COMMENTS

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Comment on "Low frequency dusty plasma modes in a uniform magnetic field" [Phys. Plasmas 9, 4396 (2002)]

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It is shown that Wang *et al.* [Phys. Plasmas **9**, 4396 (2002)] have not properly treated low-frequency dusty plasma waves in a uniform magnetic field. Their electron and ion susceptibilities are dubious. An improved description of the low-frequency dusty plasma wave spectra is presented. [DOI: 10.1063/1.1532341]

In a recent paper, Wang *et al.*[1] presented an investigation of low-frequency plasma waves in a magnetized dusty plasma ignoring dust charge fluctuations. For this purpose, they derived the dispersion relation (16) which includes the unmagnetized dust susceptibility [the second term in the left-hand side of Eq. (16)], as well as the electron and ion susceptibilities which are the third and fourth terms in Eq. (16). However, the electron and ion susceptibilities are inconsistent with the magneto-ionic and kinetic theories, and the subsequent wave modes in the intermediate and low-frequency ranges are not reliable.

Our objective here is to present correct susceptibilities and associated low-frequency wave modes in a magnetized dusty plasma. The constituents of the latter are electrons, ions, charged dust grains, and neutrals. The external magnetic field is $\hat{\mathbf{z}}B_0$, where $\hat{\mathbf{z}}$ is the unit vector along the z axis and B_0 is the strength of the magnetic field. At equilibrium, we have $en_{i0}=en_{e0}-Qn_{d0}$, where e is the magnitude of the electron charge, n_{j0} is the unperturbed number density of the particle species j (j equals e for the electrons, i for the ions, and d for the dust grains), and Q is the dust charge ($Q=-eZ_d$ for negatively charged dust grains and $Q=eZ_d$ for positively charged dust grains, where Z_d is the number of electronic charges residing on the dust grain surface). The dispersion properties of electrostatic waves in a dusty magnetoplasma are governed by

$$\epsilon(\omega,\mathbf{k})\equiv 1+\sum_{j=e,i,d}\chi_j(\omega,\mathbf{k})=0, \tag{1}$$

where the plasma susceptibility is[2–4]

$$\chi_j(\omega,\mathbf{k})=\frac{k_{Dj}^2}{k^2}\left[1+\sum_{n=-\infty}^{\infty}\Gamma_n(b_j)\frac{\omega+i\nu_{jn}}{\sqrt{2}k_zV_{Tj}}Z(\xi_j)\right]\times\left[1+\sum_{n=-\infty}^{\infty}\Gamma_n(b_j)\frac{i\nu_{jn}}{\sqrt{2}k_zV_{Tj}}Z(\xi_j)\right]^{-1}, \tag{2}$$

where $k_{Dj}^2=4\pi n_{j0}Q_j^2/T_j$, Z is the plasma dispersion function, $\xi_j=(\omega+i\nu_{jn}-n\omega_{cj})/\sqrt{2}k_zV_{Tj}$, ω and $\mathbf{k}(=\hat{\mathbf{z}}k_z+\mathbf{k}_\perp)$ are the frequency and wave vectors, respectively, $V_{Tj}=(T_j/m_j)^{1/2}$ is the thermal speed of species j, T_j is the temperature, m_j is the mass, $\omega_{cj}=|Q_jB_0/m_jc|$ is the gyrofrequency, Q_j is the charge, c is the speed of light in vacuum, ν_{jn} is the collision frequency between the species j and neutrals, $\Gamma_n=I_n(b_j)\exp(-b_j)$, with I_n being the modified Bessel function of order n, $b_j=k_\perp^2\rho_j^2$, and $\rho_j=V_{Tj}/\omega_{cj}$ is the thermal gyroradius.

We stress that high-frequency Langmuir and upper-hybrid waves are not affected by the presence of ions and dust as their frequencies are much larger than the plasma and gyrofrequencies of ions and charged dust grains. However, low-frequency waves are indeed affected by ion and dust motions, as shown below.

