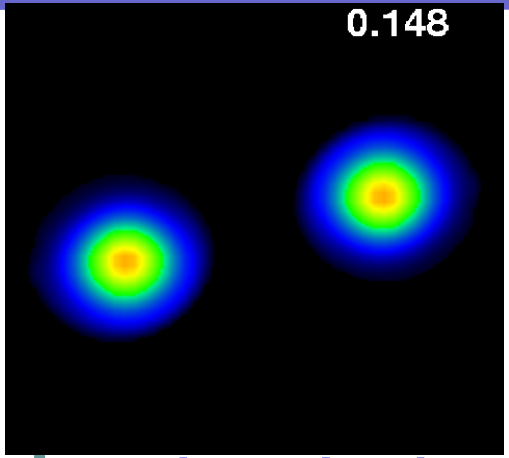
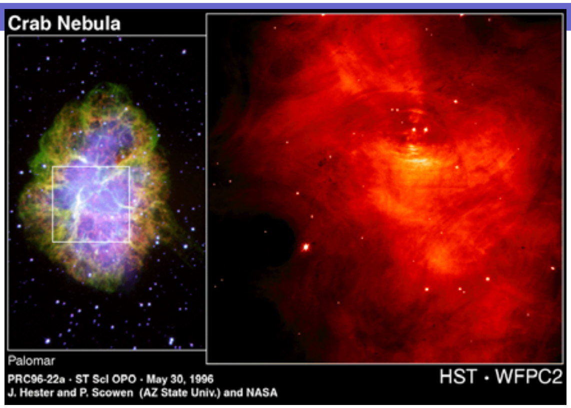
Sources of Gravitational Waves

2nd part

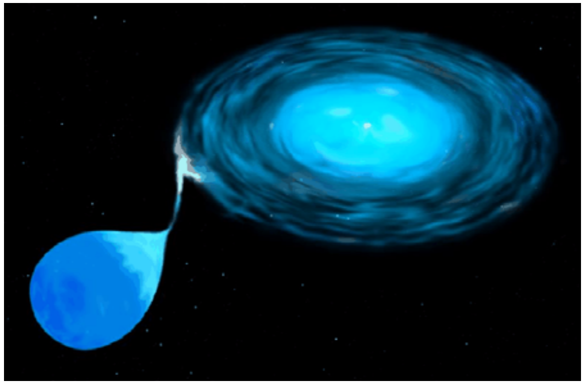
GW sources in ground-based detectors



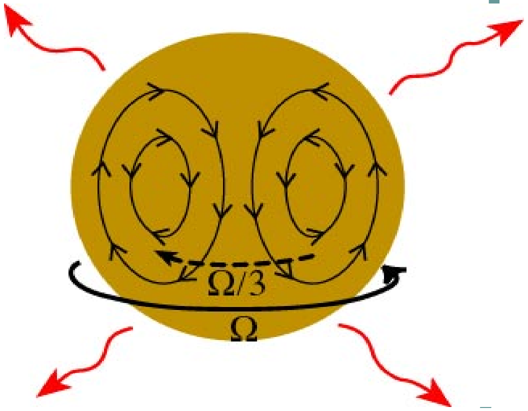
BH and NS Binaries



Supernovae, BH / NS formation

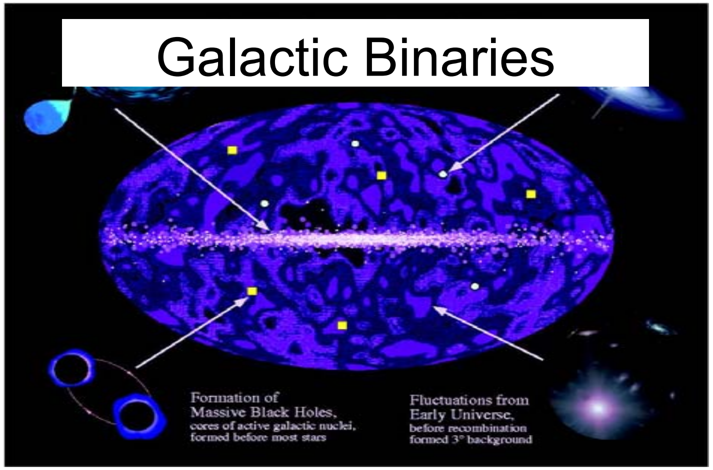


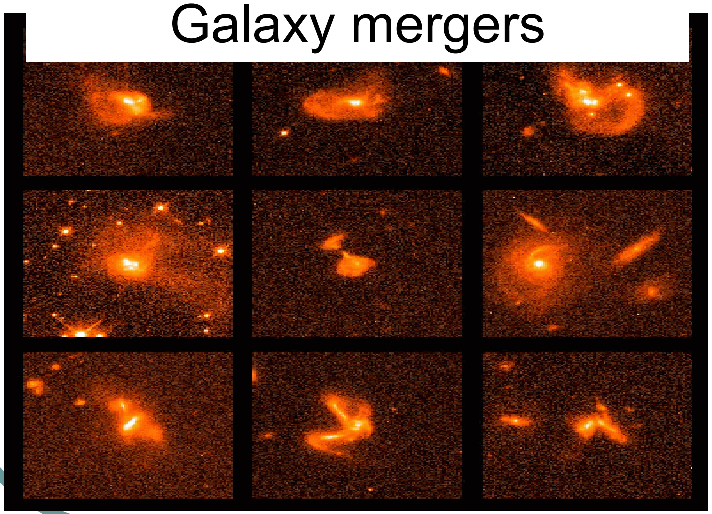
Spinning neutron stars in X-ray binaries

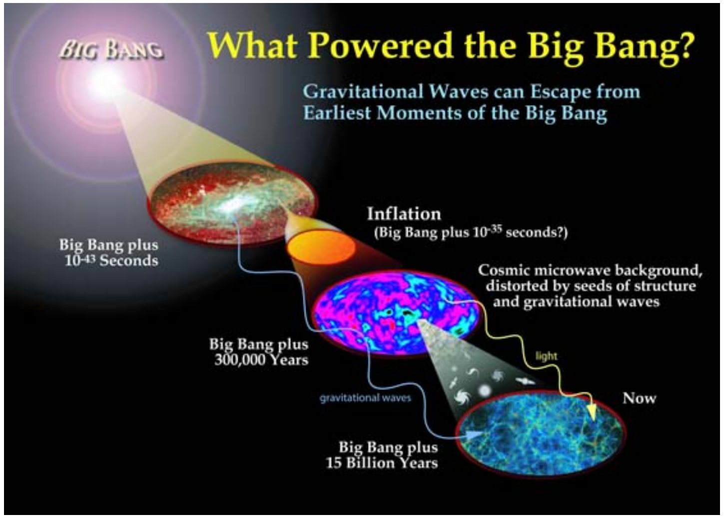


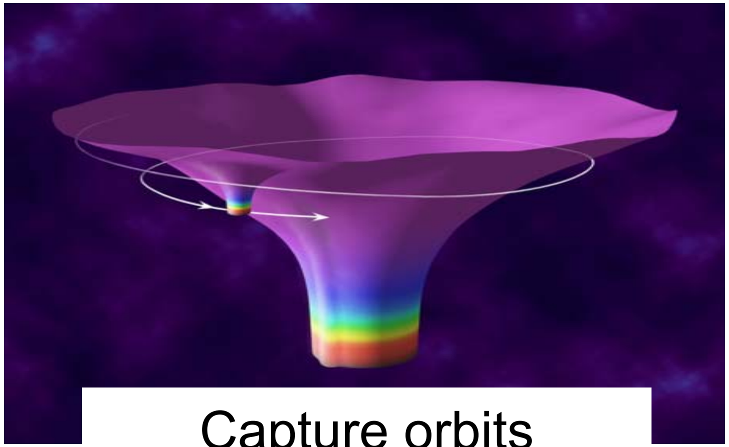
Young Neutron Stars

Sources in LISA

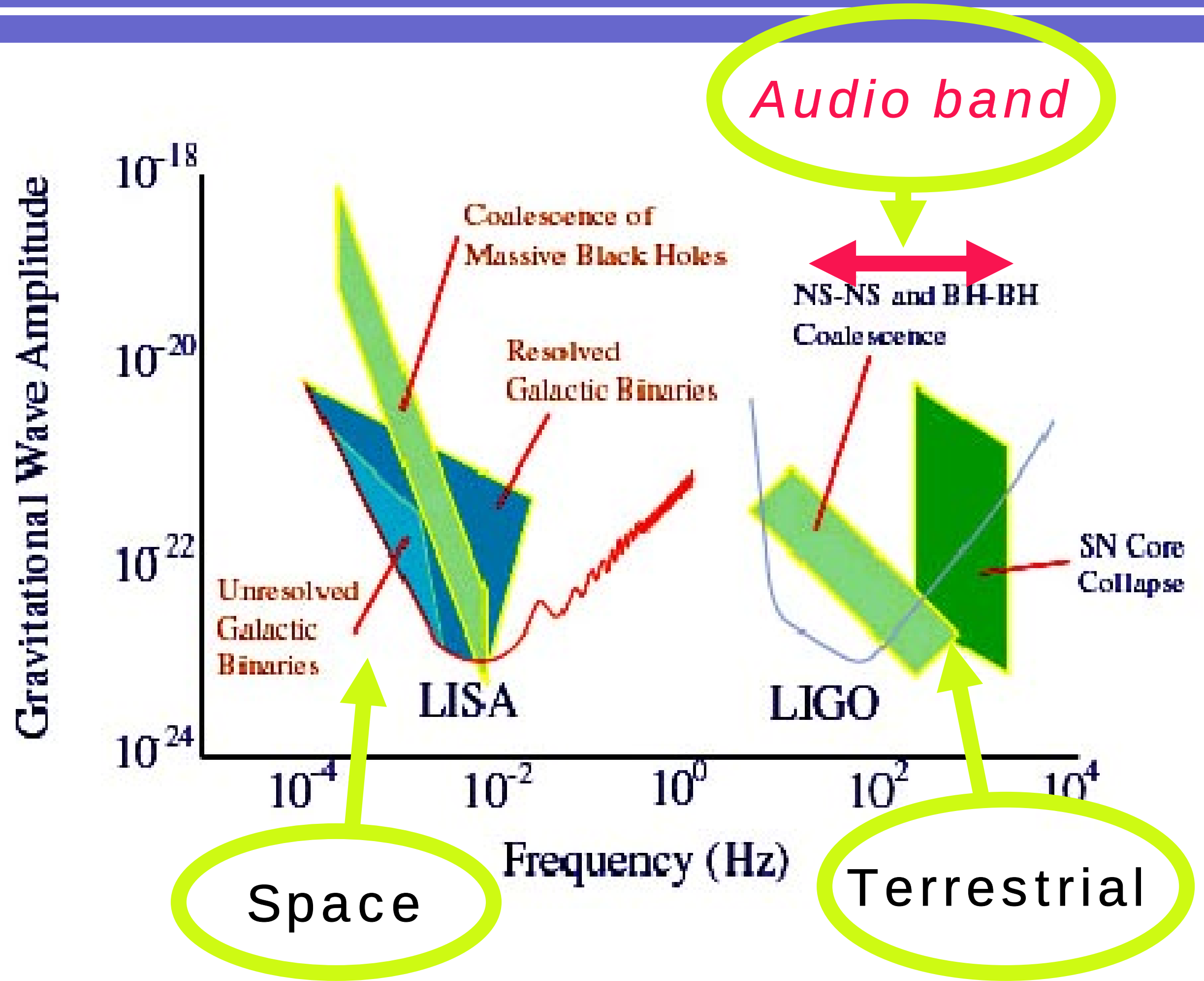








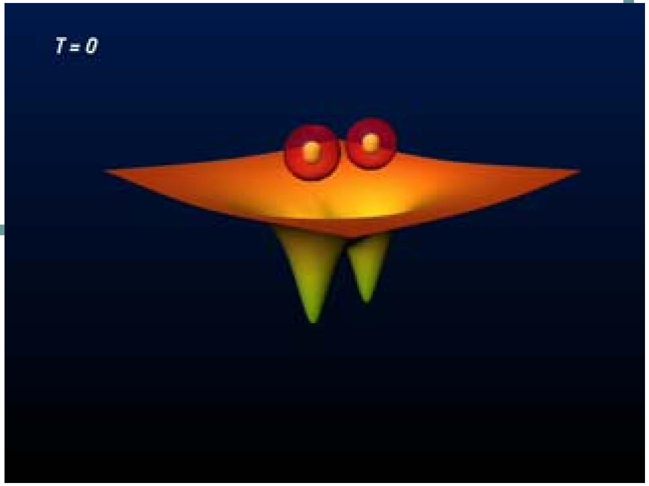
Frequency range of astrophysics sources



Gravitational Waves over ~8 orders of magnitude

Binary systems (NS/NS, NS/BH, BH/BH)

