TRANSPORT IN STELLARATORS

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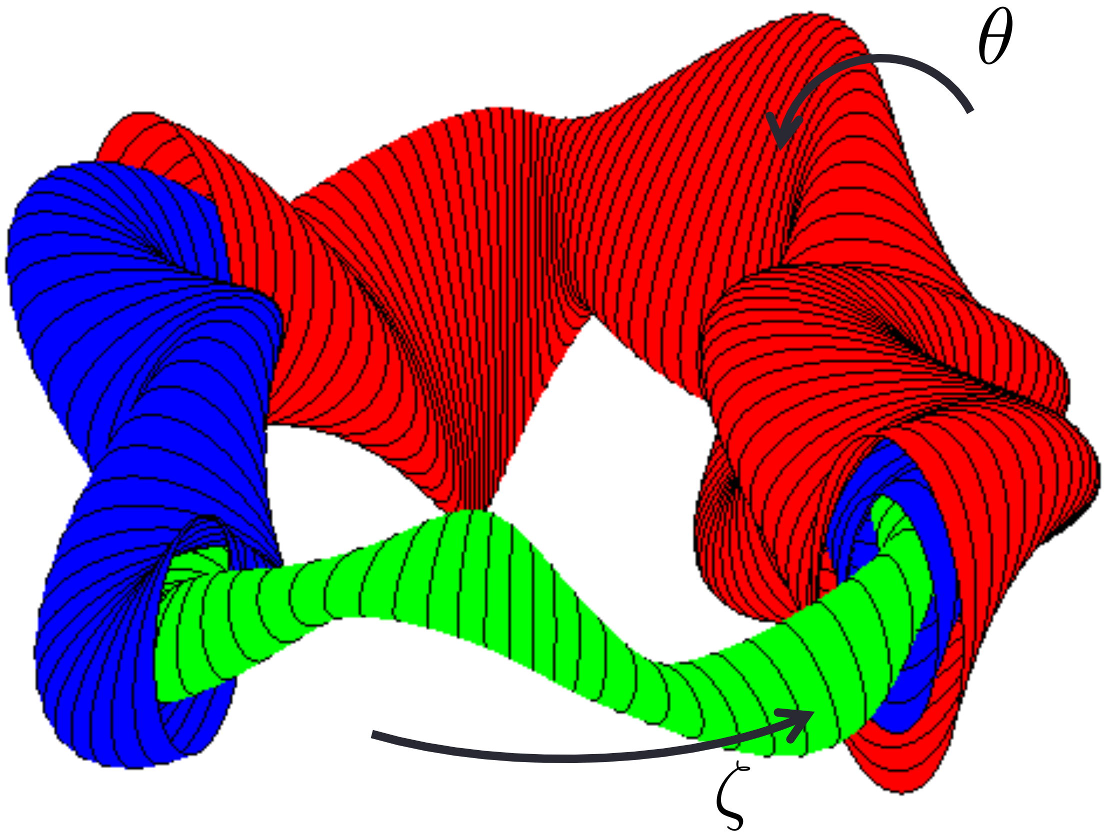
3D magnetic fields in stellarators

- Magnetic field $\mathbf{B}$ must be 3D (that is, without direction of symmetry) if we want
  - steady state
  - nested flux surfaces (surfaces || to $\mathbf{B}$)
  - $\mathbf{B}$ (mostly) generated by external currents

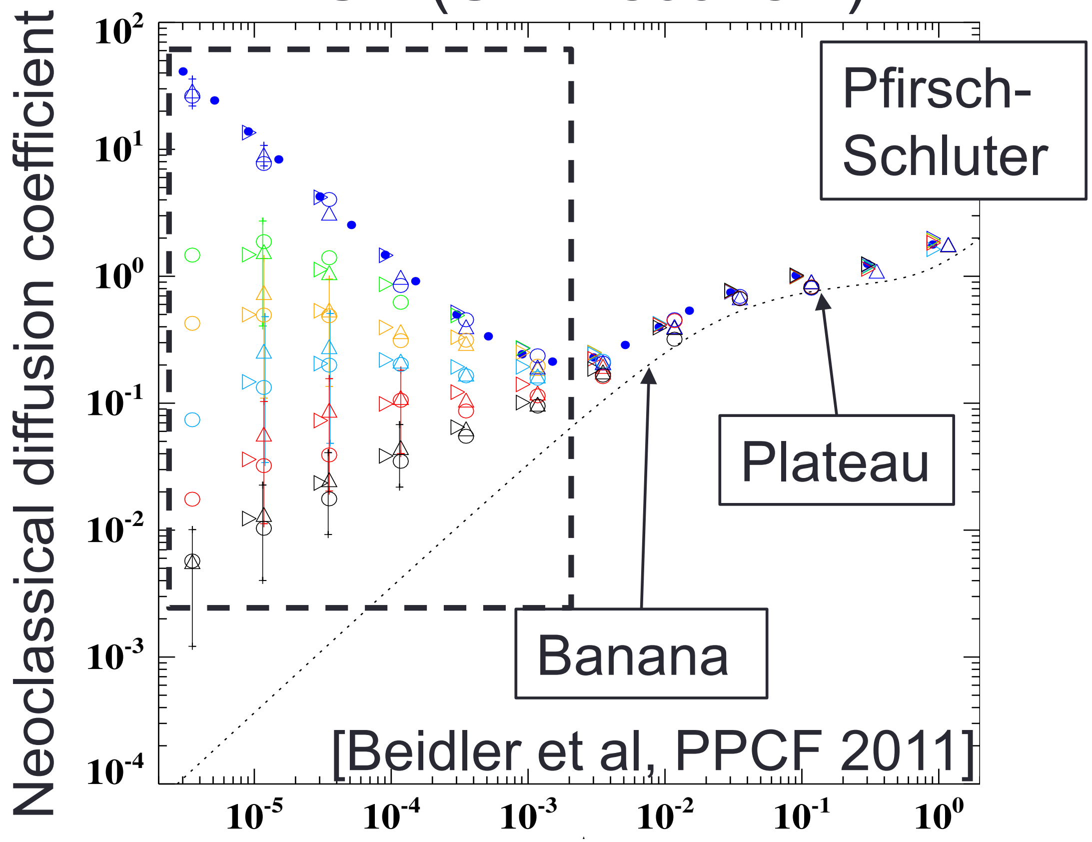


- Stellarators have inherent advantages
  - No current in the plasma $\Rightarrow$ no current drive, no current instabilities

