Gravitational Waves

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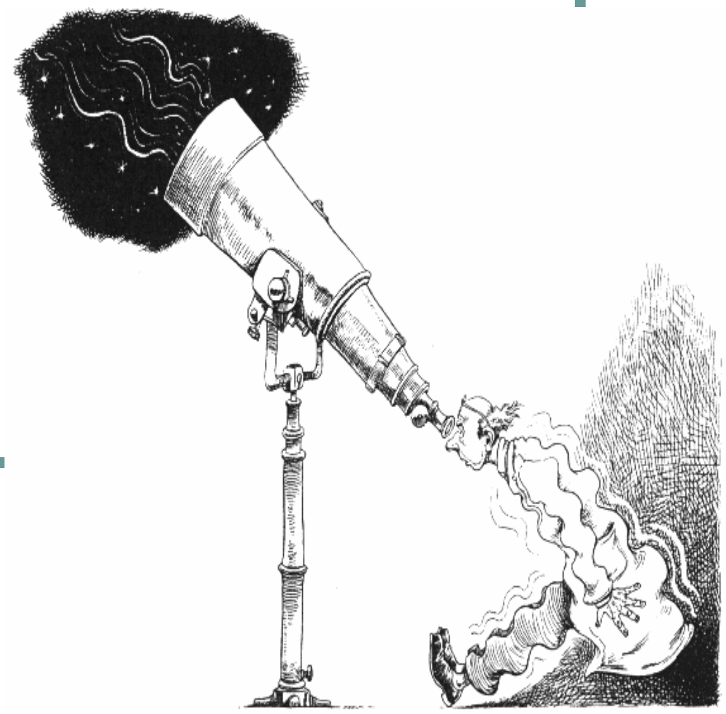
