Fast Secondary Reconnection and the Sawtooth Crash

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(EFTC-17, Athens, 9 October 2017)
Problem: why is the sawtooth crash so “fast” at large Lundquist number $S=\tau_R/\tau_A$?

$\tau_R = a^2/\eta$  
Resistive time  
$\tau_A = a/V_{\text{Alfven}}$  
Alfven time
The m=1 mode and the sawtooth crash

The m=1 mode: an essential element of the sawtooth cycle
[Von Goeler et al.(1974), Kadomtsev(1975)]
Process mediated by the restivity $\eta$. Becomes slow at large $S$, small $\eta$.

The sawtooth crash time in JET is shorter than the collision time. [Edwards et al., (1986)]
This process is too fast to be explained by resistive MHD.

Non-collisional physics (electron inertia and other effects) gives the right time scale (100 microseconds) for JET:
$\gamma \tau_A \sim (d_e/L)$ both in the linear and in the nonlinear phase.
($d_e =$ skin depth, $L \sim a \sim R =$ machine scale length. $\tau_A =$ Alfven time).
Typically $d_e/L \sim 10^{-3}$
[Ottaviani and Porcelli, (1993)].

Probably not sufficient to explain fast crashes in medium size tokamaks and in some simulations at moderate Lundquist number
Look for a new explanation
What about the stability of $m=1$ islands?

- Current sheets of large aspect ratio are developed during the nonlinear stage of primary internal-kink modes. How does their instability to secondary reconnecting modes relate to fast sawtooth crashes?

$$\Delta' = \infty$$

(Figures taken from [Q.Yu et al., Nucl. Fusion 54, 072005 (2014)])
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**Ex:**

\[ \Delta' = \infty \]

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Large $\Delta'$ regime in slab geometry

FIG. 13. Contour plots of (a) the streamfunction $\varphi$ and (b) the current density $J$ in the purely resistive case.

( Figures taken from [M. Ottaviani et al., Phys. Plasmas 2, 4104 (1995)] )
FIG. 13. Contour plots of (a) the streamfunction $\varphi$ and (b) the current density $J$ in the purely resistive case.
Due to the current sheet's large aspect ratio \( L/\delta_1 \gg 1 \) a quasi-continuum spectrum of unstable wavenumbers can be assumed.

The maximum growth rate occurs at short wavelength. This justifies the local assumption about the geometry.
Key assumptions

1) In the current sheet, the current density is comparable to the equilibrium current density which is what one gets at the end of the linear phase of the m=1 mode:

\[ J_{cs} \sim J_0 \]

2) The above is reasonable if the instability of the current sheet (what we call the secondary instability) is fast enough that its growth rate exceeds the evolution rate of the “equilibrium” (i.e. the primary reconnecting mode):

\[ \gamma_{II} > \tau_{cs}^{-1} \sim \gamma_I \]

This is verified \textit{a posteriori}. The m=1 island can be considered as \textit{static}.

3) \textit{The width of the current sheet is assumed to be of the order of the m=1 layer width.}

This is justified at the end of the linear phase/beginning of the nonlinear phase of the m=1 primary island

4) We perform an \textit{asymptotic analysis} (i.e. Lundquist number \( S \to \infty \), etc.)
Size of the transverse magnetic field in the current sheet

In a tokamak $L_0 \sim R$ and $L \sim R \sim L_0$.

And the end of the internal-kink linear phase $a \sim \delta_I = S^{-1/3} L$

$$J_{cs} \sim J_0 \Rightarrow \frac{B_{cs}}{B_0} = \frac{a}{L}$$

$$\Delta' \approx \frac{1}{k_a a^2}$$

Example of Harris-pinching equilibrium, typical for $a/L \leq ka \ll 1$

(thus valid for the tearing at small $ka$ and for the dominant mode at $\Delta' \delta \sim 1$)

Note: in several papers $B_{cs} \sim B_0$ and $J_{cs} \gg J_0$ NOT JUSTIFIED
Scaling of the reconnection rates (n/m=1/1 case of a tokamak in cylindrical approximation)

Estimation:
\[
\frac{B_{cs,\theta}^i}{B_0} \approx \frac{x}{R}, \quad \text{for} \quad x \leq \delta_I
\]

Tokamak geometry:
\[
L_0 = \frac{R}{\hat{s}_1} \quad \hat{s}_1 \equiv \frac{r_1 q'(r_1)}{q_1} \quad k = 1/r_1
\]

Primary internal-kink:
\[
\Delta'\delta_I = \infty
\]

\[
\gamma_I\tau_0 \sim S_R^{-\frac{1}{3}} \left( \frac{r_1}{R\hat{s}_1} \right)^{-\frac{2}{3}} \quad \frac{\delta_I}{R} \sim S_R^{-\frac{1}{3}} \left( \frac{r_1}{R\hat{s}_1} \right)^{-\frac{1}{3}}
\]

A broad range of unstable secondary modes with a broad range of wavenumbers k and a varying \(\Delta'\) and layer width \(\delta_{II}\)

The fastest growing mode always occurs at \(\Delta'\delta_{II} \sim 1\)

\[
k_{M}^{(II)} R \sim S_R^{\frac{1}{4}} \left( \frac{r_1}{\hat{s}_1 R} \right)^{-\frac{1}{2}} \quad \gamma_{II}^{(M)}\tau_0 \sim S_R^{-\frac{1}{6}} \left( \frac{r_1}{\hat{s}_1 R} \right)^{-\frac{1}{3}}
\]
Results: secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection.