Neoclassical transport in stellarators

- Important differences between stellarators and tokamaks at small collision frequency ν
- Stellarator particle orbits very different from tokamak orbits

HSX (U. Wisconsin)



$$\nu_* = \frac{qR\nu}{v_t}$$

Magnetized particle motion

- Assume steady state $\mathbf{E}$ and $\mathbf{B}$: $\mathbf{E} = -\nabla\phi \sim T/eL$
- Constant total energy

$$\mathcal{E} = \frac{v^2}{2} + \frac{Ze\phi}{m}$$

- Motion for $\rho_* = \rho/L << 1$
  - Magnetic moment (= adiabatic invariant) is constant

$$\mu = \frac{v_\perp^2}{2B} + O\left(\rho_* \frac{v_t^2}{B}\right)$$

  - Motion = fast parallel streaming + slow perpendicular drifts

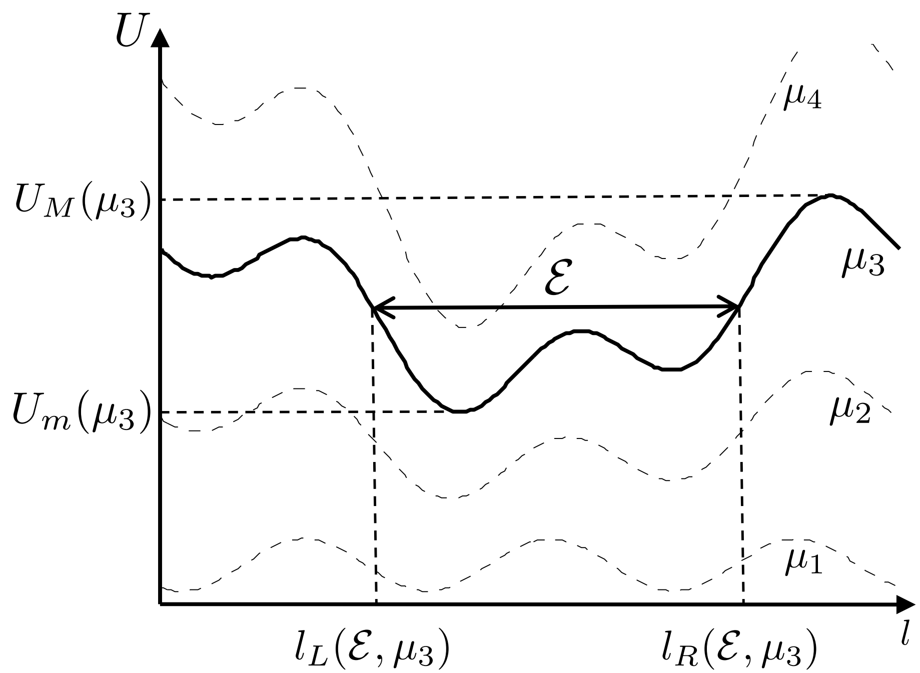
$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \underbrace{v_\parallel \hat{\mathbf{b}}}_{\sim v_t} + \underbrace{\mathbf{v}_E + \mathbf{v}_{\nabla B} + \mathbf{v}_\kappa}_{= \mathbf{v}_d \sim \rho_* v_t}$$

Parallel motion

- To lowest order, particles move along magnetic field lines

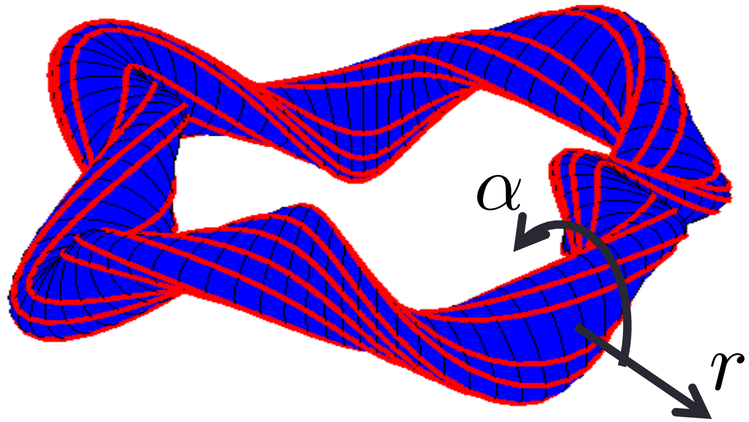
$$\frac{\mathrm{d}l}{\mathrm{d}t} = v_{\|} = \pm\sqrt{2\left(\mathcal{E} - \mu B(l) - \frac{Ze\phi(l)}{m}\right)} = \pm\sqrt{2\left(\mathcal{E} - U(l)\right)}$$

- $\mathcal{E} > U_M(\mu) \Rightarrow v_{\|}$ does not change sign: passing particles
- $\mathcal{E} < U_M(\mu) \Rightarrow v_{\|}$ vanishes at bounce points: trapped particles
  - Bounce period = τ_b