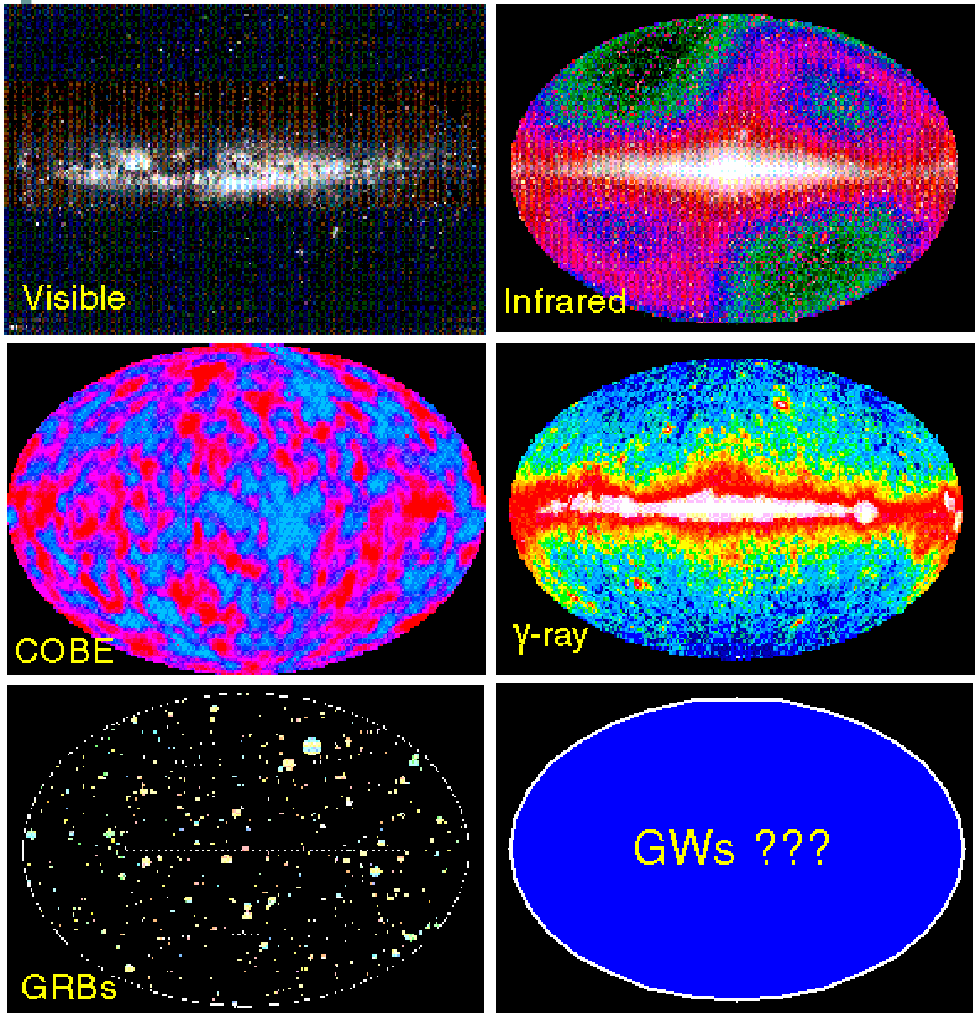
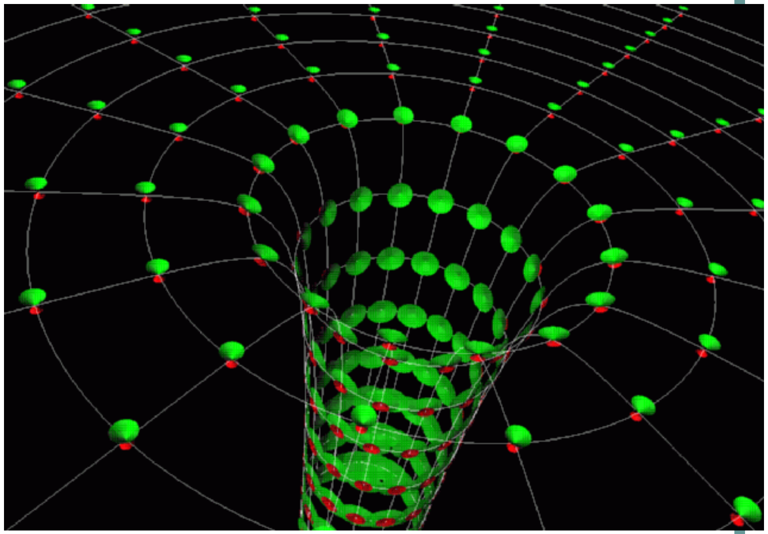


