Interaction of Gravitational Waves with Spinning Particles

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The Problem: We discuss the equations which determine the response of a gyrating spinning charged particle moving in a constant magnetic field to an incident gravitational wave, in the linearized approximation to general relativity.

Results: The equations which determine the response of a spinning charged particle moving in a uniform magnetic field to an incident gravitational wave are derived in the linearized approximation to general relativity.

We verify that:

1) the components of the 4-momentum, 4-velocity and the components of the spinning tensor, both electric and magnetic moments, exhibit resonances and

2) the co-existence of the uniform magnetic field and the GW are responsible for the resonances appearing in our equations. In the absence of the GW, the magnetic field and the components of the spin tensor decouple and the magnetic resonances disappear.

The Dixon-Souriau(DS) Equations of Motion:

The equations of motion of a spinning test particle originally derived from Papapetrou, later on by Dixon and Souriau. These equations are:

$$\frac{dx^{\mu}}{d\tau} = \upsilon^{\mu}$$

$$\begin{aligned} \frac{dp^{\mu}}{d\tau} &= -\Gamma^{\mu}_{\lambda\nu}\upsilon^{\lambda}p^{\nu} - \frac{1}{2}R^{\mu}_{\nu\lambda\rho}S^{\lambda\rho}\upsilon^{\nu} + eF^{\mu}_{\beta}\upsilon^{\beta} \\ &- \frac{\lambda}{2}S^{\kappa\rho}\nabla^{\mu}F_{\kappa\rho} \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{dS^{\mu\nu}}{d\tau} &= -\Gamma^{\mu}_{\lambda\rho}\upsilon^{\lambda}S^{\rho\nu} - \Gamma^{\nu}_{\lambda\rho}\upsilon^{\lambda}S^{\mu\rho} \\ &+ (p^{\mu}\upsilon^{\nu} - p^{\nu}\upsilon^{\mu}) \\ &+ \lambda(S^{\mu\kappa}F^{\nu}_{\kappa} - S^{\nu\kappa}F^{\mu}_{\kappa}) \end{aligned} \quad (2)$$

and $p_{\mu}S^{\mu\nu} = 0$ (Dixon's condition).

Also, Greek indices take values 0,1,2,3, Latin 1,2,3, τ is an affine parameter across a word line L which is chosen as the proper time of the charged particle, υ^{μ} is the 4-velocity of the charged particle across the world line L, $p^{\mu} = \int T^{0\mu} dV$ are the components of the 4-momentum of the spinning charged particle, $F^{\mu\nu}$ is the electromagnetic tensor, λ is an electromagnetic coupling scalar and $S^{\mu\nu}$ is the spin tensor. Unlike special relativity, p^{μ} and υ^{μ} are not generally proportional to each other. But it is well known that Eqs.(1) and (2) themselves, do not constitute an independent set of equations since they are less than the unknown quantities (3 components of the spin tensor are not determined). Therefore, several supplementary conditions have been used in the literature to remedy this problem. Here we will adopt Dixon's condition e.g. center of mass condition).

To find the trajectory of the spinning charged particle, we need to know its 4-velocity. But there are no equations of motion for this purpose. However, we may obtain indirectly a relation between v^ν and p^ν from the following equation:

$$\begin{aligned} v^\nu &= N\{u^\nu - \frac{1}{2m^2\Delta} R_{\mu\beta\lambda\rho} S^{\lambda\rho} S^{\mu\nu} u^\beta \\ &+ \frac{e}{m^2\Delta} S^{\mu\nu} F_{\mu\beta} u^\beta \\ &+ \lambda[\frac{1}{2m^3\Delta} R_{\mu\beta\lambda\rho} S^{\lambda\rho} S^{\mu\nu} S^{\beta\kappa} (F^\sigma_\kappa u_\sigma) \\ &- \frac{1}{2m^2\Delta} S^{\mu\nu} S^{\kappa\rho} \nabla_\mu F_{\kappa\rho} - \frac{1}{m} S^{\nu\kappa} (F^\sigma_\kappa u_\sigma)] \\ &- \frac{\lambda e}{m^3\Delta} S^{\mu\nu} F_{\mu\beta} S^{\beta\kappa} (F^\sigma_\kappa u_\sigma)\} \end{aligned} \qquad (3)$$

where $p^\mu = m u^\mu$, $p_\mu p^\mu = -m^2$, m is the mass of the particle and $G = c = 1$