Perpendicular motion

- Ignore drifts for passing particles
  - In ergodic flux surfaces, passing particles sample the whole surface
    - Drift parallel to flux surface small compared to fast $v_{||}$
    - Radial drift averages out
  - Rational flux surfaces are similar due to continuity
- Need drifts for trapped particles
  - Trapped particles don't leave initial region without drifts
  - Use flux coordinates: r labels flux surface, and α labels $\mathbf{B}$ lines within the flux surface
  - Use l along lines



$$\mathbf{B} = \Psi'(r)\nabla r \times \nabla \alpha$$

Kinetic equations with collisions

- Kinetic equation

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} \cdot \nabla f = v_{\|} \hat{\mathbf{b}} \cdot \nabla f + \mathbf{v}_d \cdot \nabla f = C[f]$$

- Low collisionality

$$\nu \sim \frac{\mathbf{v}_d}{L} \sim \rho_* \frac{v_t}{L} \ll \frac{v_t}{L} \Rightarrow \boxed{\nu_* \sim \rho_*}$$

  - Lower collisionality than banana regime in tokamaks

- Use $f = f^{(0)} + f^{(1)} + \ldots \simeq f^{(0)}$, with $f^{(n)} = {\rho_*}^n f$

$$v_{\|} \hat{\mathbf{b}} \cdot \nabla f^{(0)} = 0 \Rightarrow \boxed{f \simeq f^{(0)}(r, \alpha, \cancel{l}, \mathcal{E}, \mu)}$$

$$v_{\|} \hat{\mathbf{b}} \cdot \nabla f^{(1)} + \mathbf{v}_d \cdot \nabla f^{(0)} = C[f^{(0)}]$$

- Need to eliminate $f^{(1)}$ to find equation for $f^{(0)}$

Trapped and passing particles

- Passing particle $f = f_p$: cannot depend on α because most flux surfaces are traced by one single field line

$$f_p(r, \cancel{\alpha}, \mathcal{E}, \mu)$$

  - Eliminate $f^{(1)}$ using flux surface average ($f^{(1)}$ is periodic)

$$v_{||}\hat{\mathbf{b}} \cdot \nabla f^{(1)} + \mathbf{v}_d \cdot \nabla f^{(0)} = C[f^{(0)}] \Rightarrow \boxed{\left\langle \frac{B}{v_{||}} C[f_p] \right\rangle_{\text{surface}} = 0}$$

- Trapped particle $f = f_t$
  - Need to use orbit average to eliminate $f^{(1)}$: $\left\langle \ldots \right\rangle_{\text{orbit}} = \frac{1}{\tau_b} \oint (\ldots) \frac{\mathrm{d}l}{v_{||}}$
  - Trapped particles move with an average drift

$$\boxed{\left\langle \mathbf{v}_d \cdot \nabla r \right\rangle_{\text{orbit}} \frac{\partial f_t}{\partial r} + \left\langle \mathbf{v}_d \cdot \nabla \alpha \right\rangle_{\text{orbit}} \frac{\partial f_t}{\partial \alpha} = \left\langle C[f_t] \right\rangle_{\text{orbit}}}$$

Second adiabatic invariant

- Adiabatic invariant of periodic motion of trapped particles

$$J(r, \alpha, \mathcal{E}, \mu) = \oint v_{\parallel}\, \mathrm{d}l = 2\int_{l_L}^{l_R} \sqrt{2\left(\mathcal{E} - \mu B(l) - \frac{Ze\phi(l)}{m}\right)}\, \mathrm{d}l$$

- Equations based on second adiabatic invariant

$$\left\langle \mathbf{v}_d \cdot \nabla r \right\rangle_{\text{orbit}} \simeq \frac{mc}{Ze\Psi'\tau_b}\frac{\partial J}{\partial \alpha}$$

$$\left\langle \mathbf{v}_d \cdot \nabla \alpha \right\rangle_{\text{orbit}} \simeq -\frac{mc}{Ze\Psi'\tau_b}\frac{\partial J}{\partial r}$$

- Trapped particles move keeping J = const.

$$\frac{\mathrm{d}J}{\mathrm{d}t} = \left\langle \mathbf{v}_d \cdot \nabla r \right\rangle_{\text{orbit}}\frac{\partial J}{\partial r} + \left\langle \mathbf{v}_d \cdot \nabla \alpha \right\rangle_{\text{orbit}}\frac{\partial J}{\partial \alpha} = 0$$

Tokamaks

- In tokamaks, $\phi \simeq \phi(r) \Rightarrow U(l) \simeq \mu B(l)$
- Then, due to axisymmetry

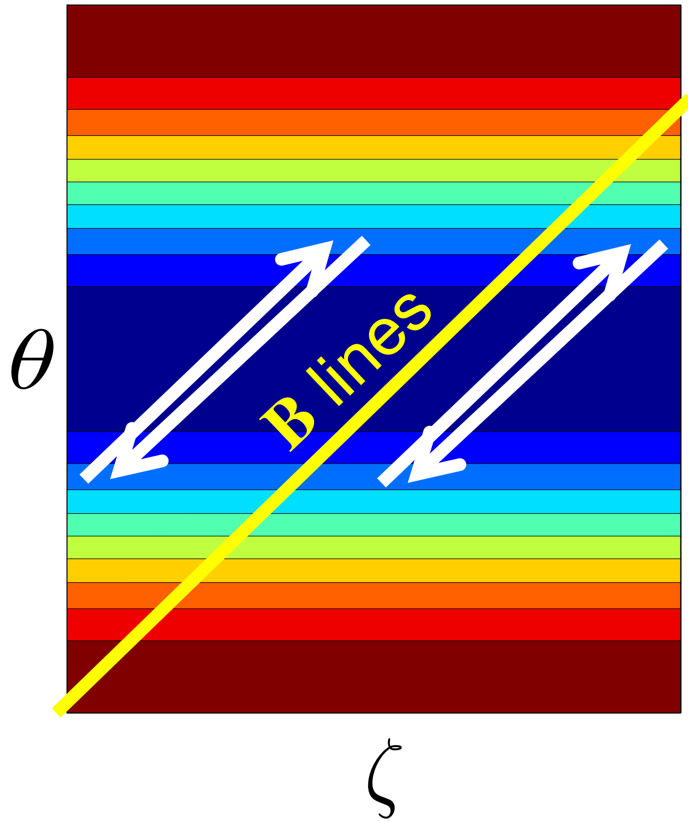
$$\frac{\partial J}{\partial \alpha} = 0 = \left\langle \mathbf{v}_d \cdot \nabla r \right\rangle_{\text{orbit}}$$