The best candidates and most reliable sources for broad band detectors



Coalescence of Compact Binaries

- During the frequency change from 100-200Hz GWs carry away $5 \times 10^{-3} M_{\odot} c^2$.
- In LIGOs band
  - **NS/NS** (~16000 cycles)
  - **NS/BH**(~3500 cycles)
  - **BH/BH**(~600 cycles)
- The GW amplitude is:

$$h \approx 7.5\times10^{-23}\left(\frac{M}{2.8M_{\odot}}\right)^{2/3}\left(\frac{\mu}{0.7M_{\odot}}\right)\left(\frac{f}{100Hz}\right)^{2/3}\left(\frac{100Mpc}{r}\right)$$

- Larger total mass improves detection probability.

events/year	LIGO-I	LIGO-II
NS/NS	~0.5	~60-500
BH/NS	~0.02	~80
BH/BH	~0.8	~2000
Total	0.8	$\gtrsim$2000

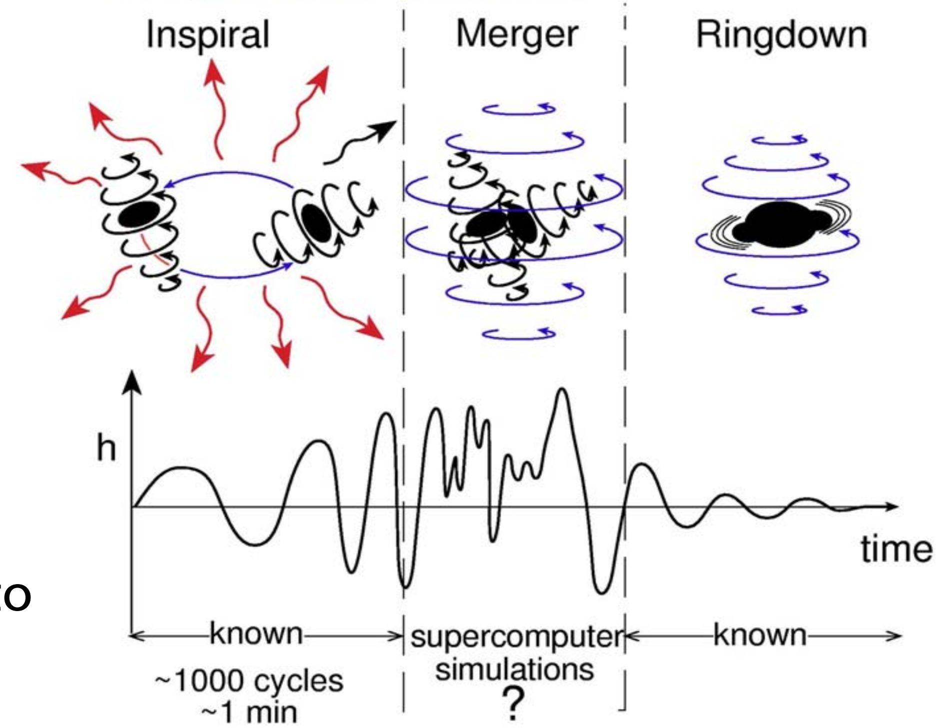
•**Phase effects are important**, if the signal and the template get out of phase their cross correlation will be reduced.
•**High accuracy templates** are needed for accurate detection.

Gravitational Waves from Binaries

Generically, there are 3 regimes in which black holes radiate:

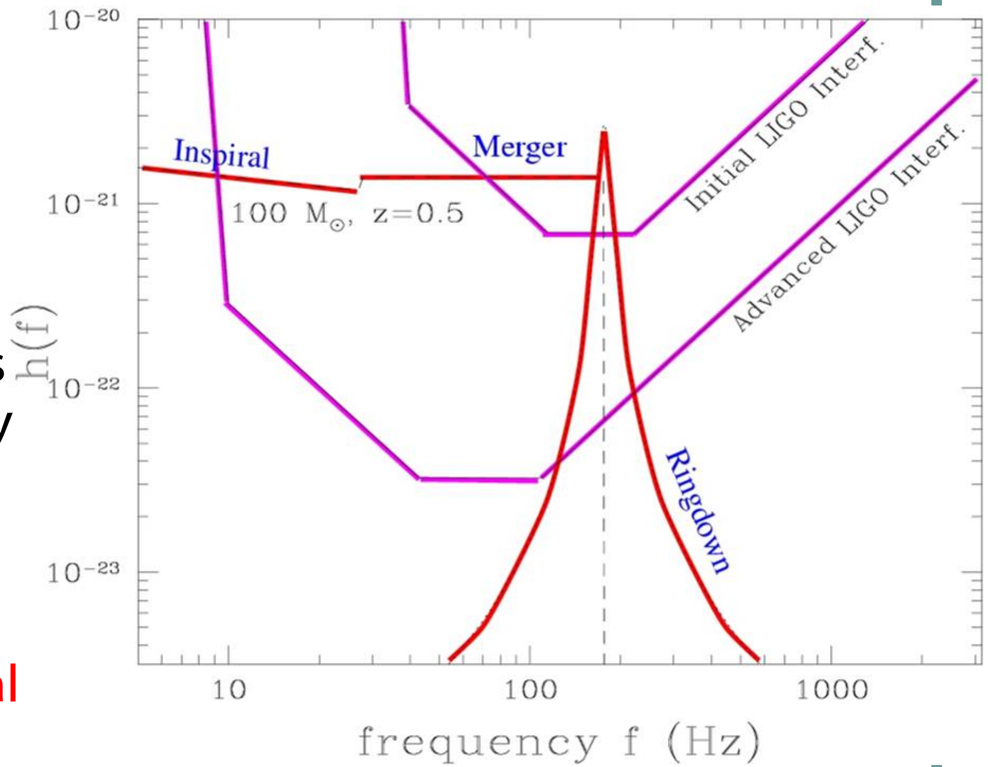
- **Orbital in-spiral**: **PN-approximations** or **point-particle orbits.**
- **Plunge/merger** after the last stable orbit: **numerical simulations** or **point-particle orbits**.
- **Ring-down** of the disturbed black hole as it settles down to a Kerr hole: **perturbation theory** of black holes.

- Merger Science: nonlinear dynamics of spacetime curvature



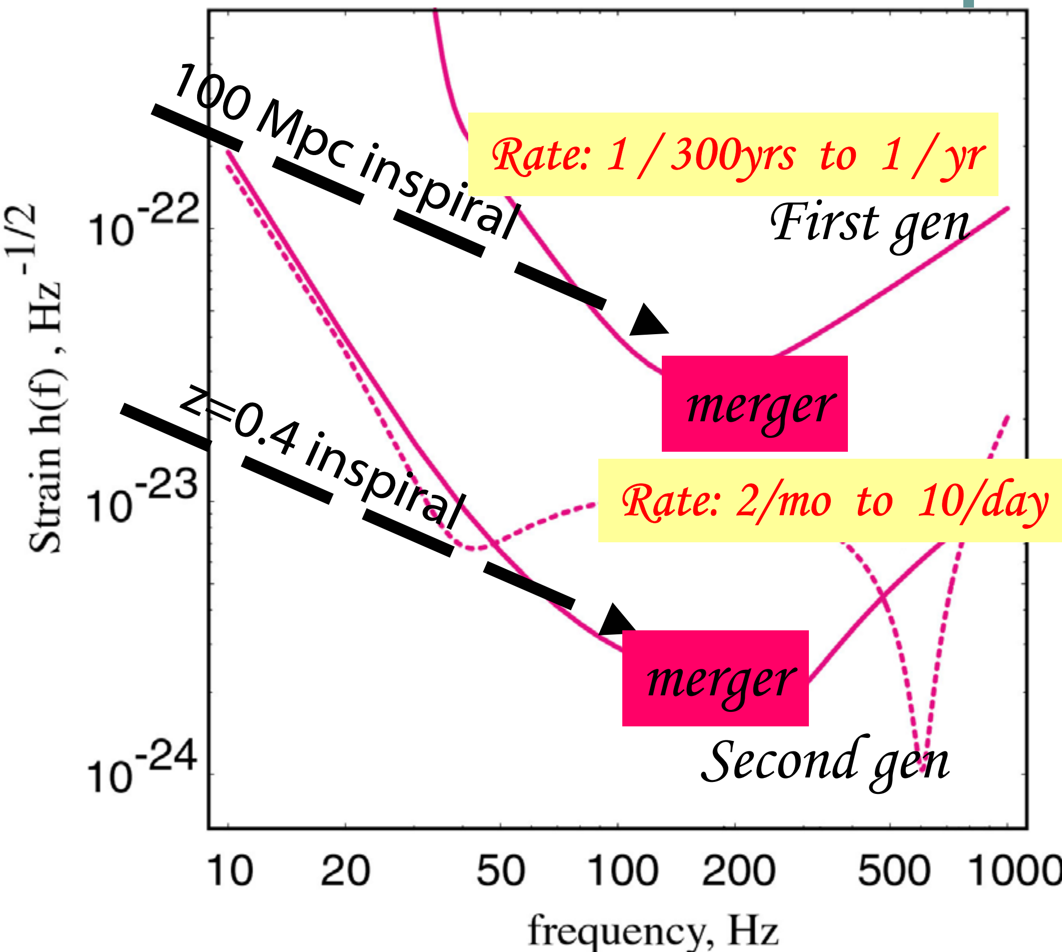
BH/BH coalescence

- The inspiral, merger, and ringdown waves from **50M$_\odot$ BH binaries** as observed by initial and advanced LIGO.
- The energy spectra are coming from crude estimates (10% of the total mass energy is radiated in merger waves and 3% in ringdown waves).
- We observe that the inspiral phase is not visible with initial LIGO, for this case Numerical Relativity is important.