$$\Delta = 1 + \frac{1}{4m^2} R_{t\mu\lambda\rho} S^{\lambda\rho} S^{t\mu} + \frac{1}{2m^2} e F_{t\mu} S^{t\mu} \quad (4)$$

$$\begin{aligned}
N &= \{1 - \Lambda^{(1)}_{\mu} \Lambda^{(1)}_{\alpha} S^{\mu\nu} S^{\alpha}_{\nu} - 2\lambda[\Lambda^{(2)}_{\mu} \Lambda^{(1)}_{\alpha} S^{\mu\nu} S^{\alpha}_{\nu} \\
&- (F^{\sigma}_{\kappa} u_{\sigma}) \Lambda^{(1)}_{\alpha} S^{\kappa\nu} S^{\alpha}_{\nu}] - \lambda^2 [\Lambda^{(2)}_{\mu} \Lambda^{(2)}_{\alpha} S^{\mu\nu} S^{\alpha}_{\nu} \\
&- 2(F^{\sigma}_{\kappa} u_{\sigma}) \Lambda^{(2)}_{\alpha} S^{\kappa\nu} S^{\alpha}_{\nu} \\
&+ (F^{\sigma}_{\kappa} u_{\sigma})(F^{\alpha}_{\beta} u_{\alpha}) S^{\kappa\nu} S^{\beta}_{\nu}] \\
&- 2\lambda e \Lambda^{(3)}_{\mu} \Lambda^{(3)}_{\alpha} S^{\mu\nu} S^{\alpha}_{\nu} \\
&- 2e\lambda^2 [-\Lambda^{(3)}_{\mu} \Lambda^{(2)}_{\alpha} S^{\mu\nu} S^{\alpha}_{\nu} \\
&+ (F^{\sigma}_{\kappa} u_{\sigma}) \Lambda^{(3)}_{\mu} S^{\mu\nu} S^{\kappa}_{\nu}] \\
&+ \lambda^2 e^3 \Lambda^{(3)}_{\mu} \Lambda^{(3)}_{\alpha} S^{\mu\nu} S^{\alpha}_{\nu}\}^{-1/2} \quad (5)
\end{aligned}$$

$$\Lambda_x^{(1)} = \frac{1}{2m^2\Delta} R_{x\sigma\lambda\rho} S^{\lambda\rho} u^{\sigma} + \frac{e}{m^2\Delta} F_{x\sigma} u^{\sigma} \quad (6)$$

$$\begin{aligned} \Lambda_x^{(2)} &= -\frac{1}{2m^3\Delta} R_{x\sigma\lambda\rho} S^{\lambda\rho} S^{\sigma\kappa} (F_{\kappa}^{\beta} u_{\beta}) \\ + & \frac{1}{m^2\Delta} S^{\kappa\rho} \nabla_x F_{\kappa\rho} \end{aligned} \quad (7)$$

$$\Lambda_x^{(3)} = \frac{1}{m^3\Delta} F_{x\sigma} S^{\sigma\kappa} (F_{\kappa}^{\beta} u_{\beta}) \quad (8)$$

where x stands for $x = \mu, \alpha$.

Upon the consideration of the assumption($\lambda = 0$), we neglect particular terms in Eqs.(1),(2) and a simplified covariant model is obtained:

The simplified DS Equations:

$$\frac{dp^\mu}{d\tau} = -\Gamma^\mu_{\lambda\nu}\upsilon^\lambda p^\nu - \frac{1}{2}R^\mu_{\nu\lambda\rho}S^{\lambda\rho}\upsilon^\nu + eF^\mu_\beta\upsilon^\beta \quad (9)$$

$$\begin{aligned}\frac{dS^{\mu\nu}}{d\tau} &= -\Gamma^\mu_{\lambda\rho}\upsilon^\lambda S^{\rho\nu} \\ &- \Gamma^\nu_{\lambda\rho}\upsilon^\lambda S^{\mu\rho} + (p^\mu\upsilon^\nu - p^\nu\upsilon^\mu)\end{aligned} \quad (10)$$

$$\begin{aligned}\upsilon^\nu &= N\{u^\nu - \frac{1}{2m^2\Delta}R_{\mu\beta\lambda\rho}S^{\lambda\rho}S^{\mu\nu}u^\beta \\ &+ \frac{e}{m^2\Delta}S^{\mu\nu}F_{\mu\rho}u^\rho\}\end{aligned} \quad (11)$$