- Trapped particle equation

$$\frac{mc}{Ze\Psi'\tau_b}\left(\frac{\partial J}{\partial \alpha}\frac{\partial f_t}{\partial r} - \frac{\partial J}{\partial r}\frac{\partial f_t}{\partial \alpha}\right) = \left\langle C[f_t]\right\rangle_{\text{orbit}}$$

- Only solution, Maxwellian that does not depend on α
- No transport!
  - Need to keep correction $f^{(1)} \sim \rho_* f_M$ to recover banana regime

$U \simeq \mu B$ map on tokamak flux surface



$1/\nu$ regime

[Galeev et al, PRL 1969]

- For $\rho_* \ll \nu_* \ll 1$, collision operator dominates

$$\left\langle \frac{B}{v_\parallel} C[f_p] \right\rangle_{\text{surface}} = 0, \quad \left\langle C[f_t] \right\rangle_{\text{orbit}} = \cdots \Rightarrow \boxed{f \simeq f_M(r, \cancel{\alpha}, \mathcal{E})}$$

small

  - f must be Maxwellian, and it cannot depend on α because f_p does not depend on α

- Using $f = f_M + f_1 + \ldots$,

$$\left\langle C^{(\ell)}[f_{1,t}] \right\rangle_{\text{orbit}} = \left\langle \mathbf{v}_d \cdot \nabla r \right\rangle_{\text{orbit}} \frac{\partial f_M}{\partial r} \Rightarrow \boxed{f_1 \sim \frac{\rho_*}{\nu_*} f_M}$$

- Very large neoclassical transport

$$Q = \left\langle \int f_1 \frac{mv^2}{2} \mathbf{v}_d \cdot \nabla r \, \mathrm{d}^3 v \right\rangle_{\text{orbit}} + \ldots \sim \frac{\rho_*^2 n T v_t}{\nu_*} = \frac{\text{gyroBohm}}{\nu_*}$$

1/ν regime

- Particles move radially until collision happens

$$\text{width} \sim \frac{\mathbf{v}_d}{\nu} \sim \frac{\rho_*}{\nu_*} L$$

- Diffusion coefficient

$$D \sim \nu (\text{width})^2 \sim \frac{\rho_*}{\nu_*} \rho v_t$$

- Optimize to reduce transport

$$\left\langle \mathbf{v}_d \cdot \nabla r \right\rangle_{\text{orbit}} \propto \frac{\partial J}{\partial \alpha} \to 0$$

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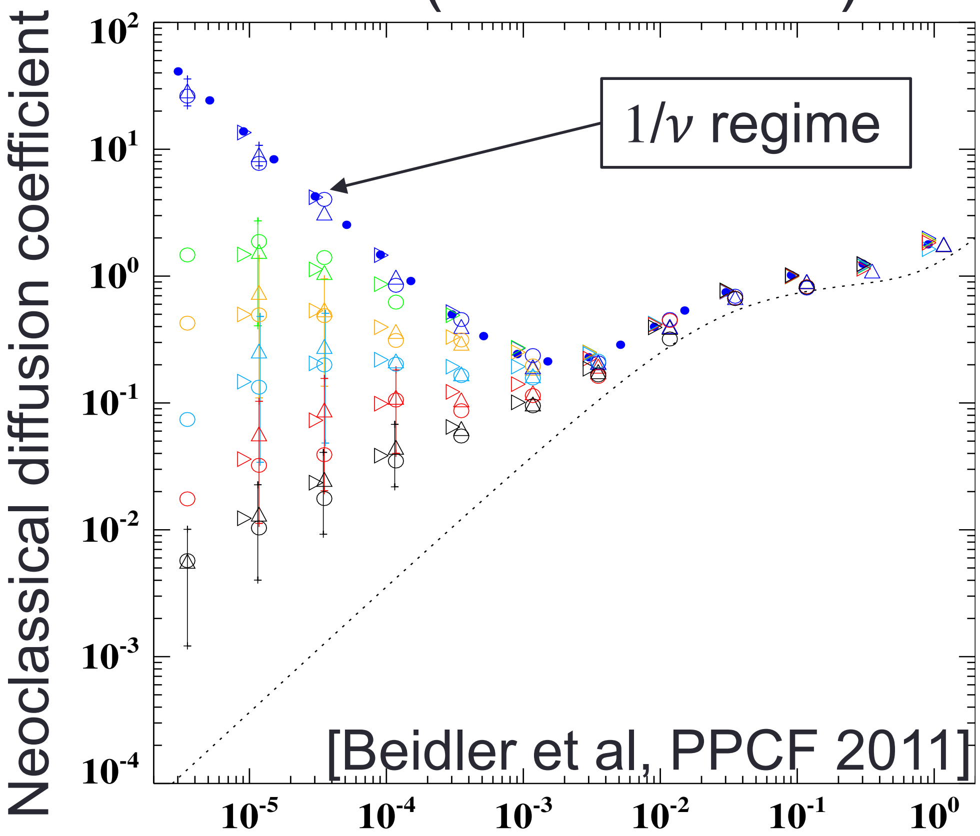


$$\nu_* = \frac{qR\nu}{v_t}$$

Very low collisionality regime

- For $\nu_* \ll \rho_*$, trapped particles follow J = const.

