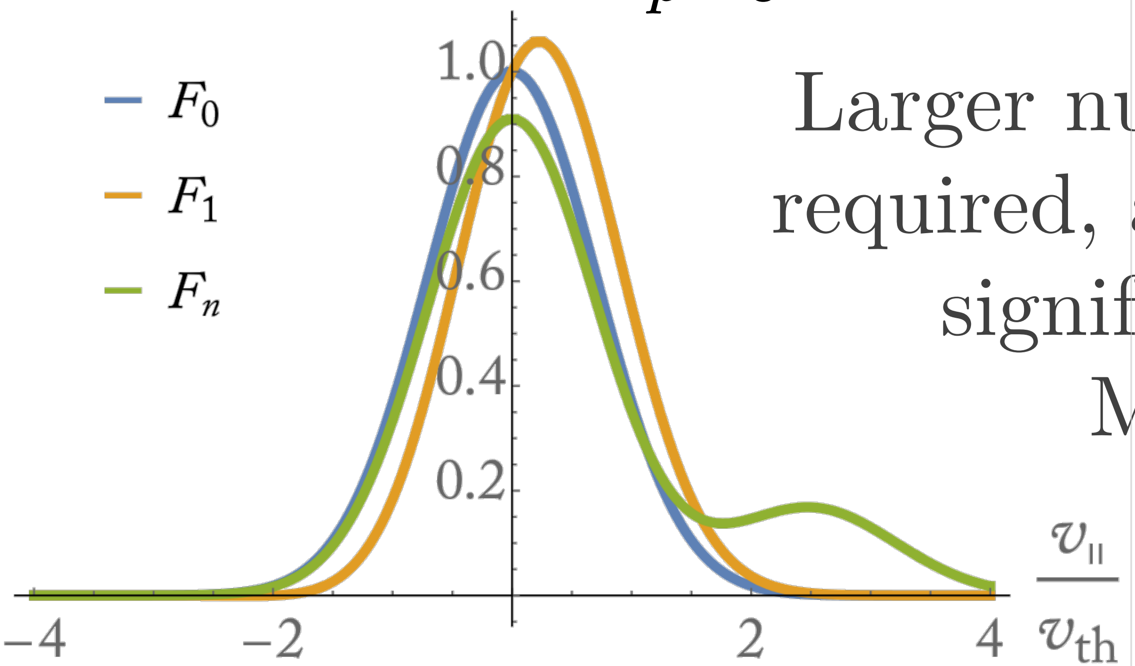
Most efficient representation of kinetic effects

$$F_0 = f_M$$

$$F_1 = f_M \times (1 + N^1 v_{\|})$$

$$\vdots$$

$$F_n = F_M \times \sum_{p=0}^{n} N^p H_p(v_{\|})$$



Larger number of moments required, as F deviates more significantly from a Maxwellian

From DK Equation to Moment Hierarchy

$$\int (\text{DK Eq.}) H_p L_j dv_\parallel d\mu$$

Spatial evolution of Moments + Fields

Fluid Operator (density, velocity, temperature)

$$\frac{\partial N^{pj}}{\partial t} + \boxed{\nabla \cdot \dot{\mathbf{R}}^{pj}} - \boxed{\frac{\sqrt{2p}}{v_{th}} \dot{v}_\parallel^{p-1j}} + \boxed{\mathcal{F}^{pj}} = \boxed{C^{pj}}$$

Forces included at p>0

Collisions (which may be complicated...)

From DK Equation to Moment Hierarchy

$$\int (\text{DK Eq.}) H_p L_j \, dv_{\|} \, d\mu$$

- Phase-Mixing $\sim N^{p+1j}, N^{p-1j}, N^{pj+1}, \ldots$
- Coupling with EM fields $\sim \boldsymbol{v}_{\boldsymbol{E}\times\boldsymbol{B}} \cdot \nabla N^{pj}$
- Lowest order fluid equations $\dfrac{\partial N^{00}}{\partial t} + \nabla \cdot (N^{00}\mathbf{u}) = 0$
- Collisions $C^{pj} \quad = \ldots$

Example – 1D Linear Drift-Kinetic

$$\frac{\partial F}{\partial t} + v_{\|} \frac{\partial F}{\partial z} + E_{\|} \frac{\partial f_M}{\partial v_{\|}} = C(F)$$

In Hermite space $\quad F = f_M \sum_p N^p H_p(v_{\|})$

Phase Mixing
(coupling with other moments)

Collisions
(which may be
complicated...)

$$\boxed{\frac{\partial N^p}{\partial t}} + \boxed{\frac{1}{2}\frac{\partial N^{p+1}}{\partial z} + p\frac{\partial N^{p-1}}{\partial z}} = \boxed{2\delta_{p,1} E_{\|}} + \boxed{C^p}$$