where

$$N = [1 - \frac{N_a}{4m^4\Delta^2}]^{-1/2} \tag{12}$$

and

$$\begin{aligned}
N_a &= (R_{\mu\sigma\lambda\rho}S^{\lambda\rho}S^{\mu\nu}u^{\sigma})[R_{\mu\sigma\lambda\rho}S^{\lambda\rho}(\eta_{\nu\beta} \\
&+ h_{\nu\beta})S^{\mu\beta}u^{\sigma}] - 2e[R_{\mu\sigma\lambda\rho}S^{\lambda\rho}(\eta_{\nu\beta} \\
&+ h_{\nu\beta})S^{\mu\beta}u^{\sigma}](F_{\kappa\gamma}S^{\kappa\nu}u^{\gamma}) \\
&- 2e(R_{\mu\sigma\lambda\rho}S^{\lambda\rho}S^{\mu\nu}u^{\sigma})[F_{\kappa\sigma}(\eta_{\nu\beta} \\
&+ h_{\nu\beta})S^{\kappa\beta}u^{\sigma}] + 4e^2[F_{\kappa\sigma}(\eta_{\nu\beta} \\
&+ h_{\nu\beta})S^{\kappa\beta}u^{\sigma}](F_{\kappa\sigma}S^{\kappa\nu}u^{\sigma})
\end{aligned} \tag{13}$$

The DS-Equations of motion in the linearized theory of gravity

To understand the Eqs.(9-13) in the linearized approximation to general relativity, we decompose the metric in the fashion

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{14}$$

By imposing the condition

$$(h_{\mu}^{\nu} - \delta_{\mu}^{\nu} h_{\rho}^{\rho})_{;\nu} = 0 \tag{15}$$

we reduce the vacuum field equations to homogeneous wave equations for all components of h_{ν}^{μ}. A coordinate transformation can now be effected to reduce the trace h_{ν}^{ν} and the mixed components $h_{0\alpha}$, to zero.

The gravitational field is then described by a symmetric traceless, divergenceless tensor with two independent space components which, for simplicity, we call $h_1 = h_+$ and $h_2 = h_\times$. Thus, the square of the line element is

$$\begin{aligned} & ds^2 = (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu \\ = \ & -(dx^0)^2 + (1 + h_1) dx^2 + (1 - h_1) dy^2 \\ + \ & dz^2 + 2h_2 dx dy \end{aligned} \tag{16}$$

where $|h_1, h_2| \ll 1$. We consider a plane GW which is characterized by the wave 3-vector

$$k_g^i = \omega_g (0, 0, 1) \tag{17}$$

and the two possible states of polarization given by

$$h_1 = h_{10} e^{i(k_g z - \omega_g t)}, \quad h_2 = h_{20} e^{i(k_g z - \omega_g t)} \tag{18}$$

where h_{10}, h_{20} are the amplitudes of the two components of the GW.

We choose the electromagnetic field to be

$$F_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -H_3 & 0 \\ 0 & H_3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{19}$$

where the background magnetic field is constant e.g $H^a = (0, 0, H^3) = const.$(from now on $H^3 = H$). In this scenario, the metric is still a solution to the Einstein equations in vacuum, because we assume that the energy density of the magnetic field, is approximately zero(i.e. no effect of the magnetic field on either the $\eta_{\mu\nu}$ or $h_{\mu\nu}$).

The Problem: We intend to discuss the electrodynamics of a spinning point-like charged particle with mass m, with an intrinsic angular momentum, in the presence of a uniform magnetic field across the z-axis, initially at rest with respect to the coordinate system in which the metric (16) is expressed.

To achieve this task, a relation between the invariant proper time τ and the coordinate time t is needed.

In the absence of external forces, this relation may be found from the expression

$$ds^2 = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu = -d\tau^2 \qquad (20)$$

because in Einstein's theory of gravity, the word lines of classical point particles in curved space times are time-like.

In the case we have external forces, we have to use the expression

$$d\tau = dt[\frac{1-\upsilon^2}{1-eS^{\mu\nu}F_{\mu\nu}/m^2}]^{1/2} \qquad (21)$$

where υ^2 is the total space velocity of the spinning particle.

Looking at the Eq.(22) we would like to make the following comments:

a)The physical meaning of the Eq.(22) is that relativistic time dilation occurs for a spinning charged particle with non-zero magnetic moments in an external magnetic field.

b) The structure of Eq.(22) results from the fact that υ^μ and u^μ differ from each other, where τ is connected with υ^μ and u^μ is normalized.