$$\frac{\partial J}{\partial \alpha}\frac{\partial f_t}{\partial r} - \frac{\partial J}{\partial r}\frac{\partial f_t}{\partial \alpha} = \underbrace{\ldots}_{\text{small}} \Rightarrow \boxed{f_t \simeq f_t(J, \mathcal{E}, \mu)}$$

- In general, J = const. does not coincide with flux surfaces $\Rightarrow$ very large heat flux!
  - Particles move across the machine at drift velocities ~ $\rho_* v_t$

$$Q \sim nT\mathbf{v}_d \sim \frac{\rho_*^2 nTv_t}{\rho_*} = \frac{\text{gyroBohm}}{\rho_*} = \text{Bohm}$$

- Two effects reduce heat flux
  - Large $\mathbf{E}\times\mathbf{B}$ drift (≈ large aspect ratio)
  - Optimization

Cases with "smaller" transport

- Basic expansion: $J \simeq J_0(r) + \delta J_1(r, \alpha)$, with $\delta << 1$

$$\left\langle \mathbf{v}_d \cdot \nabla r \right\rangle_{\text{orbit}} \frac{\partial}{\partial r} << \left\langle \mathbf{v}_d \cdot \nabla \alpha \right\rangle_{\text{orbit}} \frac{\partial}{\partial \alpha}$$

- For $\nu_* \sim \rho_*$,

$$\left\langle \mathbf{v}_d \cdot \nabla \alpha \right\rangle_{\text{orbit}} \frac{\partial f_t}{\partial \alpha} \simeq \left\langle C[f_t] \right\rangle_{\text{orbit}} \Rightarrow \boxed{f \simeq f_M(r, \mathcal{E})}$$

- For $\rho_* << \nu_* << 1$, one recovers $1/\nu$ regime
- For $\nu_* << \rho_*$, using $f = f_M + f_1 + \ldots$,

$$f_t = f_t(J, \mathcal{E}, \mu) \Rightarrow \boxed{f_{1,t} \simeq \frac{\delta J_1}{\partial J_0 / \partial r} \frac{\partial f_M}{\partial r}}$$

  - No transport! Particles don't scatter, just follow J = const.

Large $\mathbf{E}\times\mathbf{B}$ drift: $\sqrt{\nu}$ regime

[Ho & Kulsrud, PoF 1987]

- Usually justified by aspect ratio expansion $\epsilon = a/R << 1$
- Assuming $\nabla\phi \sim T/ea$ and $\phi \simeq \phi(r)$ (consistent)

$$\mathbf{v}_E = -\frac{c}{B}\nabla\phi\times\hat{\mathbf{b}} = \frac{c\phi'}{B}(\hat{\mathbf{b}}\times\nabla r) \sim \frac{\rho}{a}v_t \gg \mathbf{v}_{\nabla B} \sim \mathbf{v}_\kappa \sim \frac{\rho}{R}v_t$$

- Drift parallel to flux surface $\approx \mathbf{v}_E$ >> radial drift
- For $\nu_* = R\nu/v_t << \rho/a$, formula for $f_{1,t}$ is valid
  $\Rightarrow$ discontinuity between f_p and f_t ($\partial f_p/\partial\alpha = 0 \neq \partial f_t/\partial\alpha$)
  $\Rightarrow$ collisional boundary layer $\Rightarrow$ "enhanced" scattering

$$\nu v_t^2\frac{\partial^2}{\partial v_\parallel^2} \sim \mathbf{v}_E\cdot\nabla\alpha\frac{\partial}{\partial\alpha} \Rightarrow \boxed{\frac{\delta v_\parallel}{v_t} \sim \sqrt{\frac{\nu_*}{\epsilon^{-1}\rho_*}}} \Rightarrow \boxed{Q \sim \sqrt{\frac{\nu_*}{\epsilon^{-1}\rho_*}}\epsilon^{5/2}\underbrace{\rho_* n T v_t}_{\text{Bohm}}}$$