Time Evolution

Electric Field Drive

Projection of the Collision Operator

$$C^{pj} = \int \langle C(F) \rangle \; H_p \; L_j \; dv_{\|} d\mu$$

In general $$C(F) = \frac{\partial}{\partial \mathbf{v}} (\mathbf{A} F) + \frac{\partial^2}{\partial \mathbf{v} \partial \mathbf{v}} : (\mathbf{D} F)$$

Projection of the Collision Operator

$$C^{pj} = \int \langle C(F) \rangle \ H_p \ L_j \ dv_\parallel d\mu$$

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Example: Lenard-Bernstein Operator

Lenard & Bernstein
Phys. Rev. 112 (1958)

$\mathbf{A} = \mathbf{v}$ and $\mathbf{D} = \mathbf{I} v_{th}^2$

$$C^{pj} = -(p + 2j) N^{pj}$$

Projection of the Collision Operator

$$C^{pj} = \int \langle C(F) \rangle \; H_p \; L_j \; dv_{\|} d\mu$$

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$\mathbf{A} = \mathbf{v}$ and $\mathbf{D} = \mathbf{I} v_{th}^2$

$$C^{pj} = -(p + 2j) N^{pj}$$

Assumes constant collision frequency in velocity space...
Need for Full Coulomb collision operator!

Full Coulomb Collision Operator

$$C^{pj} = \int \langle C(F) \rangle \;\; H_p \; L_j \; dv_{\|} d\mu$$

Not immediate...

Eigenfunctions of the collision operator

If $F \sim \sum P_l\left(\frac{v_\|}{v}\right) L_k^{l+1/2}(v^2)$

Pitch Angle Scattering — Braginskii solution (collision integral)

then $C(F) \sim \sum P_l\left(\frac{v_\|}{v}\right) L_k^{l+1/2}(v^2)$

However, v and $v_\|/v$ are not DK variables...

Change of Basis

Find an analytical expression

$$P_l\left(\frac{v_\parallel}{v}\right)L_k^{l+1/2}(v^2) \sim \boxed{T_{lk}^{pj}} H_p(v_\parallel)L_j(\mu)$$

Change of basis

so that

$$C^{pj} = \int \langle C(F)\rangle \; H_p \; L_j \; dv_\parallel d\mu$$

can be analytically evaluated!

Moment of Collision Operator

$$C^{pj} = \int \langle C(F) \rangle \; H_p \; L_j \; dv_{\|} d\mu$$

After integration

$$C^{pj}_{ab} = \sum T \; N^{lk}_a \; N^{nq}_b$$

Infinite Moment Hierarchy: How to Close?

Truncation

$$N^{00} \quad N^{10} \quad N^{20} \quad \ldots \quad N^{p_{\max} j_{\max}} = 0$$

Hard to control:

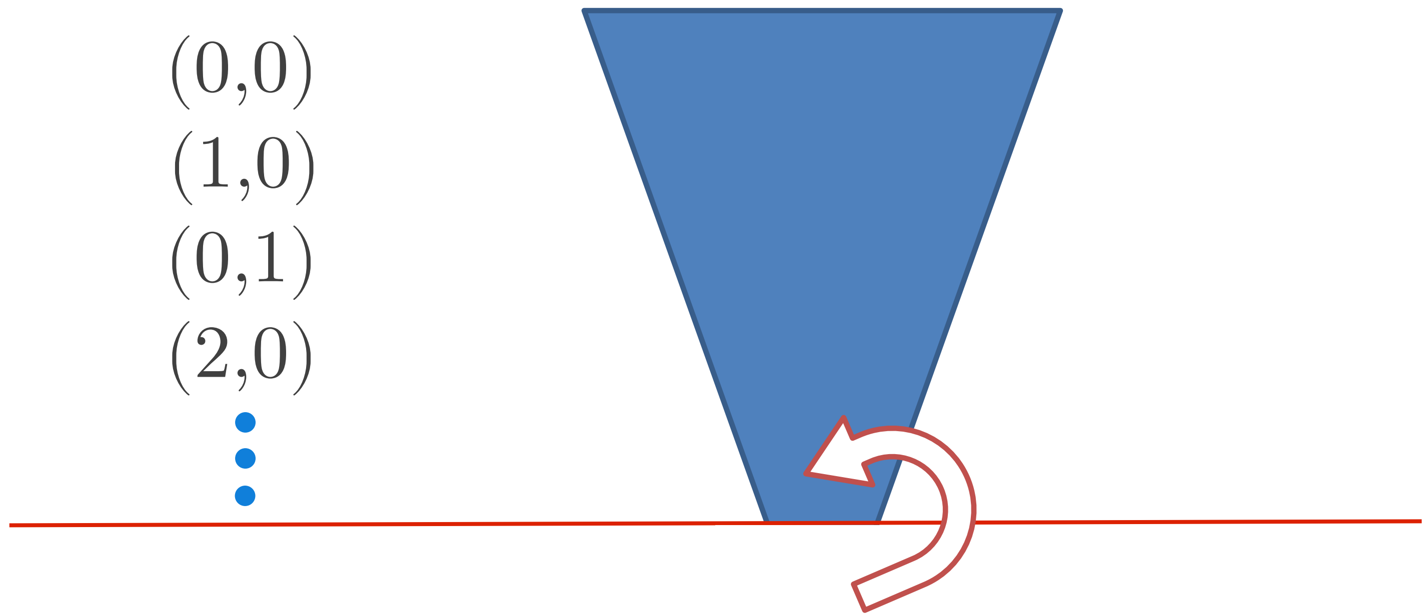
- Large number of moments needed
- Recurrence problem
- ...