Collisionless regime:
\[ \gamma \tau_0 > \nu_c \tau_0 \]

Rates, normalised to the Alfven time

\[ \nu_c \tau_0 \sim (L/d_e)^2 \ S^{-1} \]

Alfvenic limit:
\[ \gamma \tau_0 \sim O(1) \]

\[ (\gamma \tau_0)_{\text{res}} \sim S^{-1/3} \]

\[ (\gamma \tau_0)_{\text{res}} \sim S^{-1/6} \]

\[ S_1 = (L/d_e)^{12/5} \]

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Collision rate:

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\[ \tau_0 \equiv \frac{L}{c_A} \]

Ottaviani & Porcelli, 93
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Secondary instability, collisional:
\[ (\gamma_{II} \tau_0)_{res} \sim S^{-1/6} \]

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Diagram:
- Normalised rates
- Lundquist number
- Collisionless reconnection
- Normalised rates
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Collision rate
\[ \nu_c \tau_0 \sim (L/d_e)^2 \text{ S}^{-1} \]

Secondary instability, collisionless
\[ (\gamma_{II} \tau_0)_{res} \sim (d_e/L)^2 \text{ S}^{2/3} \]

\[ (\gamma_I \tau_0)_{res} \sim S^{-1/3} \]

\[ (\gamma_{II} \tau_0)_{res} \sim S^{-1/6} \]

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N.B.: for JET \( S \sim 10^9 \sim (L/d_e)^3 \) \( (L \sim R) \)

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N.B.: for JET \( S \sim 10^9 \sim (L/de)^3 \) \( (L \sim R) \)

\[ (\gamma_1 \tau_0)_{\text{res}} \sim S^{-1/3} \]

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New Theory

Faster growth rate

Normalised rates

\( (L/d_e)^2 \)

Lundquist number

\[ S_1 = (L/d_e)^{12/5} \]

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\( S \)
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N.B.: for JET $S \sim 10^9 \sim (L/d_e)^3 \quad (L \sim R)$

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New Theory

Faster growth rate
Broader collisionless range

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New Theory
Faster growth rate
Broader collisionless range

$\nu_c \tau_0 \sim (L/d_e)^2 S^{-1}$

$\tau_0 \equiv \frac{L}{c_A}$

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Numerical simulations (ongoing work)

current

t=301
current

t=321
current

t=325
current

t=331
Numerical simulations (ongoing work)

Current sheet instability and splitting
Numerical simulations (ongoing work)

Current maxima

absolute maximum

value at primary X-point

Current sheet instability and splitting

$t=301$

$t=321$

$t=325$

$t=331$
Numerical simulations (ongoing work)

Current maxima

Current sheet instability and splitting

Pseudo-spectral simulations
- Box aspect ratio $A=2$
- Lundquist number $S=10^3$

Still work to do…

Current maxima

absolute maximum

value at primary X-point

time
Summary and conclusions

- A current sheet generated by a primary instability with $\Delta' = \infty$, such as the resistive internal kink mode, becomes unstable at an early stage in its nonlinear development because of sub-Alfvenic modes.

- In the case of sawtooth phenomenon in a purely resistive framework the current sheet becomes unstable to a secondary mode with growth rate $\gamma_{\tau_A} \sim S^{-1/6}$, apparently in agreement with the numerical results of [Yu et al., Nucl. Fusion 54, 072005 (2014)].

- When finite electron inertia is included in the analysis, for $(L/d_e)^{12/5} < S < (L/d_e)^3$, regime relevant to most medium size tokamak devices, the secondary instability develops in the inertia-driven collisionless regime with a growth rate $\gamma_{\Pi} \tau_A \sim (d_e/a)^2 \sim (d_e/L)^2 S^{2/3}$ which can become near-Alfvenic.

- The overall reconnection rate is at least of order $\gamma \tau_A \sim (d_e/L)^{2/5}$, always larger than the internal kink growth rate.

THANK YOU FOR YOUR ATTENTION
On the plasmoid scaling

- A «plasmoid» scaling of superfast (faster than Alfvenic) reconnection rate of current sheets has been proposed in the literature (Loureiro et al., 2005, and subsequent studies)

\[ \gamma_{\text{plasmoid}} \tau_A \sim S^{1/4} \]

- This scaling requires two conditions (Tajima and Shibata, *Space Astrophysics*, 2002)

1. An intense current sheet such that \( B_{CS} \sim B_0 \) and \( J_{CS} / J_0 \sim L/\delta_{\text{sheet}} \gg 1 \)

AND

2. A sheet size such that \( \delta_{\text{sheet}}/L \sim S^{-1/2} \)

- As we have seen, this is not justified in the sawtooth case

- Current sheets such as \( \delta_{\text{sheet}}/L \sim S^{-1/3} \) and \( J_{CS} / J_0 \sim L/\delta_{\text{sheet}} \) produce Alfvenic rate reconnection such that

\[ \gamma \tau_A \sim I \]

which is a situation of likely interest in astrophysics where sheets can be produced by Alfvenic motion (Pucci and Velli, 2014)