A New Window on the Universe

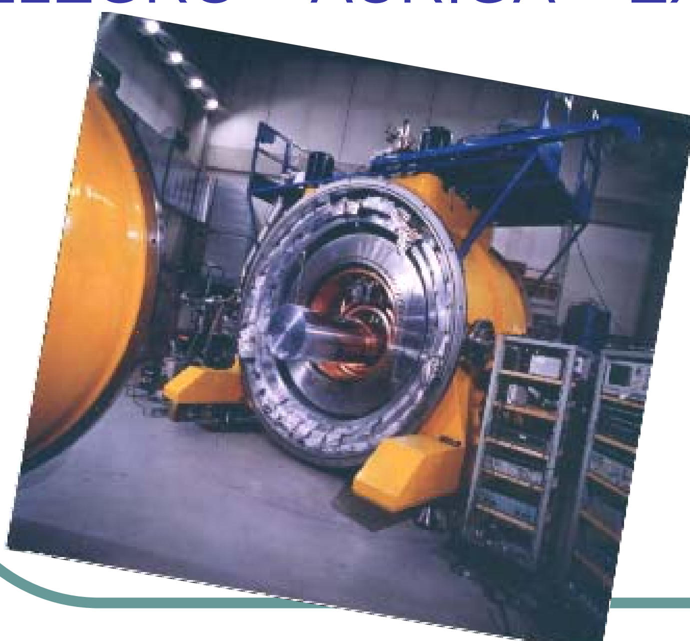
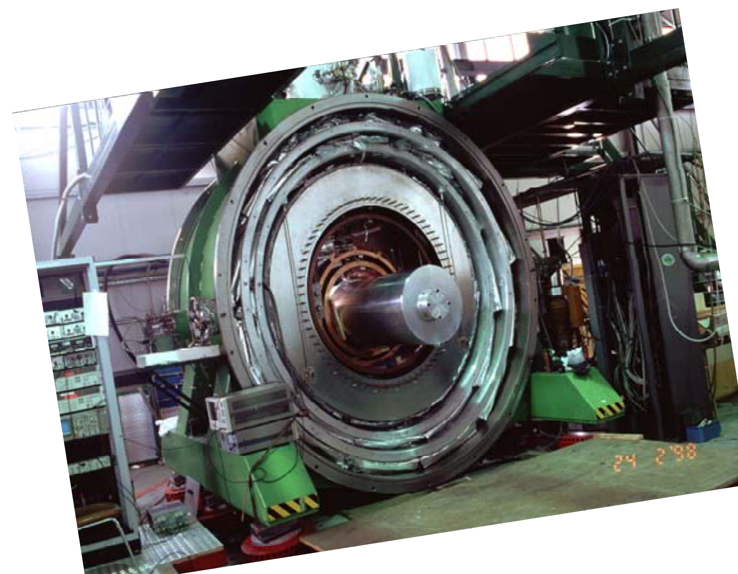