To make some further progress with the Eqs.(9-13), we decompose the particle's components of the 4-velocity, 4-momentum and spin tensor as follows:

$$v^{\mu} \simeq v^{\mu}_{0} + v^{\mu}_{1}, \;\; p^{\mu} \simeq p^{\mu}_{0} + p^{\mu}_{1},$$
$$S^{\mu\nu} \simeq S^{\mu\nu}_{0} + S^{\mu\nu}_{1} \qquad (22)$$

with v^{μ}_{1}, p^{μ}_{1} and $S^{\mu\nu}_{1}$ being of the same order as $h_{\mu\nu}$.

Initial Conditions:

We assume that the spinning particle initially is at rest, e.g., $u^{\mu}_{0} = (1,0,0,0), v^{\mu}_{0} = (1,0,0,0)$ and $p_{\mu(0)} S^{\mu\nu}_{0} = 0$

Thus, from Eqs.(9-13) and with the aid of Eqs.(21-22) we obtain the following equations:

3.a) Zero Order Equations

$$\frac{du_0^\mu}{dt} = \frac{e}{m}\eta^{\mu\nu} F_{\nu\beta} \upsilon_0^\beta \tag{23}$$

$$\frac{dS_0^{\mu\nu}}{dt} = m(u_0^\mu \upsilon_0^\nu - u_0^\nu \upsilon_0^\mu) \tag{24}$$

$$\upsilon_0^\nu = [\frac{1}{1 - eS^{\mu\nu}F_{\mu\nu}/m^2}]^{1/2}[1 - \frac{e^2 N_0}{m^4 \Delta_0}]^{-1/2}\{u_0^\nu + \frac{e}{m^2\Delta_0} S_0^{\mu\nu} F_{\mu\sigma} u_0^\sigma\} \tag{25}$$

where

$$\Delta_0 = 1 + \frac{1}{2m^2} e F_{\sigma\mu} S_0^{\sigma\mu} \tag{26}$$

$$N_0 = [F_{\kappa\sigma} S_0^{\kappa\nu} u_0^\sigma][F_{\kappa\sigma}\eta_{\nu\beta} S_0^{\kappa\beta} u_0^\sigma] \tag{27}$$

Zero Order results:

The zero order equations with the aid of the initial conditions imply:

1. The zero order electric moments of the spin-tensor vanish e.g $S_0^{0\nu} = 0$.

2. The zero order magnetic moments of the spin tensor are:

$$S_0^{12} = const., S_0^{13} = const., S_0^{23} = const. \quad (28)$$

3. After some straightforward calculations and with the aid of Eqs.(21),(26) and (27) we find; $\Delta_0 = 1 - \frac{eH}{m}(\frac{S_0^{12}}{m})$

3.b) First Order Equations

$$\frac{dp_1^\mu}{dt} = -\Gamma^\mu_{\lambda\nu} \upsilon_0^\lambda p_0^\nu - \frac{1}{2}\eta^{\mu\kappa} R_{\kappa\nu\lambda\rho} S_0^{\lambda\rho} \upsilon_0^\nu$$

$$+ \quad e[\eta^{\mu\kappa} F_{\kappa\beta} \upsilon_1^\beta + h^{\mu\kappa} F_{\kappa\beta} \upsilon_0^\beta] \qquad (29)$$

$$\frac{dS_1^{\mu\nu}}{dt} = -\Gamma^\mu_{\lambda\rho} \upsilon_0^\lambda S_0^{\rho\nu} - \Gamma^\nu_{\lambda\rho} \upsilon_0^\lambda S_0^{\mu\rho}$$

$$+ \quad [p_0^\mu \upsilon_1^\nu + p_1^\mu \upsilon_0^\nu - p_0^\nu \upsilon_1^\mu - p_1^\nu \upsilon_0^\mu] \qquad (30)$$

$$\upsilon_1^\nu = [\frac{1}{1 - eS^{\mu\nu} F_{\mu\nu}/m^2}]^{1/2} \{ \frac{1}{\Delta_0^2} [2e^2 N_{1a}$$

$$- \quad eN_{1b} - 4e^2 \frac{N_0 \Delta_1}{\Delta_0}][u_0^\nu + \frac{e}{m^2 \Delta_0} F_{\kappa\sigma} S_0^{\kappa\nu} u_0^\sigma]$$

$$+ \quad [1 - \frac{e^2 N_0}{m^4 \Delta_0^2}]^{-1/2} [u_1^\nu - \frac{eR_{\mu\sigma\lambda\rho} S_0^{\lambda\rho} S_0^{\mu\nu} u^\sigma}{2m^2 \Delta_0}$$