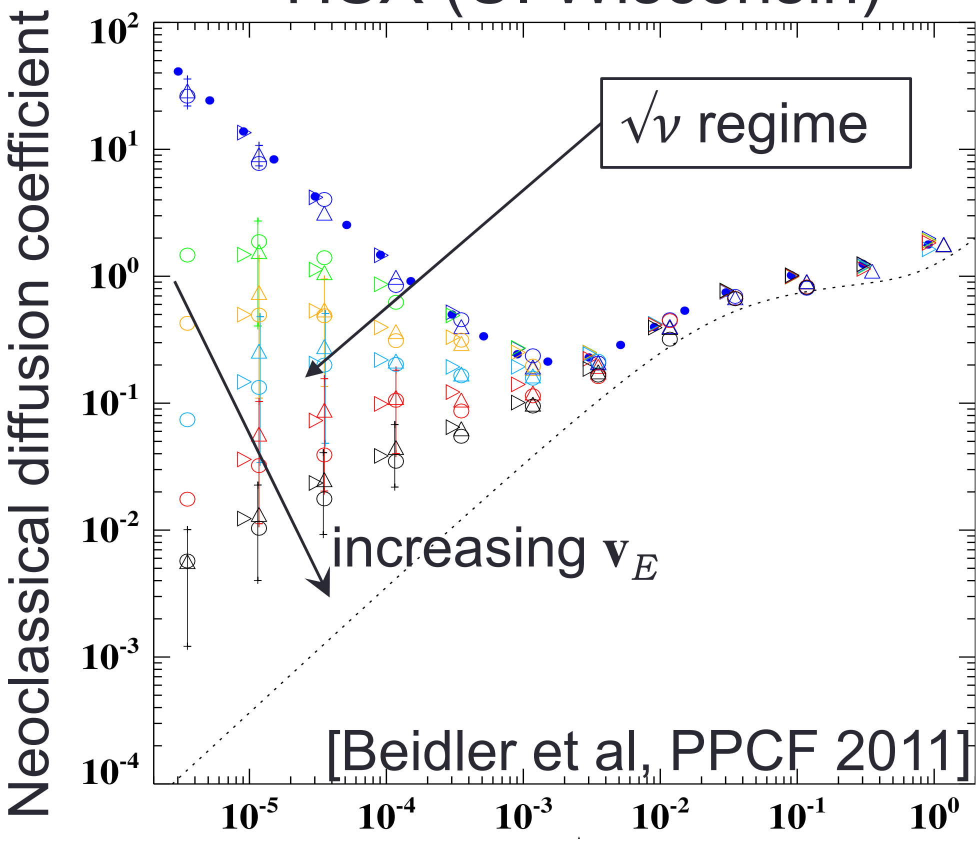
$\sqrt{\nu}$ regime

- $\mathbf{v}_E$ forces particles to move poloidally and to average over radial drift

$$\left\langle \mathbf{v}_d \cdot \nabla r \right\rangle_{\mathrm{orbit}} \propto \frac{\partial J}{\partial \alpha}$$

- $\sqrt{\nu}$ regime calculated with "fixed" δf neoclassical codes
- $1/\nu$ & $\sqrt{\nu}$ explain core transport in many shots
  - Edge dominated by turbulence

HSX (U. Wisconsin)



$$\nu_* = \frac{qR\nu}{v_t}$$

√ν regime

- $\mathbf{v}_E$ forces particles to move poloidally and to average over radial drift

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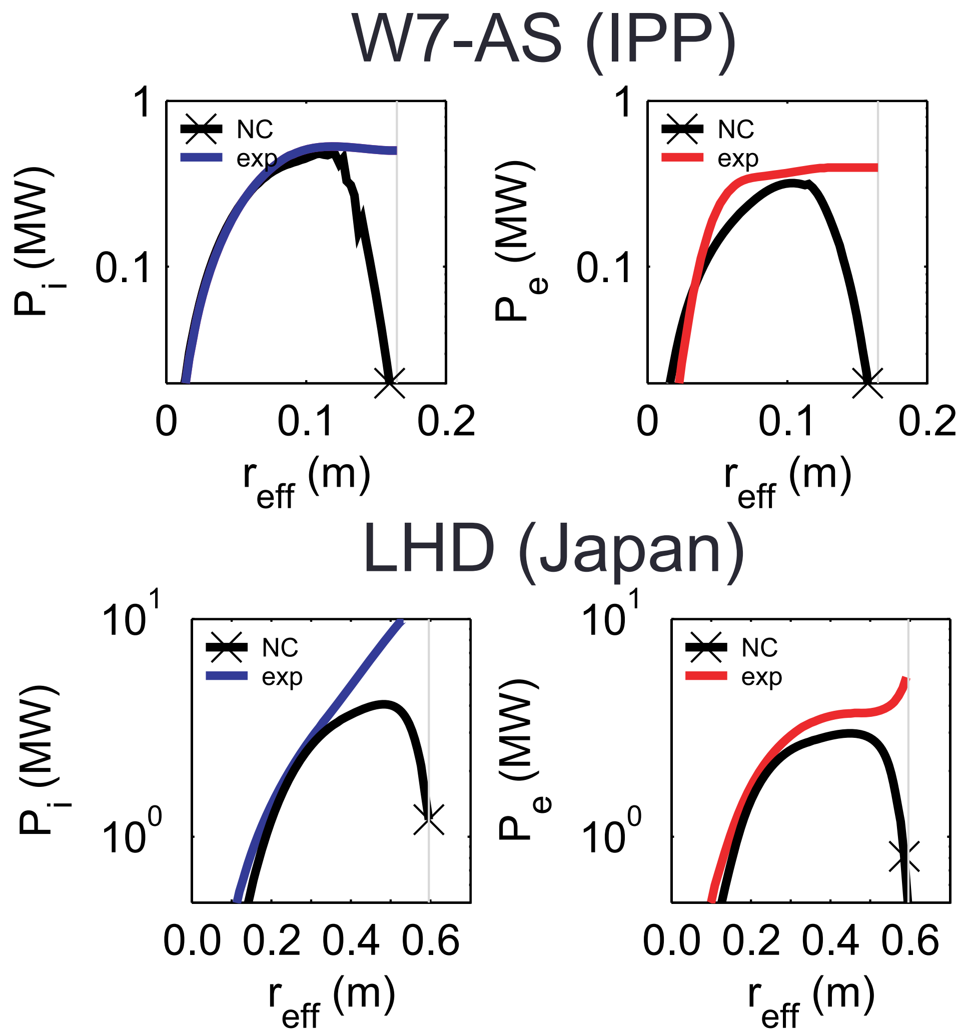


[Dinklage et al, NF 2013]

Optimized stellarators

- $\mathbf{v}_E$ is not a choice: determined by gradients of n and T
  - In tokamaks, flow moves trapped in direction of symmetry
    $\Rightarrow f \neq f(\alpha)$ and hence if $f = f_M$ at one α, $f = f_M$ everywhere
    $\Rightarrow$ toroidal rotation is undamped
  - In stellarators, trapped particles cannot follow any symmetry
    $\Rightarrow f = f(\alpha)$, and even if $f = f_M$ at one α, $f \neq f_M$ in general
    $\Rightarrow$ collisions damp flow to achieve $f = f_M$ everywhere
- Radial $\mathbf{E}$ given by neoclassical radial current = 0
- For very hot plasmas, $\mathbf{v}_E$ is not large even for $\epsilon << 1$
  - LHD “impurity hole” [Yoshimuna et al, NF ‘09; Velasco et al, NF ‘17]
- When $\mathbf{v}_E$ is not large, need to rely on optimization

$$J = J_0(r) + \delta J_1(r, \alpha)$$

  - The parameter $\delta \neq \epsilon$ measures how well we have optimized

Superbanana-plateau regime

[Shaing et al, PPCF 2009] [Calvo et al, PPCF 2017]

- f discontinuous at trapped passing boundary, but now

$$f_{1,t} \simeq \frac{\delta J_1}{\partial J_0/\partial r} \frac{\partial f_M}{\partial r}$$

Denominator can vanish!