Gravitational Waves will provide a new way to view the dynamics of the Universe



ALLEGRO AURIGA EXPLORER NAUTILUS NIOBE

Grav. Waves: an international dream

GEO600 (British-German)
Hannover, Germany

LIGO (USA)
Hanford, WA & Livingston, LA

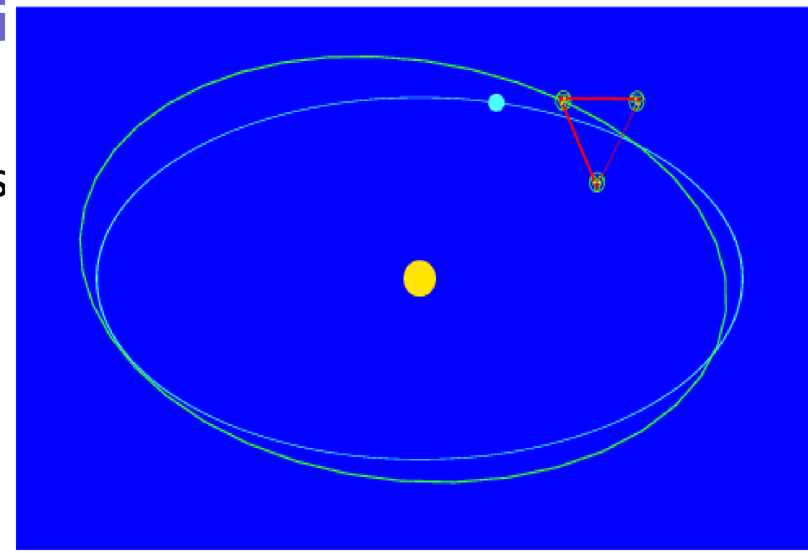
TAMA (Japan)
Mitaka

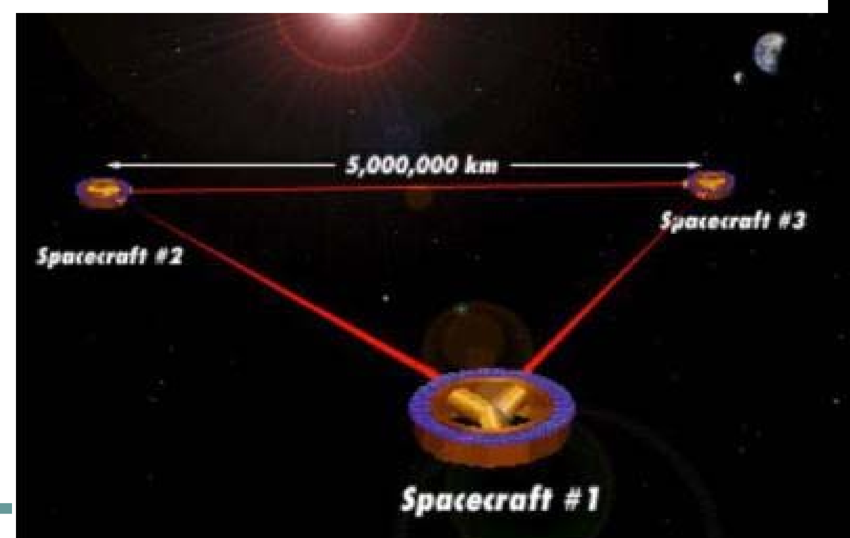
VIRGO (French-Italian)
Cascina, Italy

AIGO (Australia),
Wallingup Plain, Perth

LISA the space interferometer

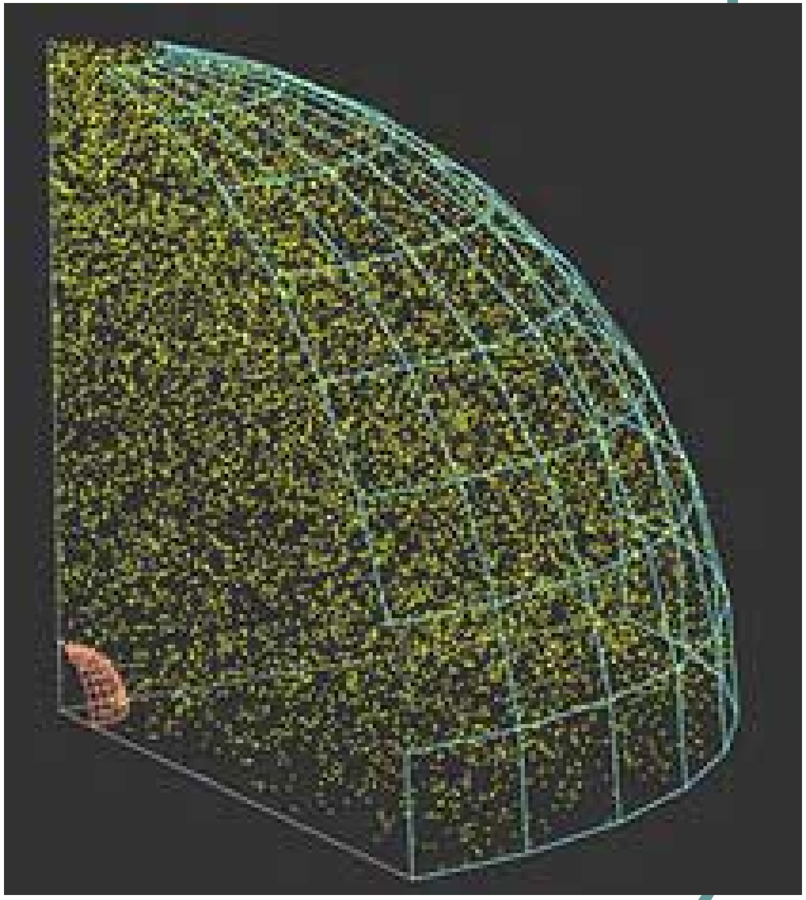
- LISA is low frequency detector.
- With arm lengths 5,000,000 km targets at 0.1mHz – 0.1Hz.
- Some sources are very well known (close binary systems in our galaxy).
- Some other sources are extremely strong (SM-BH binaries)
- LISA's sensitivity is roughly the same as that of LIGO, but at 10^5 times lower frequency.
- Since the gravitational waves energy flux scales as $F \sim f^2 \cdot h^2$, it has **10 orders** better energy sensitivity than LIGO.