$$+ \quad \frac{eF_{\kappa\sigma}(S_0^{\kappa\nu} u_1^\sigma + S_1^{\kappa\nu} u_0^\sigma)}{m^2 \Delta_0}$$

$$- \quad \frac{e\Delta_1}{m^2 \Delta_0^2} F_{\kappa\sigma} S_0^{\kappa\nu} u_0^\sigma]\} \qquad (31)$$

$$\Delta_1 = \frac{1}{4m^2} R_{\sigma\mu\lambda\rho} S_0^{\lambda\rho} S_0^{\mu\sigma} + \frac{e}{2m^2} F_{\sigma\mu} S_1^{\sigma\mu} \quad (32)$$

$$\begin{aligned} N_{1a} &= (F_{\kappa\sigma} S_0^{\kappa\nu} u_0^\sigma)[F_{\kappa\sigma}\eta_{\nu\beta}(S_0^{\kappa\beta} u_1^\sigma + S_1^{\kappa\beta} u_0^\sigma)] \\ &+ \ (F_{\kappa\sigma}\eta_{\nu\beta} S_0^{\kappa\beta} u_0^\sigma)[F_{\kappa\sigma}(S_0^{\kappa\nu} u_1^\sigma + S_1^{\kappa\nu} u_0^\sigma)] \quad (33) \end{aligned}$$

and

$$\begin{aligned} N_{1b} &= -2eR_{\mu\sigma\lambda\rho}[S_0^{\lambda\rho}\eta_{\nu\beta} S_0^{\mu\beta} u_0^\sigma (F_{\kappa\beta} S_0^{\kappa\nu} u_0^\beta) \\ &- \ S_0^{\lambda\rho} S_0^{\mu\nu} u_0^\sigma (F_{\kappa\sigma}\eta_{\nu\beta} S_0^{\kappa\beta} u_0^\sigma) \quad (34) \end{aligned}$$

Because of the initial conditions, the form of the metric (16) and the zero-order results, the first-order equations give the following results; $N_0 = N_{1a} = N_{1b} = 0, \Delta_1 = \frac{1}{2}\{h_{1,zz}[(\frac{S_0^{13}}{m})^2 - (\frac{S_0^{23}}{m})^2] + 2h_{1,zz}(\frac{S_0^{13}}{m})(\frac{S_0^{23}}{m})\} - \frac{eH}{m}(\frac{S_0^{12}}{m})$

$$\frac{du_1^0}{dt} = 0, \quad \frac{du_1^3}{dt} = 0 \tag{35}$$

$$\frac{du_1^1}{dt} + \Omega v_1^2 = \frac{1}{2m}[h_{1,tz}S_0^{13} + h_{2,tz}S_0^{23}] \tag{36}$$

$$\frac{du_1^2}{dt} - \Omega v_1^1 = -\frac{1}{2m}[h_{1,tz}S_0^{23} - h_{2,tz}S_0^{13}] \tag{37}$$

$$\frac{dS_1^{0\nu}}{dt} = m(v_1^\nu - u_1^\nu), \quad \frac{dS_1^{12}}{dt} = 0 \tag{38}$$

$$\frac{dS_1^{13}}{dt} = -\frac{1}{2}[h_{1,t}S_0^{13} + h_{2,t}S_0^{23}] \tag{39}$$

and

$$\frac{dS_1^{23}}{dt} = \frac{1}{2}[h_{1,t}S_0^{23} - h_{2,t}S_0^{13}] \tag{40}$$

The Eqs.(22) give:

$$v_1^0 = \frac{u_1^0}{\Delta_0^{1/2}} \tag{41}$$

$$v_1^1 = \frac{u_1^1}{\Delta_0^{3/2}}(1 - 2\Omega\frac{S_0^{12}}{m}) + \frac{1}{2\Delta_0^{3/2}}[h_{1,tz}\frac{S_0^{12}S_0^{23}}{m^2} - h_{2,tz}\frac{S_0^{12}S_0^{13}}{m^2}] \tag{42}$$

$$v_1^2 = \frac{u_1^2}{\Delta_0^{3/2}}(1 - 2\Omega\frac{S_0^{12}}{m}) + \frac{h_{1,tz}S_0^{12}}{2m\Delta_0^{3/2}}[\frac{S_0^{13}}{m} - \frac{S_0^{23}}{m}] \tag{43}$$