Possible First Source: Binary Black Hole Coalescence

- $10 M_\odot + 10 M_\odot$ BH/BH binary
- Event rates based on population synthesis,
- mostly globular cluster binaries.
- Totally quiet!!



(Kip Thorne)

Core-collapse Supernova

The most spectacular astronomical event with exciting physics

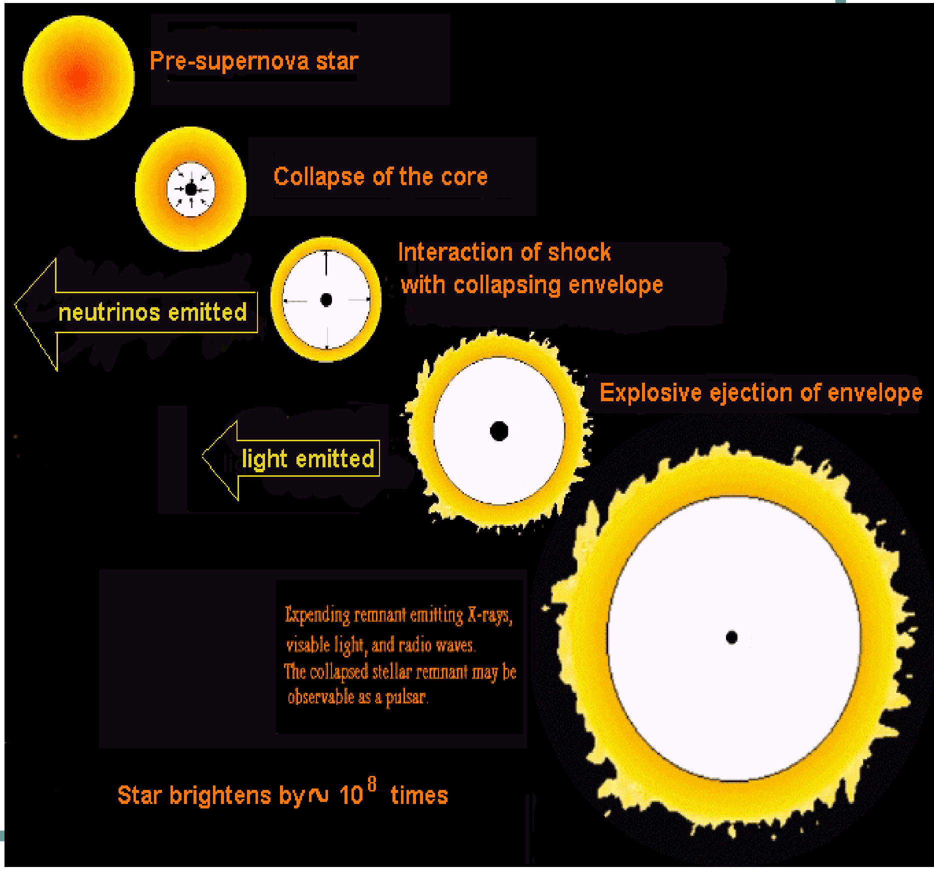
Supernovae/gravitational collapse

Supernova core collapse was the primary source of GW detectors.
GW amplitude uncertain by factors of 1,000's?

Rate 1/30yr in a typical galaxy

Detection would provide unique insight into SN physics:
– optical signal hours after collapse
– neutrinos after several seconds
– GWs emitted during collapse

Simulations suggest low level of radiation $(<10^{-6}\ M_{\odot}c^2?)$, but
– rotational instabilities possible
– observational evidence for asymmetry from speeding final neutron stars (release of $10^{-6}\ M_{\odot}\ c^2$ could explain 1000 km/s?)
– convective "boiling" observable to LMC



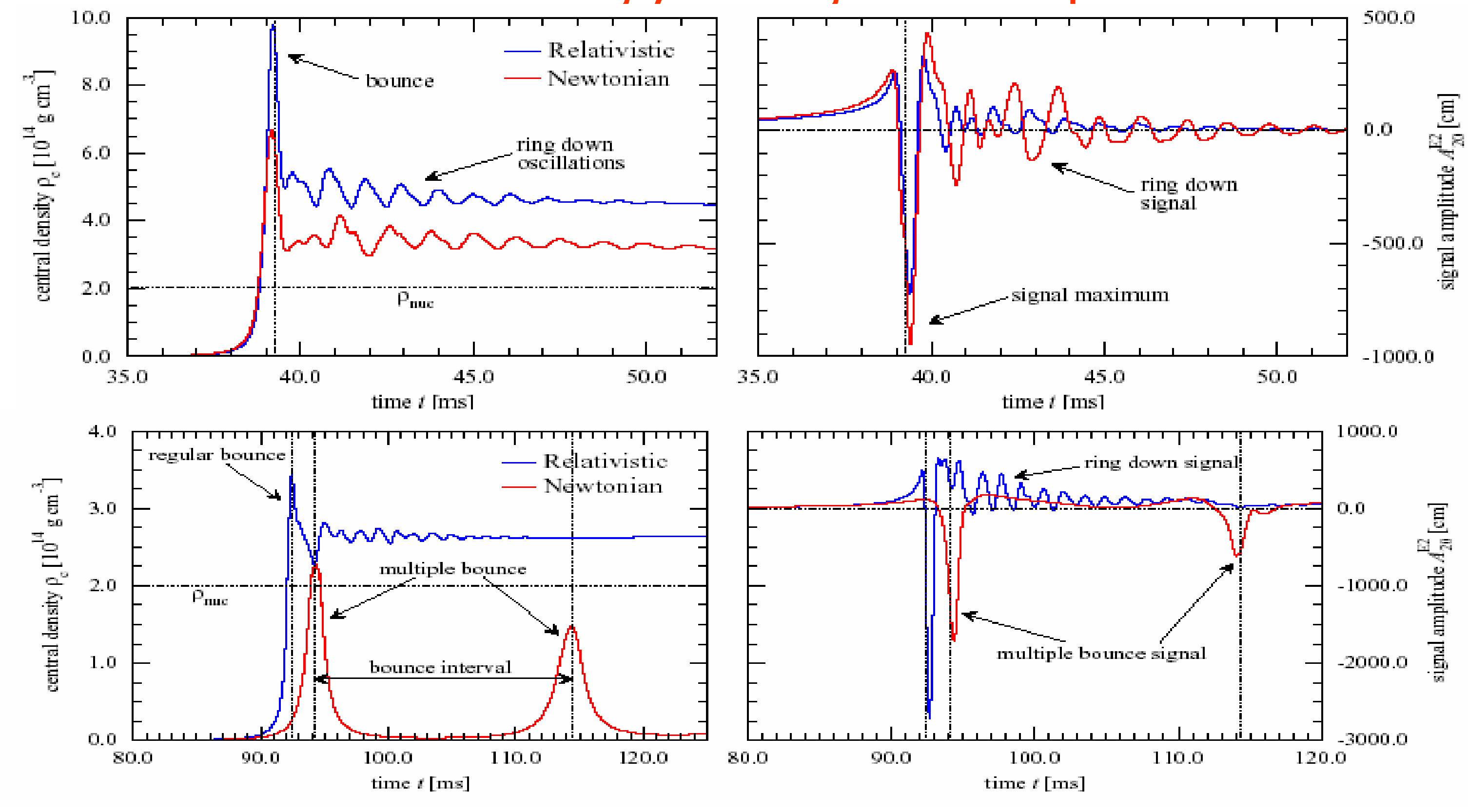
Core-Collapse Supernovae I

- **Stars more massive than ~8$M_\odot$ end in core collapse (~90% are stars with masses ~8-20$M_\odot$).**
- **Most of the material is ejected**
- **If $M>20M_\odot$ more than 10% falls back and pushes the PNS above the maximum NS mass leading to the formation of BHs (type II collapsars).**
- **If $M>40M_\odot$ no supernova is launched and the star collapses to form a BH (type I collapsars)**
- **Formation rate**:
  - 1-2 per century in the Galaxy (**Cappellaro & Turatto**)
  - 5-40% of them produce BHs through the fall back material
  - Limited knowledge of the rotation rate! Initial periods probably <20ms.
  - **Chernoff & Cordes** fit the initial spin with a Gaussian distribution peaked at 7ms. This means that 10% of pulsars are born spinning with millisecond periods.

Core-Collapse Supernovae II

Dimmelmeir, Font & Muller 2002

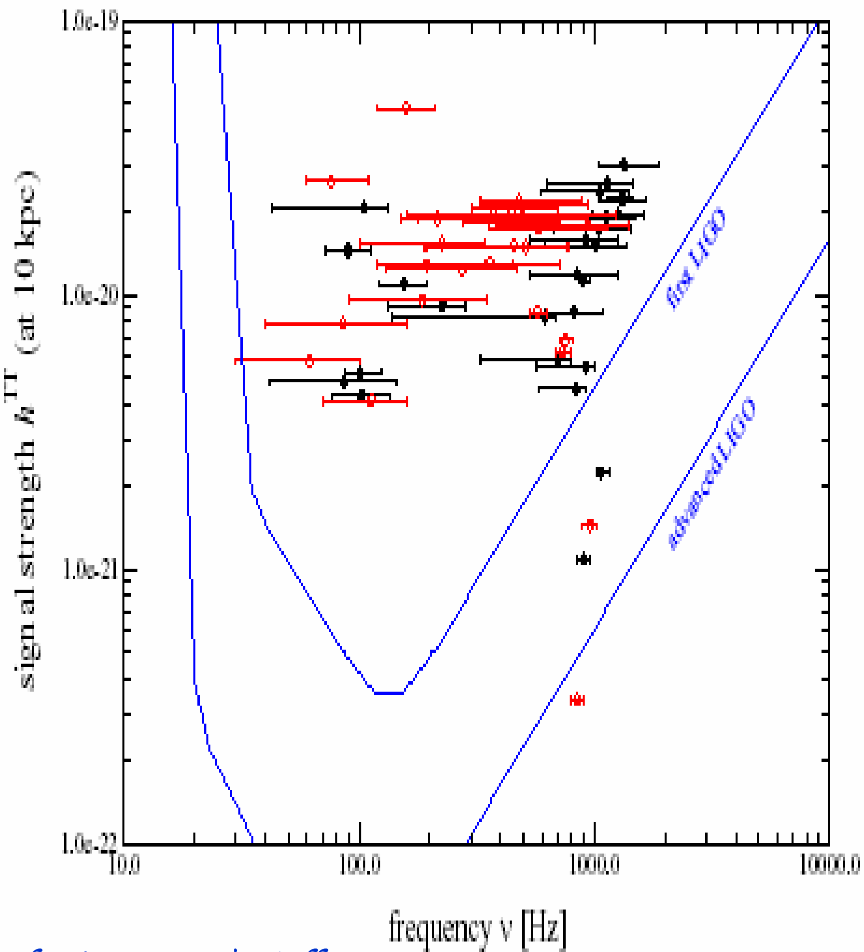
- Rotation increases strongly during the collapse.



•Multiple bounces are possible for a few models.