For the electrons, we consider the limit $\xi_e\gg 1$, with $b_e\ll 1$ and $\omega\ll\omega_{ce}$, so that $\Gamma_0\approx 1-b_e$. Then, from Eq. (2) we obtain[3] for $b_e\gg k_z^2V_{Te}^2/(\omega+i\nu_{en})^2$ the electron susceptibility[3]

$$\chi_e(\omega,\mathbf{k})\approx\frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{\omega+i\nu_{en}}{\omega+i\nu_{en}b_e}\frac{k_\perp^2}{k^2}, \tag{3}$$

where $\omega_{pe}=(4\pi n_{e0}e^2/m_e)^{1/2}$ is the electron plasma frequency. On the other hand, for $b_e\ll k_z^2V_{Te}^2/(\omega+i\nu_{en})^2$ and $|\nu_{en}\omega|\gg k_z^2V_{Te}^2$, we have[3]

$$\chi_e(\omega,\mathbf{k})\approx-\frac{\omega_{pe}^2}{\omega(\omega+i\nu_{en})}\frac{k_z^2}{k^2}, \tag{4}$$

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which reduces to

$$\chi_e(\omega,\mathbf{k}) \approx i \frac{\omega_{pe}^2}{\omega \nu_{en}} \frac{k_z^2}{k^2}, \tag{5}$$

when $\nu_{en} \gg |\omega|$. For $|\omega| \gg \nu_{en}$, we have the usual collisionless fluid result

$$\chi_e(\omega,\mathbf{k}) \approx \frac{k_\perp^2}{k^2} \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pe}^2 k_z^2}{\omega^2 k^2}. \tag{6}$$

Furthermore, for $\xi_e \ll 1$, $b_e \ll 1$ and $|\omega| \ll \nu_{en} \ll k_z V_{Te}$ we have

$$\chi_e \approx \frac{k_{De}^2}{k^2}. \tag{7}$$

For $\xi_i \gg 1$ and $b_i \ll 1$ the ion susceptibility is of the form

$$\chi_i(\omega,\mathbf{k}) \approx -\frac{\omega_{pi}^2(\omega + i\nu_{in})k_\perp^2}{\{\omega[(\omega+i\nu_{in})^2 - \omega_{ci}^2] - 3(\omega+i\nu_{in})k_\perp^2 V_{Ti}^2\}k^2} - \frac{\omega_{pi}^2 k_z^2}{[\omega(\omega+i\nu_{in}) - 3k_z^2 V_{Ti}^2]k^2}, \tag{8}$$

where $\omega_{pi} = (4\pi n_{i0} e^2/m_i)^{1/2}$ is the ion plasma frequency, and V_{Ti} is the ion thermal speed. On the other hand, for $b_i \gg 1$ and $\nu_{in}, \omega \ll k_z V_{Ti}$, the ions follow the straight-line orbit and the corresponding susceptibility is

$$\chi_i \approx \frac{k_{Di}^2}{k^2}. \tag{9}$$

Finally, for $\xi_d \gg 1$ and $b_d \ll 1$ the susceptibility of the cold dust fluid is

$$\chi_d(\omega,\mathbf{k}) \approx -\frac{\omega_{pd}^2(\omega+i\nu_{dn})k_\perp^2}{\omega[(\omega+i\nu_{dn})^2 - \omega_{cd}^2]k^2} - \frac{\omega_{pd}^2 k_z^2}{\omega(\omega+i\nu_{dn})k^2}, \tag{10}$$

where $\omega_{pd} = (4\pi n_{d0} Q_d^2/m_d)^{1/2}$ is the dust plasma frequency. When the wave frequency is much larger than ω_{cd}, Eq. (10) reduces to

$$\chi_d \approx -\frac{\omega_{pd}^2}{\omega(\omega+i\nu_{dn})}. \tag{11}$$

Let us now discuss the properties of numerous dusty plasma modes that can be deduced from Eq. (1). First, we focus on long wavelength (in comparison with the electron and ion gyroradii) waves with $|\omega| \gg \nu_{en}, \omega_{ci}, \nu_{dn}$. Here, we obtain from Eqs. (1), (6), (8), and (11) the frequency of the lower-hybrid waves[5,6]

$$\omega \approx \frac{\omega_{pi}\omega_{ce}}{(\omega_{ce}^2 + \omega_{pe}^2 k_\perp^2/k^2)^{1/2}} \left(1 + \frac{\omega_{pe}^2 k_z^2}{\omega_{pi}^2 k^2}\right)^{1/2}. \tag{12}$$

Second, we consider the limit $\nu_{en}, \nu_{in} \ll |\omega| \ll \omega_{ci}$ and obtain from Eqs. (1), (6) and (11) the dust lower-hybrid frequency[7] for a high density plasma with $\omega_{pi} \gg \omega_{ci}$

$$\omega = \omega_{dlh} \frac{k}{k_\perp} \left(1 + \frac{\omega_{pe}^2 k_z^2}{\omega_{pd}^2 k^2}\right)^{1/2}, \tag{13}$$

where $\omega_{dlh} = \omega_{pd}\omega_{ci}/\omega_{pi}$.