$$
\begin{aligned}
v_1^3 = \frac{u_1^3}{\Delta_0^{1/2}} + \frac{1}{\Delta_0^{3/2}}\{h_{1,tz}[(\frac{S_0^{13}}{m})^2 \\
- \quad (\frac{S_0^{23}}{m})^2] + 2h_{2,tz}(\frac{S_0^{23}}{m})^2\} \\
- \quad \frac{\Omega}{\Delta_0^{3/2}}[u_1^2\frac{S_0^{13}}{m} - u_1^1\frac{S_0^{23}}{m}]
\end{aligned}
\qquad (44)
$$

where comma means partial differentiation and $\Omega = \frac{eH}{m}$ is the cyclotron frequency.

We solve the Eqs.(35-44) and find:

$$u_1^0 = const., \;\; u_1^3 = const. \tag{45}$$

$$\begin{aligned} u_1^1 &= \frac{\Delta_0^{3/2}}{D_+}\{[B_{10}\sin(k_g z) \\ &+ B_{20}\cos(k_g z)][\cos(At) - \cos(\omega_g t)] \\ &+ [B_{10}\cos(k_g z) - B_{20}\sin(k_g z)][\sin(At) \\ &- \sin(\omega_g t)]\} \end{aligned} \tag{46}$$

and

$$\begin{aligned} u_1^2 &= \frac{\Delta_0^{3/2}}{D_+}\{[B_{10}\sin(k_g z) \\ &+ B_{20}\cos(k_g z)]\sin(At) - [B_{10}\cos(k_g z) \\ &- B_{20}\sin(k_g z)]\cos(At)\} \\ &+ \frac{\sin(\omega_g t)}{D_-}[B_{10}\sin(k_g z) \\ &+ B_{20}\cos(k_g z)] + \frac{\cos(\omega_g t)}{D_+}[B_{10}\cos(k_g z) \\ &- B_{20}\sin(k_g z)] \end{aligned} \tag{47}$$

where

$$B_{10} = \frac{\omega_g k_g}{2}[h_{10}(\frac{S_0^{13}}{m}) + h_{20}(\frac{S_0^{23}}{m})]$$
$$- \quad \frac{\Omega \omega_g k_g}{2\Delta_0^{3/2}} h_{10}(\frac{S_0^{12}}{m})[\frac{S_0^{13}}{m} - \frac{S_0^{23}}{m}] \qquad (48)$$

$$B_{20} = -\frac{\omega_g k_g}{2}[h_{10}(\frac{S_0^{23}}{m})$$
$$- \quad h_{20}(\frac{S_0^{13}}{m})][1 - \frac{\Omega}{\Delta_0^{3/2}}(\frac{S_0^{12}}{m})] \qquad (49)$$

$$A = \frac{\Omega}{\Delta_0^{3/2}}[1 - 2\Omega(\frac{S_0^{12}}{m})] \qquad (50)$$

and

$$D_{\pm} = \Omega - 2\Omega^2(\frac{S_0^{12}}{m}) \pm \omega_g \Delta_0^{3/2} \qquad (51)$$

From Eqs.(41-44) and Eqs.(45-47) we find: $v_0^1 = u_0^1 = const.$ and

$$\begin{aligned}
v_1^1 = [\frac{1 - 2\Omega(\frac{S_0^{12})}{m}}{D_+}]\{[\cos(At) \\
- \quad & \cos(\omega_g t)][B_{10}\sin(k_g z) + B_{20}\cos(k_g z)] \\
+ \quad & [\sin(At) + \sin(\omega_g t)][B_{10}\cos(k_g z) \\
- \quad & B_{20}\sin(k_g z)]\} + \frac{\omega_g k_g}{2\Delta_0^{3/2}}[h_{10}(\frac{S_0^{12}}{m})(\frac{S_0^{23}}{m}) \\
- \quad & h_{20}(\frac{S_0^{12}}{m})(\frac{S_0^{13}}{m})]e^{i(k_g z - \omega_g t)} \qquad (52)
\end{aligned}$$

$$
\begin{aligned}
v_1^2 &= [\frac{1-2\Omega(\frac{S_0^{12})}{m}}{D_+}]\{\sin(At)[B_{10}\sin(k_g z) \\
&+ B_{20}\cos(k_g z)] - [\cos(At) \\
&- \cos(\omega_g t)][B_{10}\cos(k_g z) - B_{20}\sin(k_g z)]\} \\
&+ [\frac{1-2\Omega(\frac{S_0^{12}}{m})}{D_-}]\sin(\omega_g t)[B_{10}\sin(k_g z) \\
&+ B_{20}\cos(k_g z)] + \frac{\omega_g k_g}{2\Delta_0^{3/2}}(\frac{S_0^{12}}{m})[(\frac{S_0^{13}}{m}) \\
&- (\frac{S_0^{23}}{m})]h_{10}e^{i(k_g z-\omega_g t)} \qquad (53)
\end{aligned}
$$