  - $\mathbf{v}_E$, $\mathbf{v}_{\nabla B}$ and $\mathbf{v}_\kappa$ can cancel each other for some particles $\Rightarrow$ radial drift does not average out $\Rightarrow$ superbananas
- Collisional boundary layer around particles with $\partial J_0/\partial r = 0$

$$\nu v_t^2 \frac{\partial^2}{\partial v_\parallel^2} \sim \frac{mc}{Ze\Psi'\tau_b} \frac{\partial J_0}{\partial r} \simeq \frac{mc}{Ze\Psi'\tau_b} \delta v_\parallel \frac{\partial^2 J_0}{\partial v_\parallel \partial r} \Rightarrow \boxed{\frac{\delta v_\parallel}{v_t} \sim \left(\frac{\nu_*}{\rho_*}\right)^{1/3}}$$

- Large heat flux: $Q \sim \delta^2$ Bohm
- Importantly, $\phi \neq \phi(r)$
  - Particles with $\partial J_0/\partial r = 0$ tend to have same bounce points, and they spend a long time in bounce points, giving large n perturbations

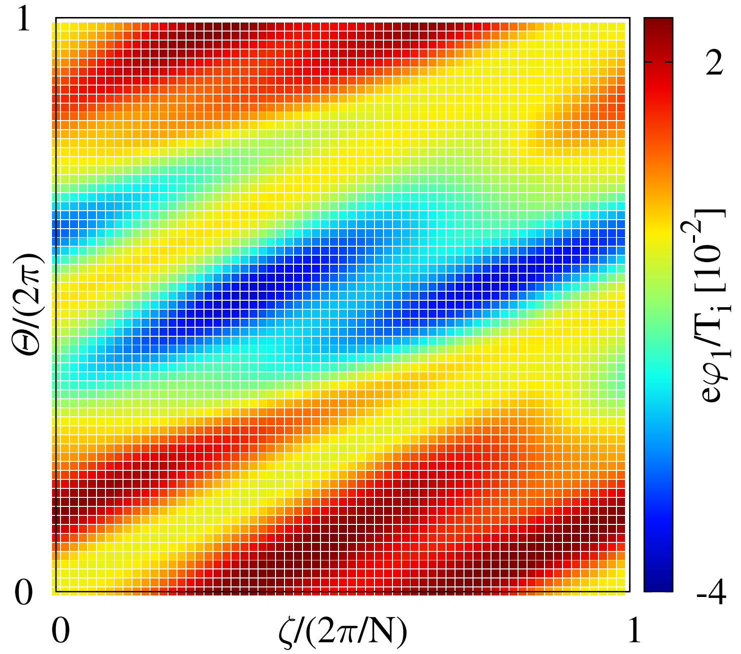
Poloidal and toroidal electric field

- For small $\mathbf{v}_E$, poloidal and toroidal electric field must be taken into account

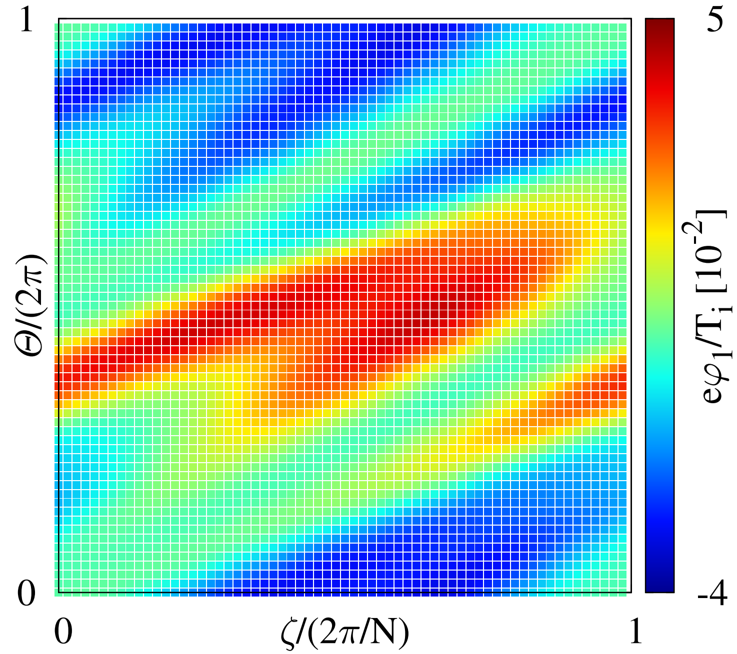
LHD (Japan)

[Velasco, ISHW 2017]

Assuming $\mathbf{v}_d \cdot \nabla\alpha \simeq \mathbf{v}_E \cdot \nabla\alpha$