Core-Collapse Supernovae III

- GW amplitude $$h^{TT} \simeq 10^{-23} \frac{10Mpc}{d}$$
- Signals from Galactic supernova **detectable.**
- Frequencies **~1 kHz**
- The numerical estimates **are not conclusive.** A number of effects *(GR, secular evolution, non-axisymmetric instabilities)* **have been neglected!** (Axisymmetric collapse, Mathews-Wilson approximation…)
- Kicks suggest that a fraction of newly born NSs (and BHs) may be strongly asymmetric.
- Polarization of the light spectra in SN indication of asymmetry.

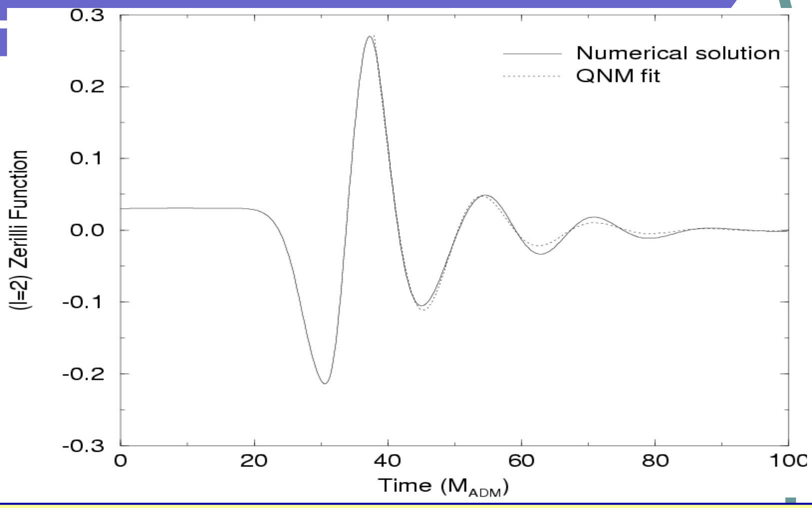


Dimmelmeir, Font & Muller 2002

Fragmentation and Fallback

- **A significant amount of remnants can fallback, subsequently spinning up and reheating the nascent NS.**
- **Instabilities can be excited again during such a process.**
- **BH-QNMs can be excited for as long as the process lasts.**
- **"Collapsars" accrete initially (for about ~2-3s) at rates ~1-10$M_\odot$ /sec ! Later at a rate ~0.1$M_\odot$ /sec for a few tenths of secs.**
- **Typical frequencies:** ~2kHz.
- Oscillation of matter surrounding the black hole *(Zanetti et al 2002)*
- If disk mass is :~1$M_\odot$ self-gravity becomes important and gravitaional instabilities (spiral arms, bars) might develop and radiate GWs *(Davies et al 2002, Fryer et al 2002)*
- The collapse material might fragments into clumps, which orbit for some circles like a binary system **(Fragmentation Instability).** *Needs density distribution to peaks off the center (maybe in Population III stars).*

Black-Hole Ringing I



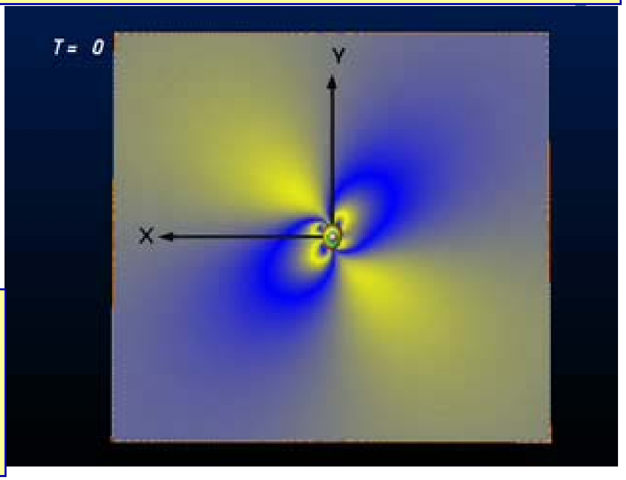
- The newly formed BH is ringing till settles down to the stationary Kerr state (QNMs).
- The ringing due to the excitation by the fallback material might last for secs
- Typical frequencies: ~1-3kHz

$$f_{m=2} \simeq 3.2\mathrm{kHz}\ M_{10}^{-1}[1-0.63(1-a/M)^{3/10}]$$

$$Q = \pi f \tau \approx 2(1-a)^{-9/20}$$

- The amplitude of the ringdown waves and their energy depends on the distortion of the BH.
- Energy emitted in GWs by the falling material: $\Delta E > 0.01 \mu c^2 (\mu/M)$

$$h_c \approx 2\times10^{-21}\left(\frac{\varepsilon}{0.01}\right)\left(\frac{d}{10Mpc}\right)^{-1}\left(\frac{\mu}{M_\odot}\right)$$

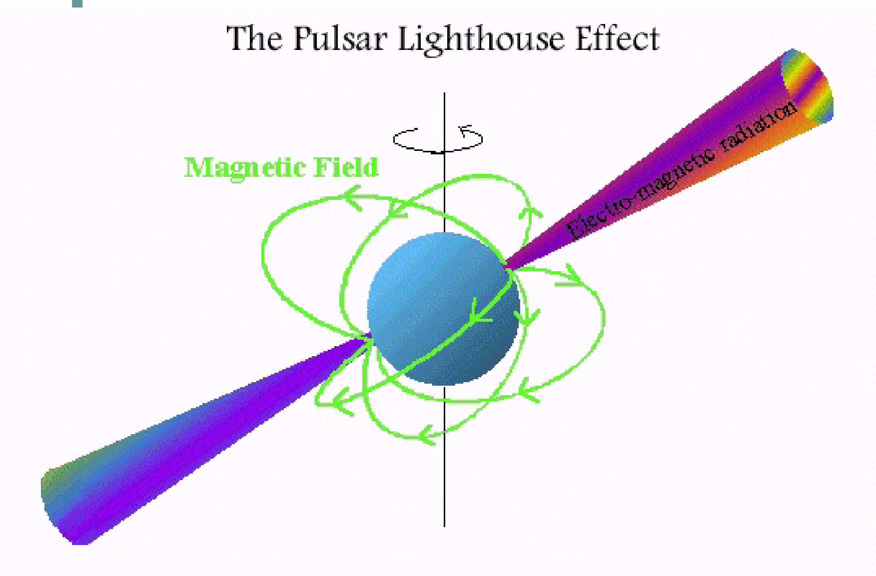


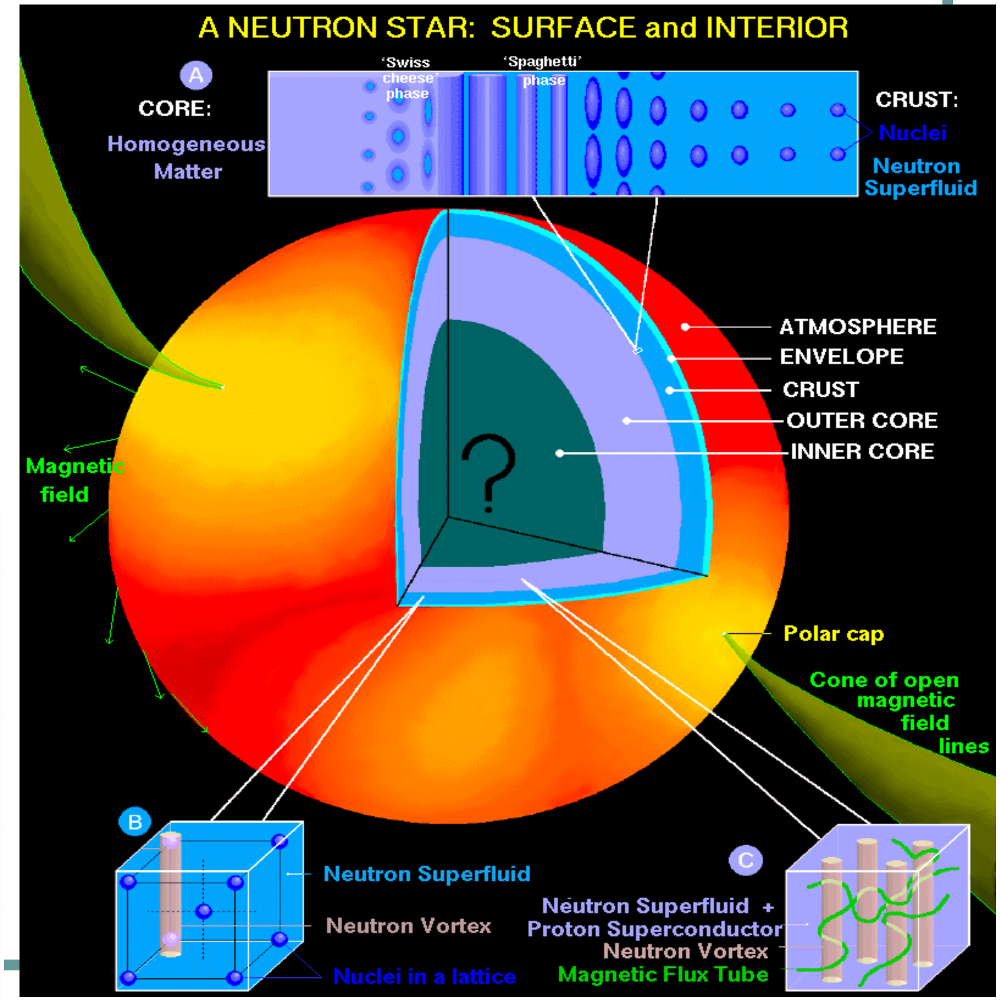
Oscillations & Instabilities

The end product of gravitational collapse

Neutron Stars

- Suggested: 1932
- Discovered: 1967
- Known: 1070+
- Mass: ~ 1.3-1.8 $M_{\odot}$
- Radius: ~ 8-14 Km
- Density: ~10^{15}gr/cm^3

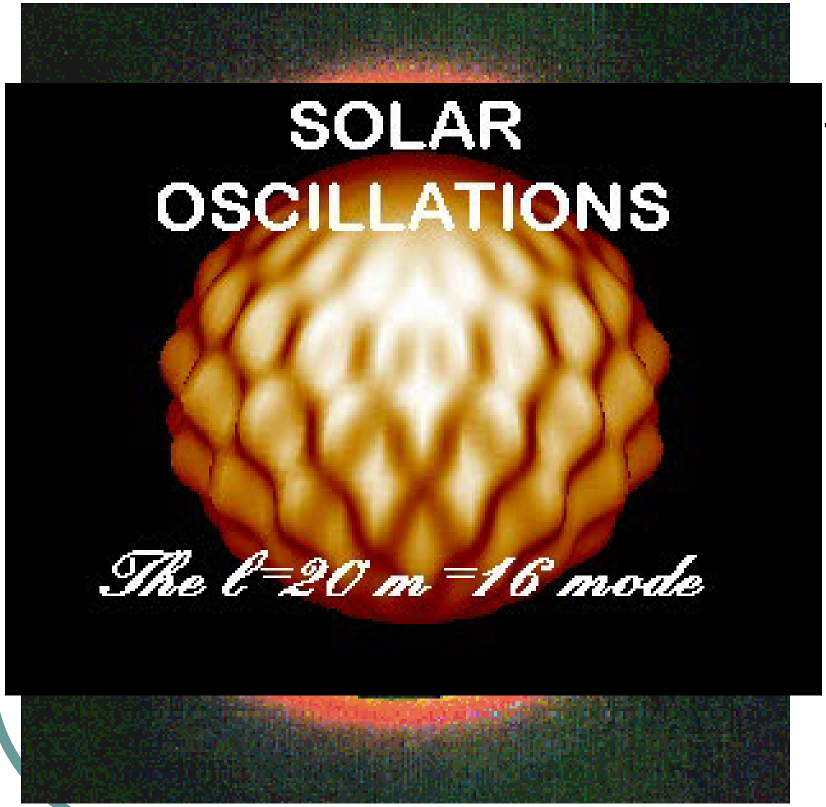




Stellar pulsation primer

For spherical stars we can (in the Cowling approximation) write the Euler equations as

$$\frac{\partial^2 \xi^i}{\partial t^2} = -\nabla^i \left(\frac{\delta p}{\rho} \right) + \frac{p\Gamma_1}{\rho} A^i (\nabla_j \xi^j)$$