$$
\begin{aligned}
v_1^3 &= \frac{u_1^3}{\Delta_0^{1/2}} - \frac{\Omega S_0^{13}}{mD_+}\{\sin(At)[B_{10}\sin(k_g z) \\
&+ B_{20}\cos(k_g z)] - \cos(At)[B_{10}\cos(k_g z) \\
&- B_{20}\sin(k_g z)]\} + \frac{\Omega}{D_+}(\frac{S_0^{23}}{m})\{[\cos(At) \\
&- \cos(\omega_g t)][B_{10}\sin(k_g z) + B_{20}\cos(k_g z)] \\
&+ [\sin(At) + \sin(\omega_g t)][B_{10}\cos(k_g z) \\
&- B_{20}\sin(k_g z)]\} \\
&- \frac{\Omega S_0^{13}}{mD_-}\sin(\omega_g t)[B_{10}\sin(k_g z) + B_{20}\cos(k_g z)] \\
&- \frac{\Omega S_0^{13}}{mD_+}\cos(\omega_g t)[B_{10}\cos(k_g z) \\
&- B_{20}\sin(k_g z)] + \frac{\omega k_g}{2\Delta_0^{3/2}}\{h_{10}[(\frac{S_0^{13}}{m})^2 \\
&- (\frac{S_0^{23}}{m})^2] + 2h_{20}(\frac{S_0^{13}}{m})(\frac{S_0^{23}}{m})\}e^{i(k_g z - \omega_g t)} \qquad (54)
\end{aligned}
$$

Furthermore,with v^{μ} given from the above equations, the spinning's particle trajectories with respect to the coordinate system we consider and the initial conditions $t = 0, X_1^{\mu}(t = 0) = 0$ can easily obtained. Also, integrating Eqs.(38-40) we find the components of the $S_1^{\mu\nu}$ tensor. The exact formulas of the spinning's particle trajectories and those of the spin's tensor are a little lengthy.

However,from those equations we verify that because of the GW we have non-zero first order electric and magnetic moments of the spinning charged particle and in the absence of the GW, all these components disappear. The electric moments particularly, exhibit resonances because of the expressions $\Delta_0 = 1 - \frac{eH}{m}\frac{S_0^{12}}{m}$, $A = \frac{\Omega}{\Delta_0^{3/2}}[1 - 2\Omega(\frac{S_0^{12}}{m})]$ and $D_\pm = \Omega - 2\Omega^2(\frac{S_0^{12}}{m}) \pm \omega_g \Delta_0^{3/2}$, appearing in the equations. The above expressions become zero for certain values of the Larmor frequency $\Omega = \frac{eH}{m}$, the ratio $\frac{S_0^{12}}{m}$ and angular frequency of the GW, ω_g. At this stage we have to point out that while the electric moments of the spinning charged particle exhibit such an interesting behavior, the magnetic moments are independent from the magnetic field and the S_0^{12} zero order component of the spin tensor. Also, for the same reasons mentioned above the obtained solutions exhibit resonaces and in the neighborhood of those resonaces the charged spinning particle gains energy from the GW and accelerates radiating.

Discussion:

Dealing with the interaction of a GW with a spinning particle in the presence of a uniform magnetic field in the linearized theory of general relativity, we found the following results:

1) In the case where the GW and magnetic field are across the z axis, the components of the 4-velocity, 4-momentum and the spin tensor $S^{\mu\nu}$, exhibit resonance at $\Omega = (\frac{S_0^{12}}{m})^{-1}$ ($\Delta_0 = 0$). Due to the coexistence of the constant magnetic field with GW, a strong coupling between the frequency $\Omega = \frac{eH}{m}$ and the magnetic moment S_0^{12} of the charged spinning particle occur. This coupling gives rise to the above resonance.