Taking the full $\mathbf{v}_d \cdot \nabla\alpha$



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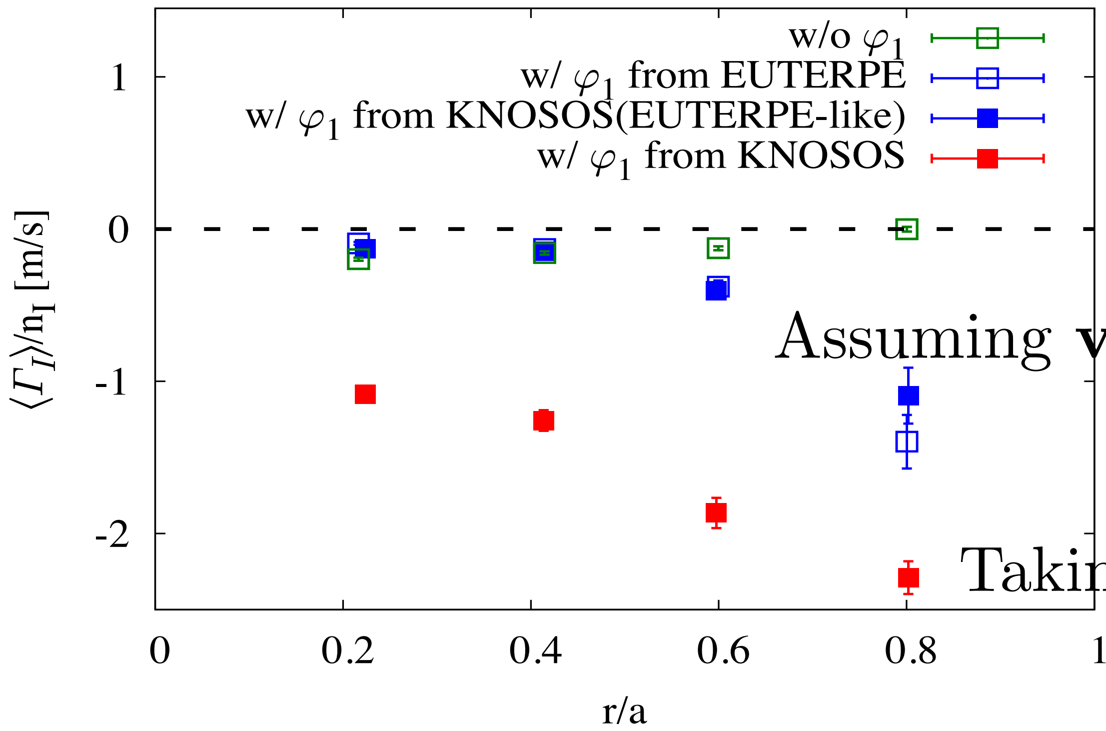
Impurity flux



Assuming $\mathbf{v}_d \cdot \nabla\alpha \simeq \mathbf{v}_E \cdot \nabla\alpha$

Taking the full $\mathbf{v}_d \cdot \nabla\alpha$

Summary of neoclassical transport

- Large neoclassical transport for small collisionality
  - Can explain transport in core of stellarators
- Neoclassical theory for small $\mathbf{v}_E$ under study
  - Probably relevant for some hot stellarator plasmas
  - The effect of poloidal and toroidal electric field seems important
- Topics that I haven't mentioned
  - Impurity accumulation: stellarator neoclassical transport seems to tend to pinch impurities in
  - Bootstrap current: similar to tokamak current ($\propto$ orbit width), but theory does not match simulations too well
  - Flow damping: can stellarators be optimized to have flow? (HSX)

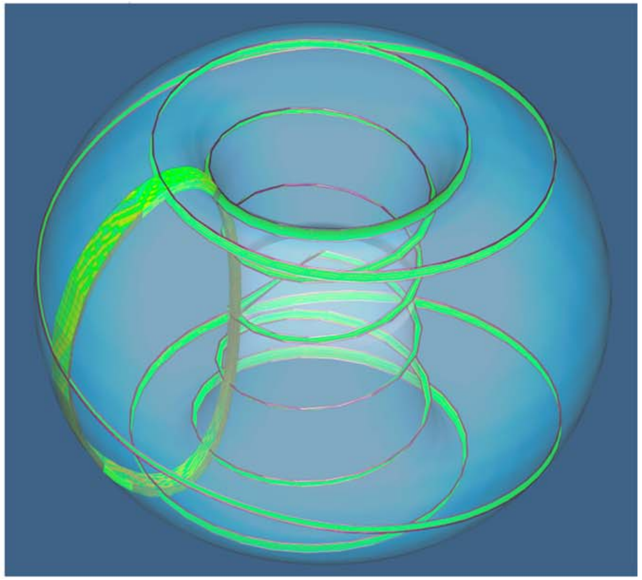
Turbulent transport in stellarators

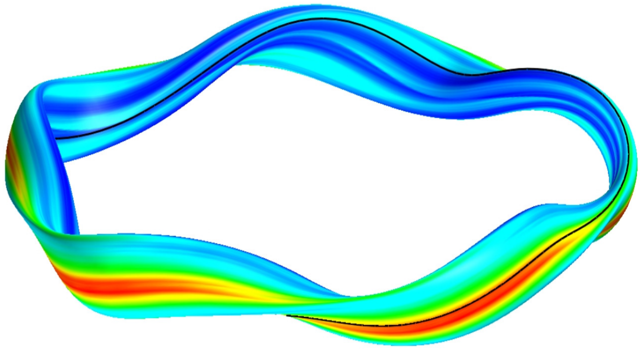
- Much less studied
- Very costly: cannot use symmetry to run cheap flux tubes

$\Rightarrow$ full flux surface simulations

  - Can still assume radially local

- Within the same flux surface, different flux tubes see different turbulence drive, different shear…
  - "Global" effects due to full flux surface seem to quench turbulence
- Zonal flow efficiently damp at large scales





[Xanthopoulos et al, PRL 2014]