Two main restoring forces, the pressure and the buoyancy associated with internal composition /temperature gradients, lead to:

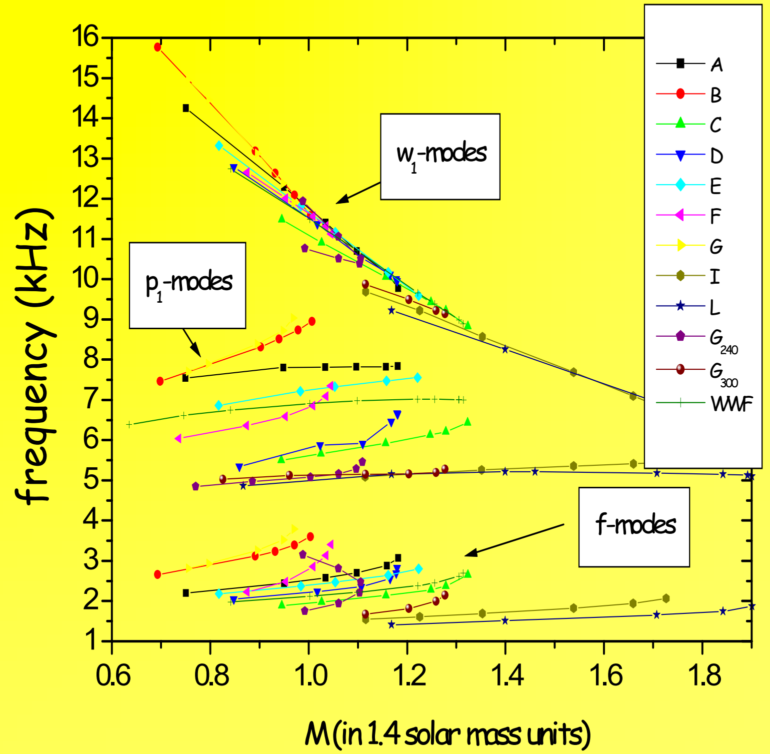
$$(\delta p \propto Y_{lm}(\theta, \varphi))$$

$$\omega^2 \approx \frac{l(l+1)c_s^2}{r^2}$$ p-modes

$$\omega^2 \approx -gA = \frac{A_i \nabla^i p}{\rho}$$ g-modes

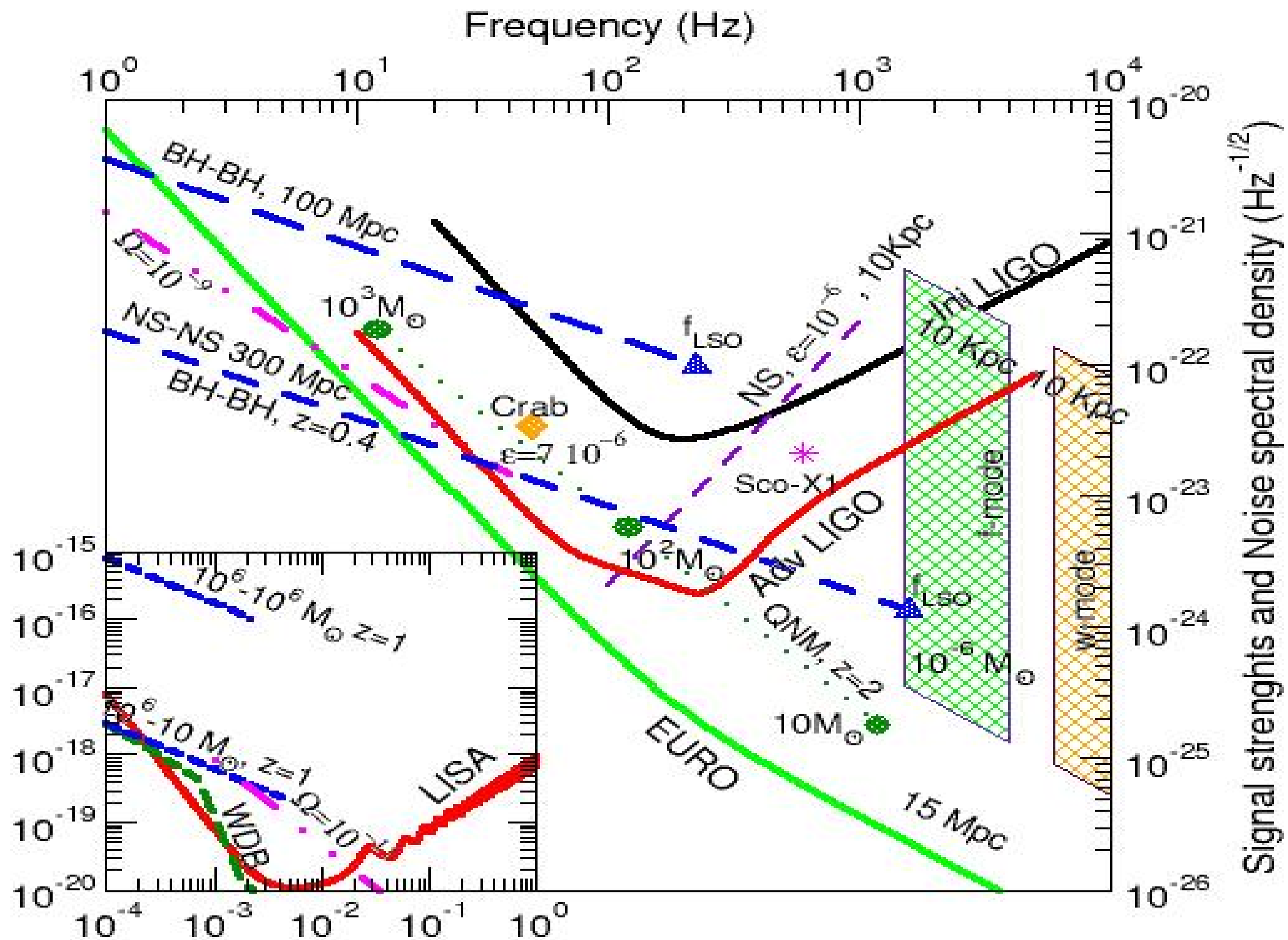
NS ringing : Stellar Modes

- **P-modes: main restoring force is the pressure**
- G-modes: main restoring force is the buoyancy force
- **F-mode: has an inter-mediate character of p- and g-mode**
- **W-modes: pure space-time modes (only in GR) (KK & Schutz)**
- **Inertial modes (r-modes) :main restoring force is the Coriolis force**
- Superfluid modes: Deviation from chemical equilibrium provides the main restoring agent



Each type of mode is sensitive to the physical conditions where the amplitude of the mode is greatest.

f- and *w*- modes in Interferometers



Stability of Rotating Stars

Non-Axisymmetric Perturbations

A general criterion is: $\beta = \frac{T}{W}$

T : rot. kinetic energy
W : grav. binding energy

Dynamical Instabilities

- Driven by hydrodynamical forces (bar-mode instability)
- Develop at a time scale of about one rotation period

$$\beta \geq 0.27$$

Secular Instabilities

- **Driven by dissipative forces (*viscosity, gravitational radiation*)**
- **Develop at a time scale of several rotation periods.**
- **Viscosity driven instability causes a Maclaurin spheroid to evolve into a non-axisymmetric Jacobi ellipsoid.**
- **Gravitational radiation driven instability causes a Maclaurin spheroid to evolve into a stationary but non-axisymmetric Dedekind ellipsoid.**

$$\beta \geq 0.14$$

The bar-mode instability I

For rapidly (differentially!) rotating stars with:

$$\beta = \frac{T}{|W|} > \beta_{dyn} \approx 0.27$$

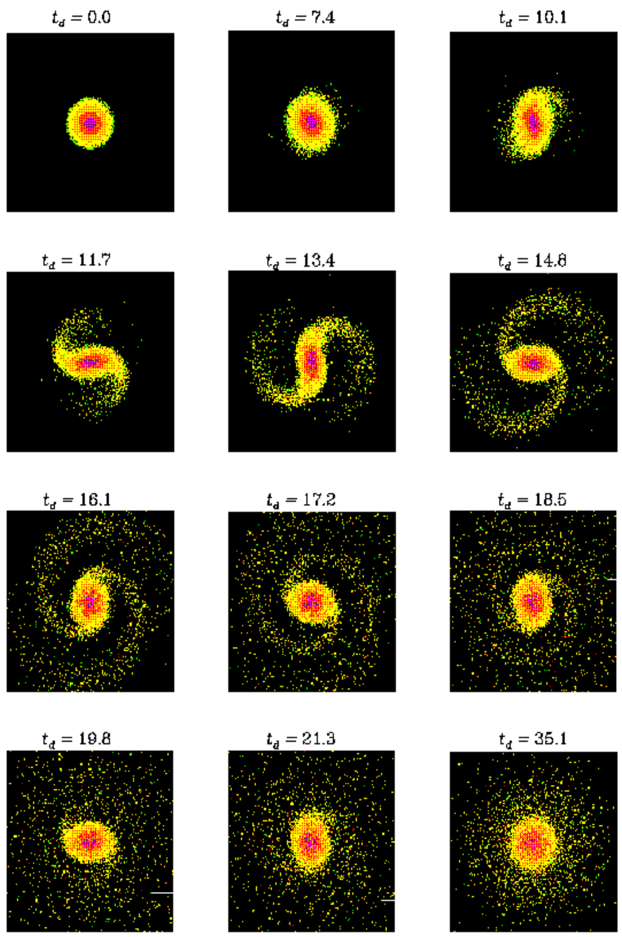
the "bar-mode" grows on a dynamical timescale.

$$h \approx 9\times10^{-23}\left(\frac{\varepsilon}{0.2}\right)\left(\frac{f}{3\text{ kHz}}\right)^2\left(\frac{15\text{ Mpc}}{\text{d}}\right)M_{1.4}R_{10}^2$$

If the bar persists for many (~10-100) rotation periods, the signal will be easily detectable from at least Virgo cluster.