Also, for the same reasons mentioned above, some other resonances appear in the solutions describing the $X^{\mu}(t)$ and $S^{\mu\nu}$, which are roots to 4-order polynomial in terms of Ω; $D_{\pm} = 0 \Rightarrow 4\Omega^4(\frac{S_0^{12}}{m})^2 - 4\Omega^3[\omega_g^2(\frac{S_0^{12}}{m})^3 - 4\frac{S_0^{12}}{m})] + \Omega^2[1 - 3\omega_g^2(\frac{S_0^{12}}{m})^2] + 3\omega_g^2(\frac{S_0^{12}}{m})\Omega - \omega_g^2 = 0$.

2) It is interesting to notify that in the absence of the GW, the magnetic field and the components of the spin tensor decouple and the magnetic resonances disappear. In this case, where the GW does not exist, the motion of a spinning charged point-particle of mass m and charge q is described in an 4-dimensional Minkowski space time by its position $X^{\mu}(t)$, defining the particle's world line, its 4-velocity u^{μ}, which is tangent to the world-line and its polarization tensor $D_{\mu\nu}(t)$, an antisymmetric 4-tensor which combines the intrinsic magnetic dipole moment **M**(a pseudo 3-vector) with the intrinsic electric dipole moment **d** (a real 3-vector) at every given point of the world-line through the relations $D_{ij} = \frac{1}{c}\epsilon_{ijk}M_k$

and $-iD_{i4} = d_i$ where $(i, j, k) = 1, 2, 3$. In the absence of external fields, the intrinsic dipole moments are found from the values of **M** and **d** (rest frame of the free particle). Usually we are interested in charged particle with no intrinsic electric dipole moment in the rest frame of the free particle. This may expressed by the condition $D_{\mu\nu} u^{\nu} = 0$. On the other hand, the polarization tensor is relate to an intrinsic angular momentum tensor $S_{\mu\nu}$(spin tensor) through the expression $D_{\mu\nu} = (q/mc) S_{\mu\nu}$. From the above mentioned equations we have the condition $S_{\mu\nu} u^{\nu} = 0$. When this relation holds, $S_{\mu\nu}$ is space-like with only 3 non-zero components in the rest frame of the free particle e.g. $S_{ij}^{(0)} = \epsilon_{ijk} s^k$ and $S_{i0}^{(0)} = 0$. Besides, we have to point out that in the case of the unperturbed Minkowski space-time the classical spin is introduced somehow indirectly, via the electromagnetic polarization tensor, because the empirical meaning of classical magnetic and electric dipole moments is clear.

3) In the case that the GW does exist and in the limit of the high frequency approximation, the charged particle which initial is at rest, starts to have a combination of an orbital and spinning motion, described by the equations $X^\mu = X_0^\mu + X_1^\mu$ and the spin tensor $S^{\mu\nu} = S_0^{\mu\nu} + S_1^{\mu\nu}$. In this case, the charge particle, exhibits electric and magnetic moments eventhough initially had magnetic moments only. We verify that the electric moments exhibit the same resonances as the components of the 4-momentum and in the neighborhood of those resonaces energy is transferred from the GW to the spinning particle. The magnetic moments do not depend neither on the magnetic field nor on the component S^{12}. Under these circumstances one could hope to detect such GW.

A possible astrophysical environment where the interaction studied in this paper maybe be of relevance is the binary neutron star merger. In this scenario, two magnetized neutron stars merge, forming (if the equation of state allows it) a very massive, differentially rotating object and a possible low-mass disk around it (the object could survive for hundreds of seconds before collapsing to a black hole. The magnetosphere of this object will be rotating rapidly and be filled with plasma, while near the object, gravitational waves of large amplitude will be emitted. It would be interesting to study the conditions under which the interaction studied in the present paper could lead to observable phenomena during such a binary neutron star merger.

To make some further comments related to possible astrophysical application of the obtained solutions, we have to consider the Pauli-Lubanski covariant spin vector formula $S_\sigma = \frac{1}{2}\epsilon_{\rho\mu\nu\sigma}u^\rho S^{\mu\nu}$, which gives $S_0^{12} = S_0^3, \quad S_0^{13} = -S_0^2, \quad S_0^{23} = S_0^1$ and assume, for simplicity, that e.g. $S_0^3 = S_0^1 = S$, while $S_0^2 = 0$, then some typical values for this scenario of some astrophysical importance may be met when, for example, the amplitude of the GW is $h \approx 10^{-10}$, $H \approx 10^6 G$, and for an electron one has roughly $S \approx 10^{-13}$m. Such conditions can be found around various compact objects, for example near neutron stars which posses a magnetic field of the order $10^8 - 10^{12} G$ and emit GW due to glitches or rotational instabilities excited by accretion.

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