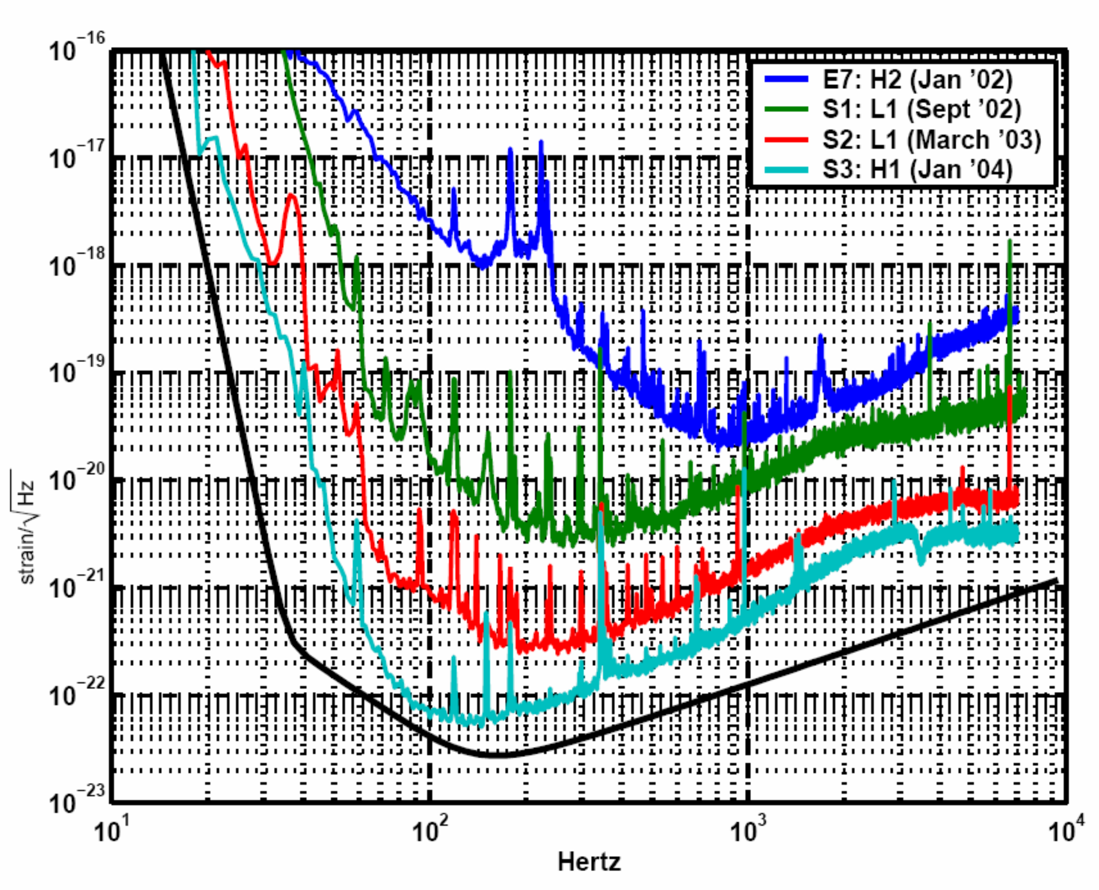
Future Gravitational Wave Antennas

- **Advanced LIGO**
  - 2006-2008; planning under way
  - 10-15 times more sensitive than initial LIGO
- **High Frequency GEO**
  - 2008? Neutron star physics, BH quasi-normal modes
- **EGO: European Gravitational Wave Observatory**
  - 2012? Cosmology