–A considerable number of events per year in Virgo: ≤10^{-2} /yr/Galaxy

–**Frequencies ~1.5-3.5kHz**

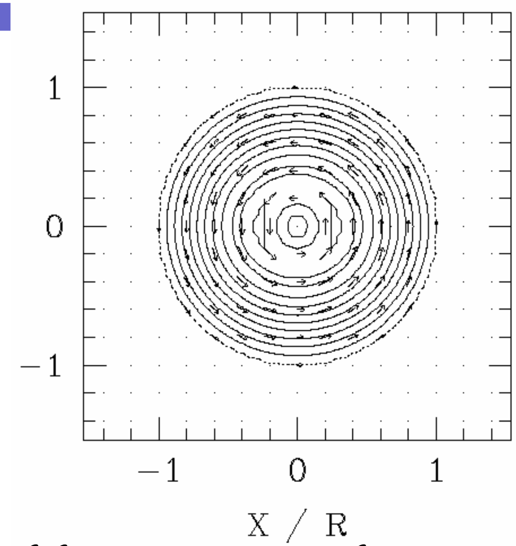


Remember mini-Grail: f_0~3.2kHz

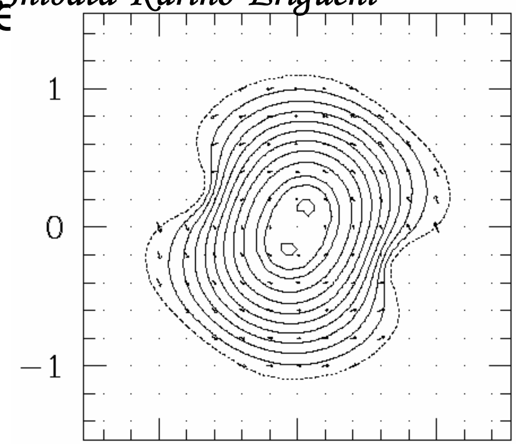
Bar Modes IV ???

- Bars can be also created during the merging of NS-NS, BH-NS, BH-WD and Collapsars (type II).
- GR enhances the onset of the instability ($\beta_{dyn} \gtrsim 0.24$) and β decreases with increasing M/R.
- Bar-mode instability might happen for much smaller β if centrifugal forces produce a peak in the density off the source's rotational center.
- Highly differentially rotating stars are shown to be dynamically unstable for significantly lower β (even when $\beta \gtrsim 0.01$).

$$h_{eff} \simeq 3\times 10^{-22}\left(\frac{f}{800Hz}\right)^{1/2}\left(\frac{R_{eq}}{30km}\right)\left(\frac{M}{1.4M_{\odot}}\right)^{1/2}\left(\frac{100Mpc}{d}\right)$$



Shibata-Karino-Eriguchi

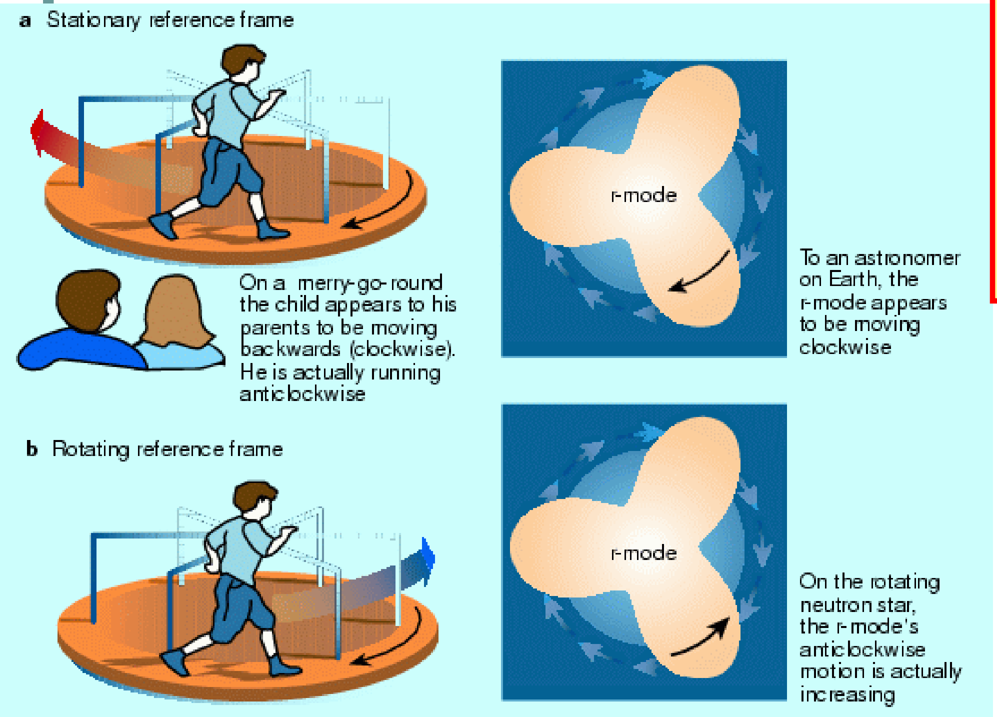


The CFS instability

Chandrasekhar 1969: *Gravitational waves lead to a secular instability*

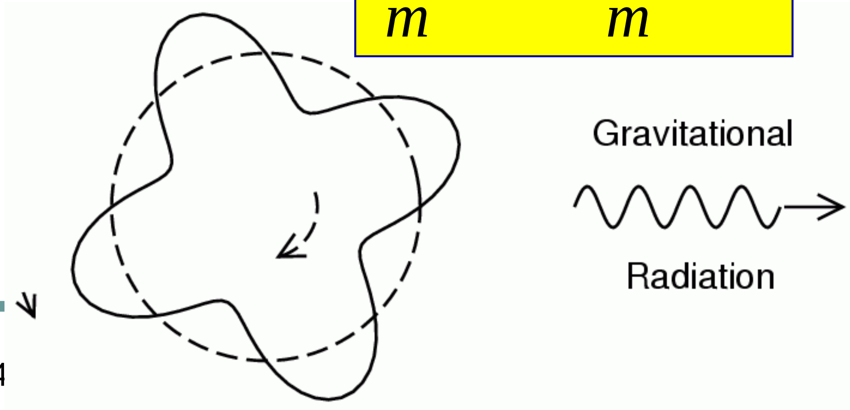
Friedman & Schutz 1978: *The instability is generic, modes with sufficiently large m are unstable.*

A **neutral mode** of oscillation signals the onset of CFS instability.



Radiation drives a mode unstable if the mode pattern moves backwards according to an observer on the star, but forwards according to someone far away.

$$\frac{\omega_{in}}{m} = -\frac{\omega_{rot}}{m} + \Omega$$



The r-mode instability I

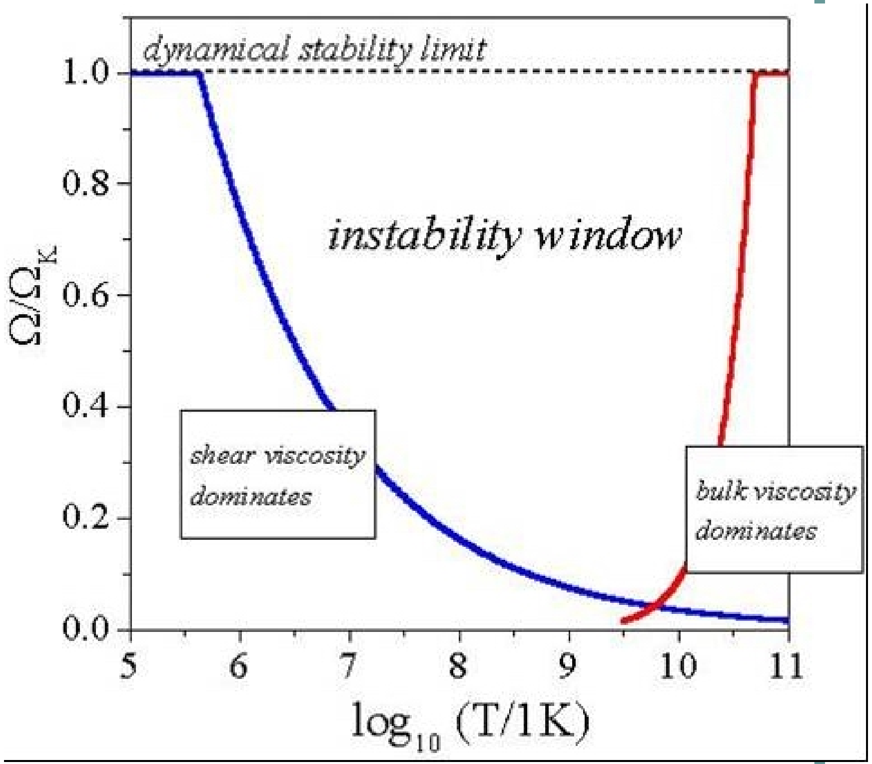
In a frame rotating with the star, the r-modes have frequency

$$\omega_{\text{rot}} = \frac{2m}{l(l+1)}\Omega$$

Meanwhile in the inertial frame

$$\frac{\omega_{\text{inertial}}}{m} = -\frac{\omega_{\text{rot}}}{m} + \Omega = \Omega\left(1 - \frac{2}{l(l+1)}\right)$$

The r-modes are unstable to the emission of GWs at all rotation rates!



The $l=m=2$ r-mode grows on a timescale

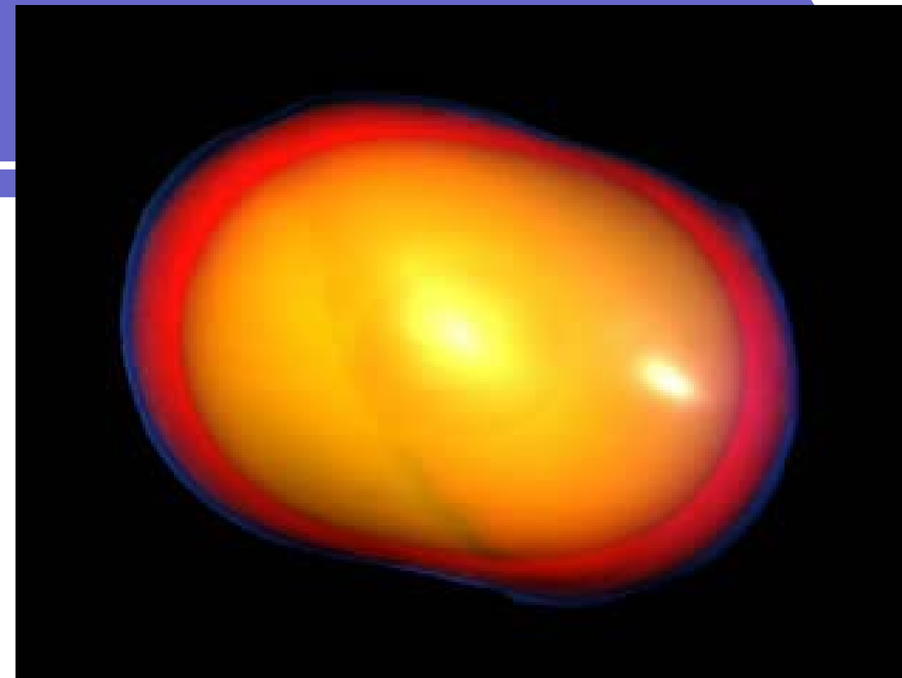
$$t_{gw} \approx 20 - 40 \left(\frac{1.4M_\odot}{M}\right)\left(\frac{10\text{ km}}{R}\right)^4\left(\frac{P}{1\text{ ms}}\right)^6\text{ s}$$