Third, for $\omega_{pd}, \nu_{in} \ll |\omega|$ and $T_i = 0$, we obtain from Eqs. (1), (7), and (8)

$$(\omega^2 - \omega_{ci}^2)(\omega^2 - k_z^2 C_A^2) = \omega^2 k_\perp^2 C_A^2, \tag{14}$$

which shows the coupling between the ion-cyclotron waves and dust ion-acoustic waves. Here, $C_A = \omega_{pi}/k_{De}$ is the dust ion-acoustic speed. For $|\omega| \ll \omega_{ci}$, Eq. (14) reduces to

$$\omega = \frac{k_z C_A}{(1 + k_\perp^2 \rho_s^2)^{1/2}}, \tag{15}$$

where $\rho_s = C_A/\omega_{ci}$.

Fourth, for $|\omega| \gg \nu_{en}, \nu_{dn}$ we have from Eqs. (1), (9) and (11) the modified dust acoustic wave frequency[3]

$$\omega = \frac{k\omega_{pd}}{(k_{Di}^2 + k^2 + \omega_{pe}^2 k_\perp^2/\omega_{ci}^2)^{1/2}}. \tag{16}$$

Fifth, for $\nu_{dn} \ll |\omega|$ and $k \ll k_D$, we obtain from Eqs. (1), (7), (9) and (10) the dispersion relation

$$(\omega^2 - \omega_{cd}^2)(\omega^2 - k_z^2 C_D^2) = \omega^2 k_\perp^2 \rho_D^2, \tag{17}$$

which shows the coupling between the dust-cyclotron and dust acoustic waves in a magnetoplasma. Here, we have denoted $k_D = (k_{De}^2 + k_{Di}^2)^{1/2}$ as the effective Debye wave number of a dusty plasma, $C_D = \omega_{pd}/k_D$ is the dust acoustic speed, and $\rho_D = C_D/\omega_{cd}$.

Sixth, for $\nu_{in} \ll |\omega| \ll \omega_{ci}$ we have from Eqs. (1), (6), and (8) the modified convective cell frequency

$$\omega \approx \left(\frac{n_{e0}}{n_{i0}}\right)^{1/2} (\omega_{ce}\omega_{ci})^{1/2} \frac{k_z}{k_\perp} \left(1 + \frac{Q_d^2 n_{d0}}{e^2 n_{e0}} \frac{m_e k^2}{m_d k_z^2}\right)^{1/2}. \tag{18}$$

We observe that the dusty plasma modes, given by Eqs. (12)–(18) suffer collisional damping.

Finally, inserting Eqs. (5), (9) and (11) into (1) we obtain for $\nu_{dn} \ll |\omega = \omega_r + i\omega_i|$ and $k/k_{Di} \ll 1$ the real and imaginary parts of the wave frequency[3]

$$\omega_r \approx \frac{k\omega_{pd}}{k_{Di}}, \tag{19}$$

and

$$\omega_i \approx -\frac{\omega_r^2 \omega_{pe}^2 k_z^2}{2\nu_{en}\omega_{pd}^2 k^2} - \frac{\nu_{dn}}{2}. \tag{20}$$

In summary, we have presented appropriate electron, ion and dust susceptibilities that are required for studying the properties of low-frequency (in comparison with the electron gyrofrequency) electrostatic modes in a dusty magnetoplasma. We have presented conditions under which the general plasma susceptibilities reduce in simple forms that are useful for analytical studies. The simplified susceptibilities are then used to investigate the frequency of the dusty plasma wave modes, some of which are already known in the literature.[5–8]

Thus, we have an improved physical picture of the various low-frequency dusty plasma modes in a uniform magnetic field, contrary to those reported in Ref. 1.

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