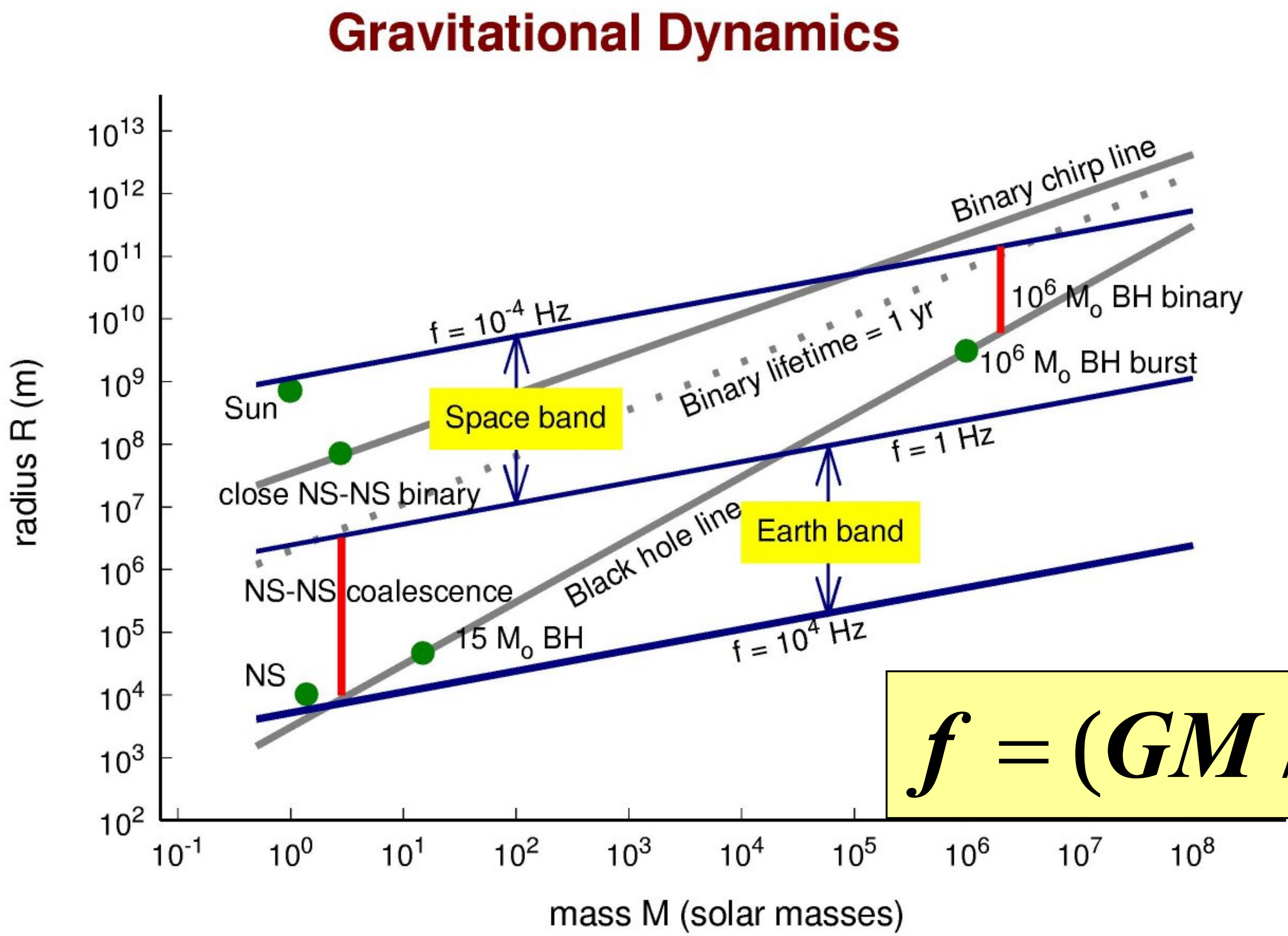


Current Sensitivity of GEO and LIGO

- Very good progress over the past 2 years and we are fast approaching the designed sensitivity goals
- TAMA reached the designed sensitivity (2003)
- LIGO (and GEO600) have reached within a factor of ~2 of their design sensitivity
- Currently focussing on:
  - upper limit studies
  - testing data analysis pipelines
  - detector characterization
- VIRGO is following very fast



Gravitational dynamics



$$f = (GM / R^3)^{1/2}$$

GW Frequency Bands

- High-Frequency: 1 Hz - 10 kHz
  - (Earth Detectors)
- Low-Frequency: 10^{-4} - 1 Hz
  - (Space Detectors)
- Very-Low-Frequency: 10^{-7} - 10^{-9} Hz
  - (Pulsar Timing)
- Extremely-Low-Frequency: 10^{-15}-10^{-18} Hz
  - (COBE, WMAP, Planck)

Gravitational vs E-M Waves

- **EM waves are radiated by individual particles, GWs are due to non-spherical bulk motion of matter.** I.e. the information carried by EM waves is stochastic in nature, while the GWs provide insights into coherent mass currents.
- **The EM will have been scattered many times. In contrast, GWs interact weakly with matter and arrive at the Earth in pristine condition.** Therefore, GWs can be used to probe regions of space that are opaque to EM waves. Stiil, the weak interaction with matter also makes the GWs fiendishly hard to detect.
- Standard astronomy is based on **deep imaging of small fields of view**, while **gravitational-wave detectors cover virtually the entire sky**.
- **EM radiation has a wavelength smaller than the size of the emitter, while the wavelength of a GW is usually larger than the size of the source.** Therefore, we cannot use GW data to create an image of the source. GW observations are more like audio than visual.