Infinite Moment Hierarchy: How to Close?

Option 2

Semi-collisional closure

$$\frac{\partial N^{pj}}{\partial t} \Big/ C^{pj}$$



Dissipation range

Use result of dissipation range in previous phase-mixing terms

Infinite Moment Hierarchy: How to Close?

Chapman-Enskog, Braginskii Closure

$$F = F_M(1 + \delta F)$$

$$\frac{\delta F}{F_M} \sim \frac{\lambda_{\mathrm{mfp}}}{L_{||}}$$

$$\sum_{p,j} N^{pj} H_p L_j$$

Infinite Moment Hierarchy: How to Close?

Chapman-Enskog, Braginskii Closure

$$F = F_M(1 + \delta F)$$

$$\frac{\delta F}{F_M} \sim \frac{\lambda_{\mathrm{mfp}}}{L_{||}}$$

$$\delta F \to \sum_{p,j} N^{pj} H_p L_j$$

$$\frac{d}{dt}\begin{pmatrix} n \\ u_{||} \\ T \\ \mathrm{Drive} \\ \mathrm{Drive} \\ 0 \end{pmatrix} + \frac{d}{dt}\begin{pmatrix} 0 \\ 0 \\ 0 \\ N^{30} \\ N^{11} \\ N^{21} \end{pmatrix} = \nu \begin{pmatrix} 0 \\ C^{10} \\ C^{01} \\ C^{30} \\ C^{11} \\ C^{21} \end{pmatrix}$$

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$$\frac{\delta F}{F_M} \sim \frac{\lambda_{\text{mfp}}}{L_{\|}} \qquad \delta F \rightarrow \sum_{p,j} N^{pj} H_p L_j$$

$$\overset{\sim\,\omega}{\frac{d}{dt}\begin{pmatrix} n \\ u_{\|} \\ T \\ \text{Drive} \\ \text{Drive} \\ 0 \end{pmatrix}} + \overset{\sim\,\omega\delta}{\frac{d}{dt}\begin{pmatrix} 0 \\ 0 \\ 0 \\ N^{30} \\ N^{11} \\ N^{21} \end{pmatrix}} = \overset{\sim\,\nu\delta}{\nu \begin{pmatrix} 0 \\ C^{10} \\ C^{01} \\ C^{30} \\ C^{11} \\ C^{21} \end{pmatrix}}$$

Infinite Moment Hierarchy: How to Close?

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$$F = F_M(1 + \delta F)$$

$$\frac{\delta F}{F_M} \sim \frac{\lambda_{\mathrm{mfp}}}{L_{\|}} \qquad \delta F \rightarrow \sum_{p,j} N^{pj} H_p L_j$$

$$C^{30} = v_{th} \nabla_{\|} T$$

$$C^{21} = 0 \qquad \longrightarrow \qquad N^{30} \sim q_{\|} = -\chi_{\|} \nabla_{\|} T$$

$$C^{12} = 0$$

$$\vdots$$

Infinite Moment Hierarchy: How to Close?

Chapman-Enskog, Braginskii Closure

$$F = F_M(1 + \delta F)$$

$$\frac{\delta F}{F_M} \sim \frac{\lambda_{\mathrm{mfp}}}{L_{||}}$$

$$\rightarrow \sum_{p,j} N^{pj} H_p L_j$$

- Improved Drift-Reduced Braginskii Equations
- Full Coulomb collision effects
- Proper treatment of particle density vs. guiding-center density leads to
  - Transport coefficients with parallel/perpendicular temperature dependence
  - Polarization effects due to particle moments vs. guiding center moments

Summary

Systematic inclusion of kinetic effects in a 3D model in the low/high collisionality regime

- Tune the number of moments according to the level of collisionality
- Most efficient representation of kinetic effects (deviation from a Maxwellian)
- Set of moment equations with reasonable computationally cost
- Improvement over drift-reduced Braginskii equations
- Generalizable to a gyrokinetic theory

arXiv:1709.01411 – accepted for publication in JPP

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