The instability will grow if

$$\tau_{\text{visc}} \geq \tau_{\text{inst}}$$

R-modes III

- **GW amplitude depends on α (the saturation amplitude).**
- **Mode coupling might not allow the growth of instability to high amplitudes (Arras et al)**
- **The existense of *crust*, hyperons in the core, magnetic fields, affect the efficiency of the instability.**
- **For newly born neutron stars might be quite weak ; unless we have the creation of a strange or hyperon star**
- **Old accreting neutron stars, probably the best source!**

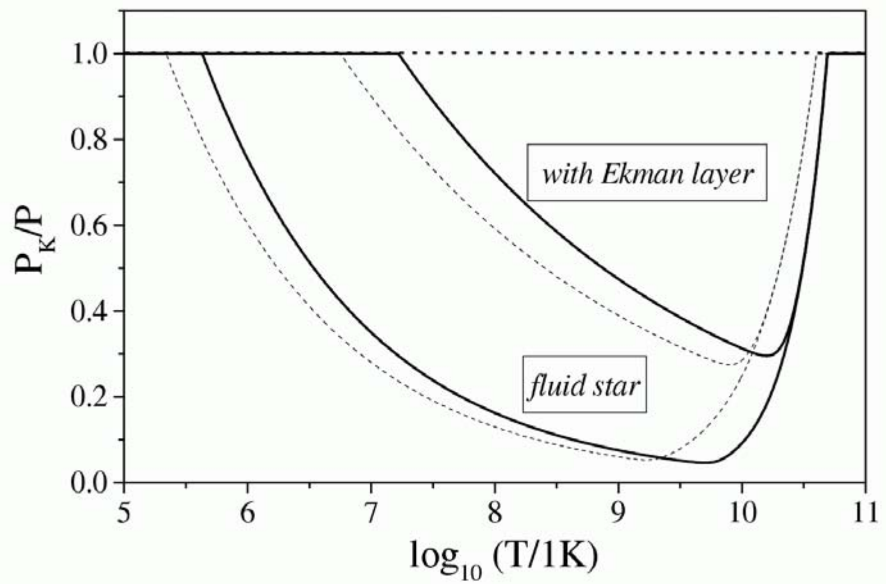


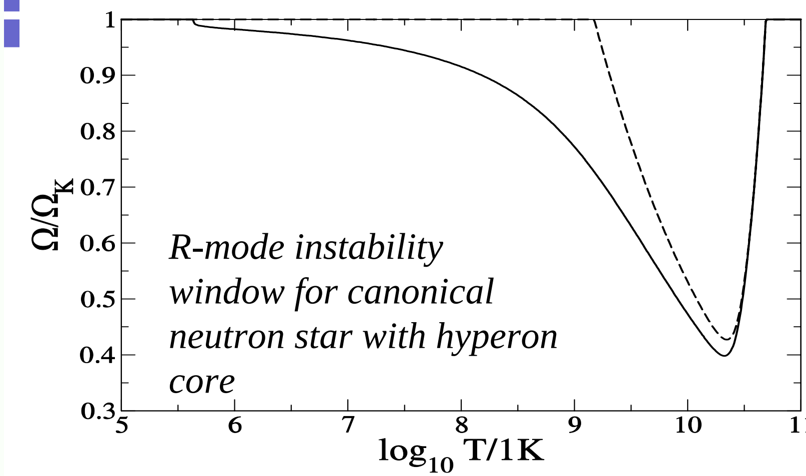
Lindblom-Vallisneri-Tohline

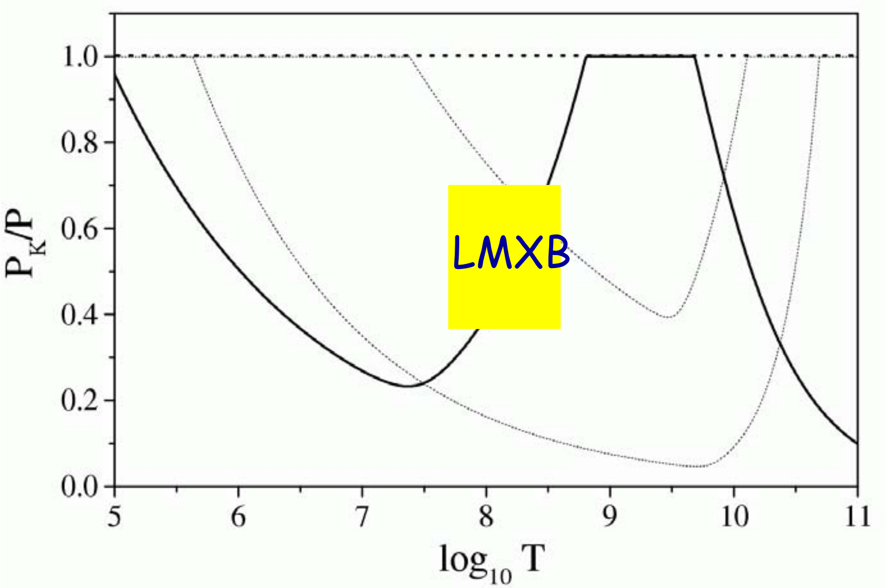
$$h(t) \approx 10^{-23} \alpha \left(\frac{\Omega}{1\ kHz} \right) \left(\frac{10 Mpc}{d} \right)$$

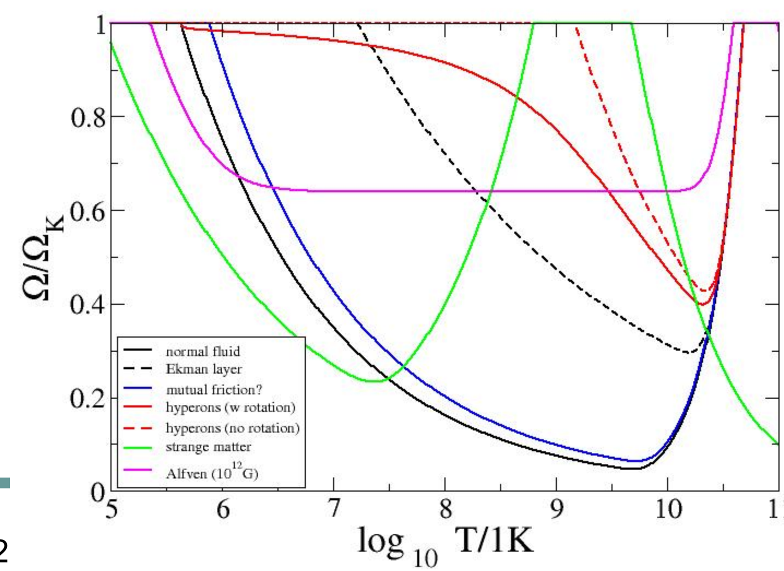
$$\alpha \simeq 10^{-2} - 10^{-4}$$

R-mode instability vs EOS

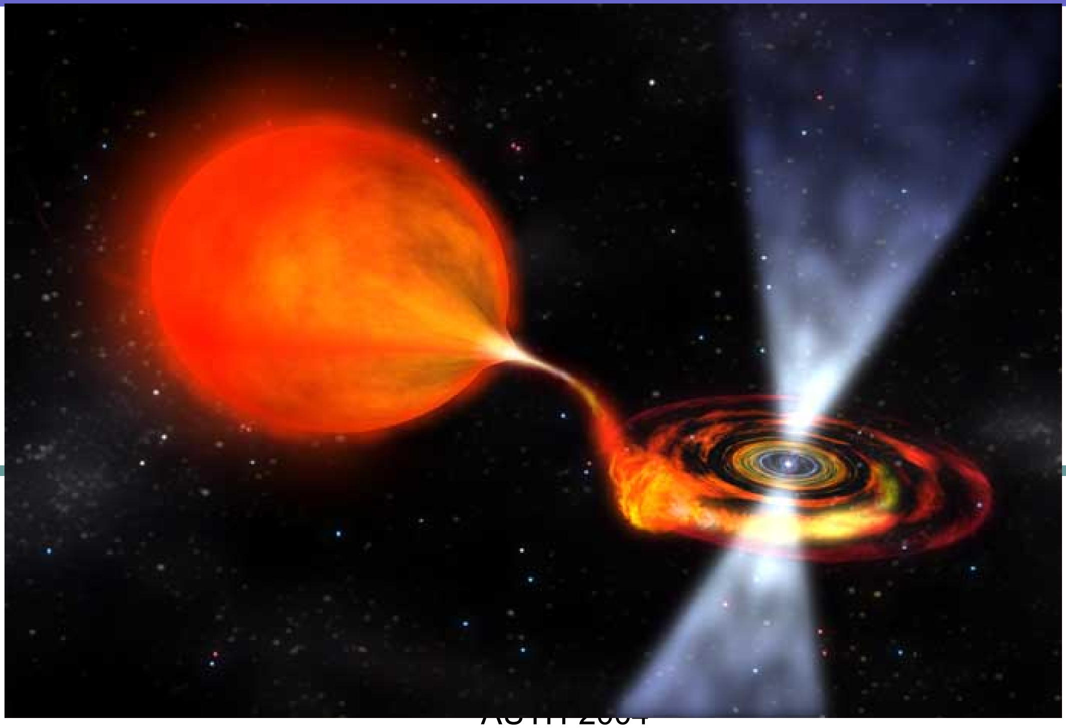




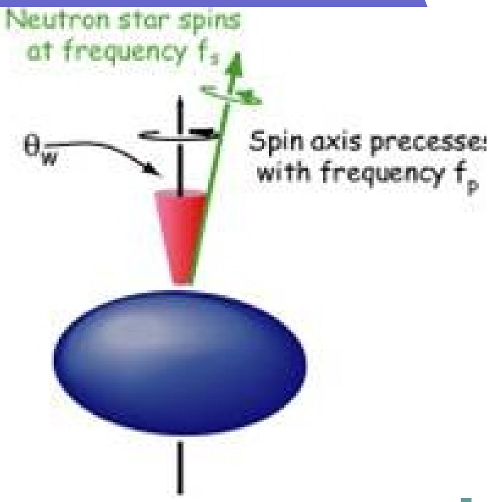




Isolated & Old NS

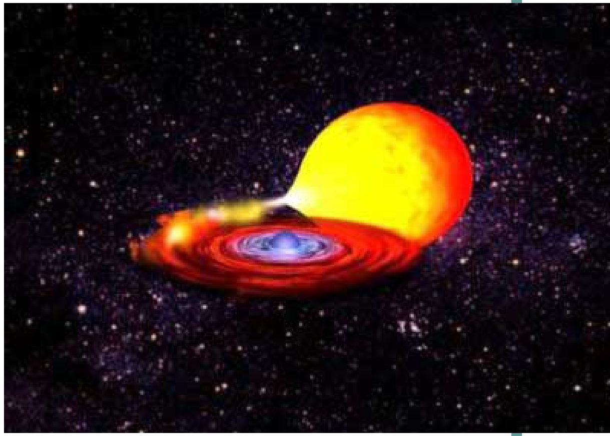


Isolated NS



- Wobbling or Deformed NS (many interesting features but highly uncertain the degree of deformation)

$$\varepsilon \geq 2\times10^{-8}\left(\frac{1kHz}{f}\right)^2\left(\frac{r}{10kpc}\right)$$



- LMXBs : if accretion spin-up torque on NS is counterbalanced by GW emission then Sco X-1 and a few more might be detectable around 500-700 Hz.