Morale: GWs carry information which would be difficult to get by other means.

Uncertainties and Benefits

- **Uncertainties**
  - The strength of the sources (may be orders of magnitude)
  - The rate of occurrence of the various events
  - The existence of the sources

- **Benefits**
  - **Information about the Universe that we are unlikely ever to obtain in any other way**
  - Experimental tests of fundamental laws of physics which cannot be tested in any other way
  - **The first detection of GWs will directly verify their existence**
  - By comparing the arrival times of EM and GW bursts we can measure their speed with a fractional accuracy **$\sim 10^{-11}$**
  - **From their polarization properties of the GWs we can verify GR prediction that the waves are transverse and traceless**
  - **From the waveforms we can directly identify the existence of black-holes.**

Information carried by GWs

- **Frequency**

$$f \sim 10^4\,\mathrm{Hz} \rightarrow \rho \sim 10^{16}\,\mathrm{gr/cm^3}$$
$$f \sim 10^{-4}\,\mathrm{Hz} \rightarrow \rho \sim 1\,\mathrm{gr/cm^3}$$

$$f_{dyn} \sim \left(\frac{GM}{R^3}\right)^{1/2} \sim (G\rho)^{1/2}$$

- **Rate of frequency change**

$$\dot{f}/f \sim (M_1, M_2)$$

- **Damping**

$$\tau \sim M^3/R^4$$

- **Polarization**
  - Inclination of the symmetry plane of the source
  - Test of general relativity
- **Amplitude**
  - Information about the strength and the distance of the source ($h \sim 1/r$).
- **Phase**
  - Especially useful for detection of binary systems.

Gravitation & Spacetime Curvature

Newton

$$\nabla^2 U = 4\pi G \rho$$

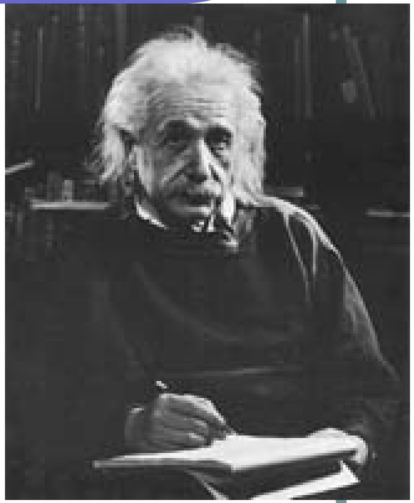
$$\frac{d^2 \vec{x}}{dt^2} = \nabla U$$

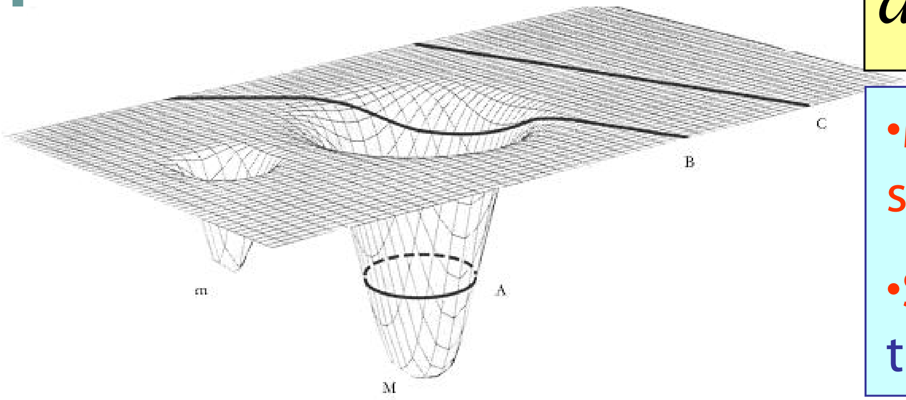
Einstein

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\frac{d^2 \vec{x}}{ds^2} \sim f(g^{\mu\nu})$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$





- Matter dictates the degree of spacetime deformation.
- Spacetime curvature dictates the motion of matter.