LMXBs might be as robust source of GWs as the binary systems!

The Wagoner mechanism (1984) Papaloizou &Pringle (1978)

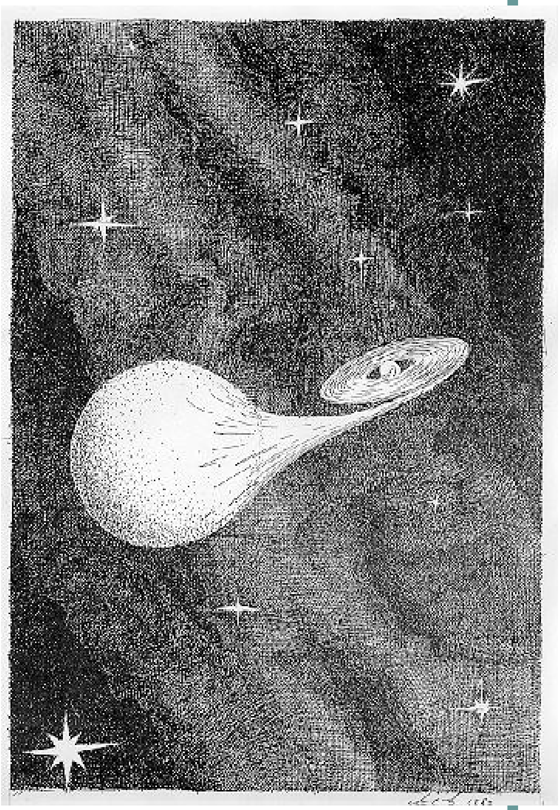
Key idea: Emission of GW balances accretion torque. Strength of waves can be inferred from X-ray flux.

Requires deformation:

$$\varepsilon = 4.5\times10^{-8}\left(\frac{\dot{M}}{10^{-9}M_\odot/yr}\right)^{1/2}\left(\frac{300\text{ Hz}}{\nu_s}\right)^{5/2}$$

Observational evidence (?):

clustering of spin-frequencies in LMXB (250-590 Hz)



Possible GW mechanisms:

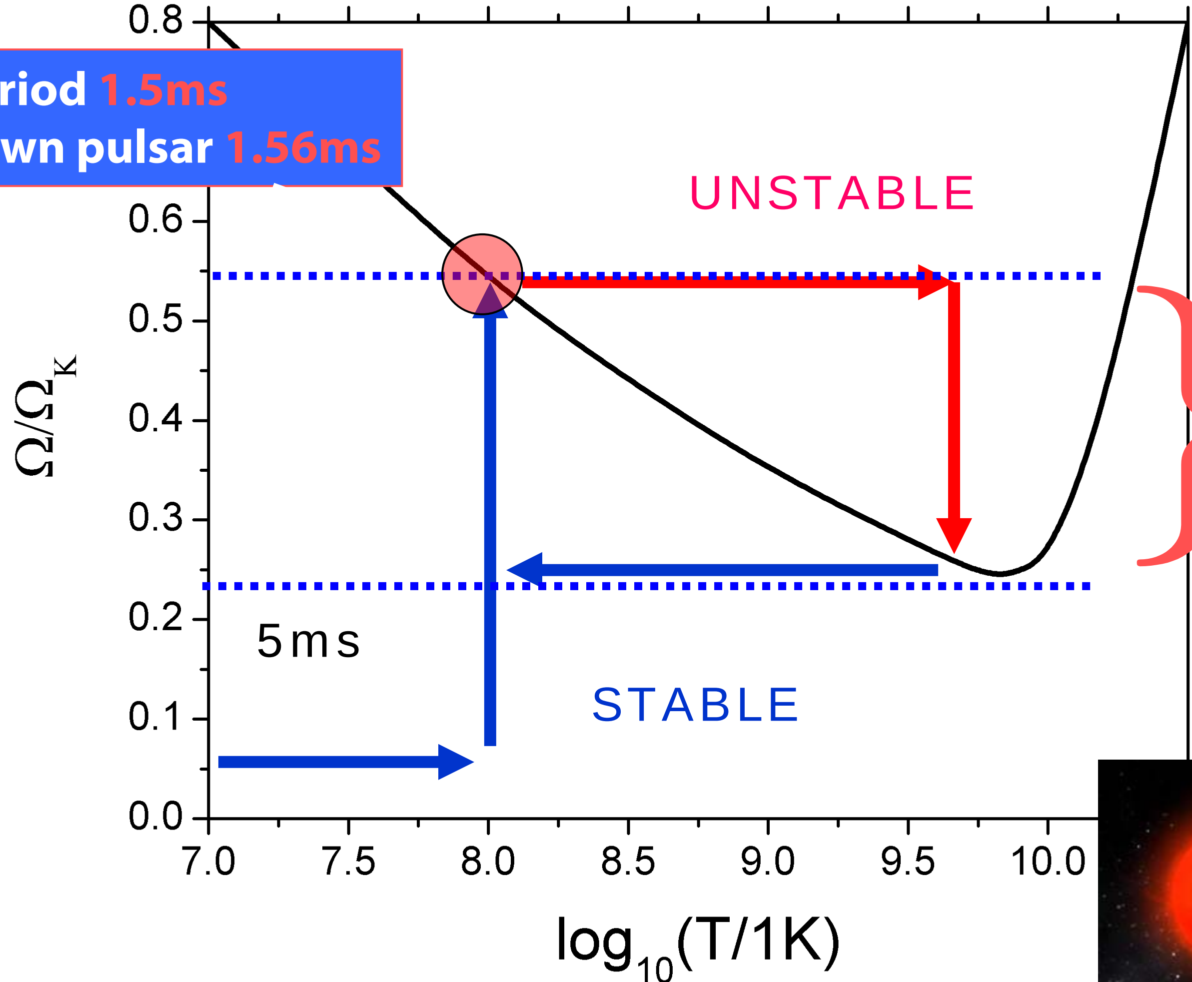
– accretion induced asymmetry

– unstable r-modes: strong bulk viscosity may shift instability window to lower temperatures; accreting stars can reach quasi-equilibrium state

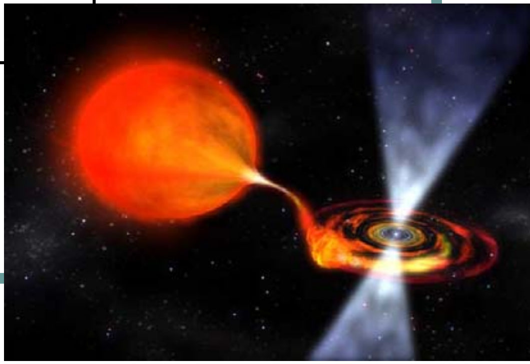
Variable accretion rate: coherent integration of signal only meaningful for 20 days or so.

LMXBs & r-modes

Limiting Period 1.5ms
Fastest known pulsar 1.56ms



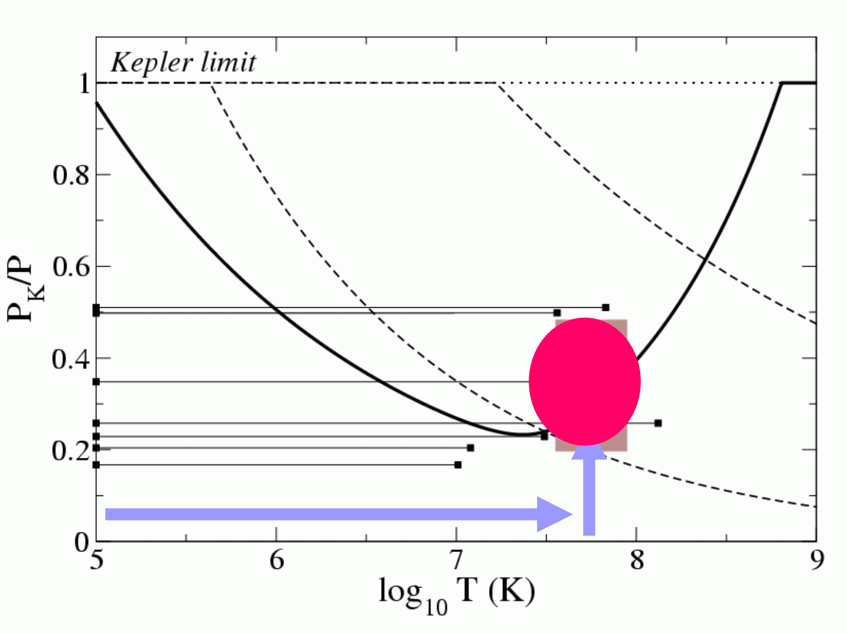
Period clustering of ms pulsars

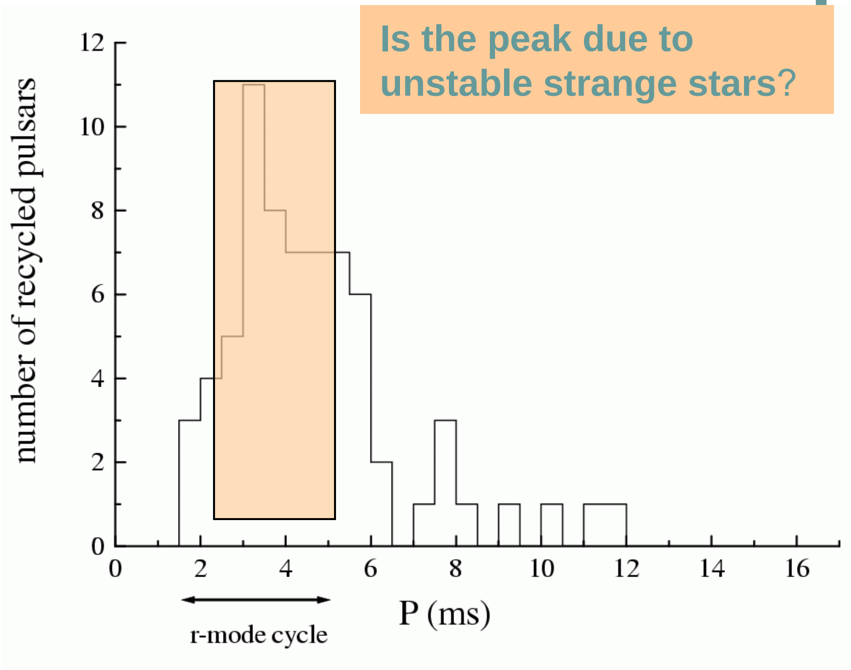


Andersson, KK, Stergioulas `99
Andersson, Jones, KK, Stergioulas `00

Clustering of millisecond pulsar periods
Strange & Hyperon Stars

- **Strange and Hyperon Stars have a quite different instability window.**
- **They can be persistent sources of detectable GWs**





Andersson, Jones, KK 2002
Andersson, KK, Miller 2004

LIGO narrow banding

- **LIGO-I phase**
  - The only detectable source is BBHs (10M⊙)
- **LIGO-II phase (2006)**
  - Many sources…

Narrow banding for LMXBs