GWs fundamental part of Einstein's theory

Linearized Gravity

- Assume a **small perturbation** on the background metric:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

- The **perturbed Einstein's equations** are:

$$h_{\alpha\beta;\mu}{}^{;\mu} + g_{\alpha\beta} h^{\mu\nu}{}_{;\nu\mu} - 2h_{\mu(\alpha}{}^{;\mu}{}_{;\beta)} + 2R_{\mu\alpha\nu\beta} h^{\mu\nu} - 2R_{\mu(\alpha} h_{\beta)}{}^{\mu} = kT_{\alpha\beta}$$

- Far from the source (**weak field limit**)…
- And by **choosing a gauge**:

$$\tilde{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} n_{\mu\nu} h_{\alpha}{}^{\alpha}$$

$$\tilde{h}^{\mu\nu}{}_{;\mu} = 0$$

- **Simple wave equation**:

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2 \right) \tilde{h}^{\mu\nu} \equiv \partial_\lambda \partial^\lambda \tilde{h}^{\mu\nu} = kT^{\mu\nu}$$

Transverse-Traceless (TT)-gauge

- **Plane wave solution**

$$\tilde{h}^{\mu\nu} = A^{\mu\nu} e^{ik_a x^a}$$

$$A^{\mu\nu} k_\mu = 0$$

$$k^\mu k_\mu = 0$$

- TT-gauge (wave propagating in the z-direction)

$$A^{\mu\nu} = h_+ \varepsilon_+^{\mu\nu} + h_\times \varepsilon_\times^{\mu\nu}$$

$$\varepsilon_+^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \varepsilon_\times^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- **Riemann tensor**
- **Geodesic deviation**

$$R_{j0k0}{}^{TT} = -\frac{1}{2}\frac{\partial^2}{\partial t^2} h_{jk}{}^{TT}$$

$$\frac{d^2\xi_k}{dt^2} \approx -R_{k0j0}{}^{TT}\xi^j = \frac{1}{2}\frac{\partial^2 h_{jk}{}^{TT}}{\partial t^2}\xi^j$$

- **…and the tidal force**

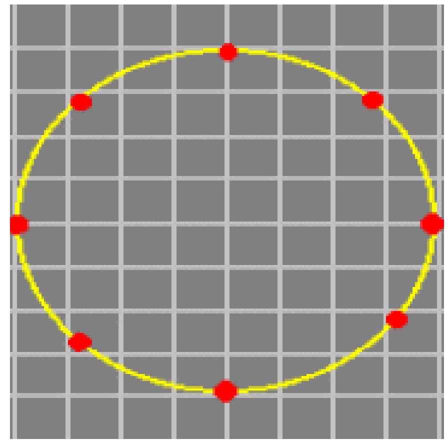
$$f^k \simeq m \cdot R^k_{0j0} \cdot \xi^j$$

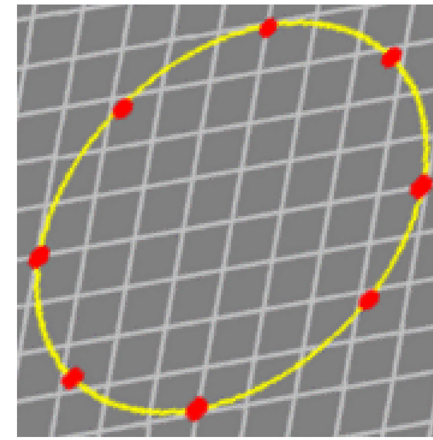
Gravitational Waves

$$h^{\mu\nu} = h_{+}\varepsilon_{+}^{\mu\nu} \cos[\omega(t - z)]$$

$$\Delta x = -\frac{1}{2} h_{+} \cos\left[\omega(t - z)\right] x$$

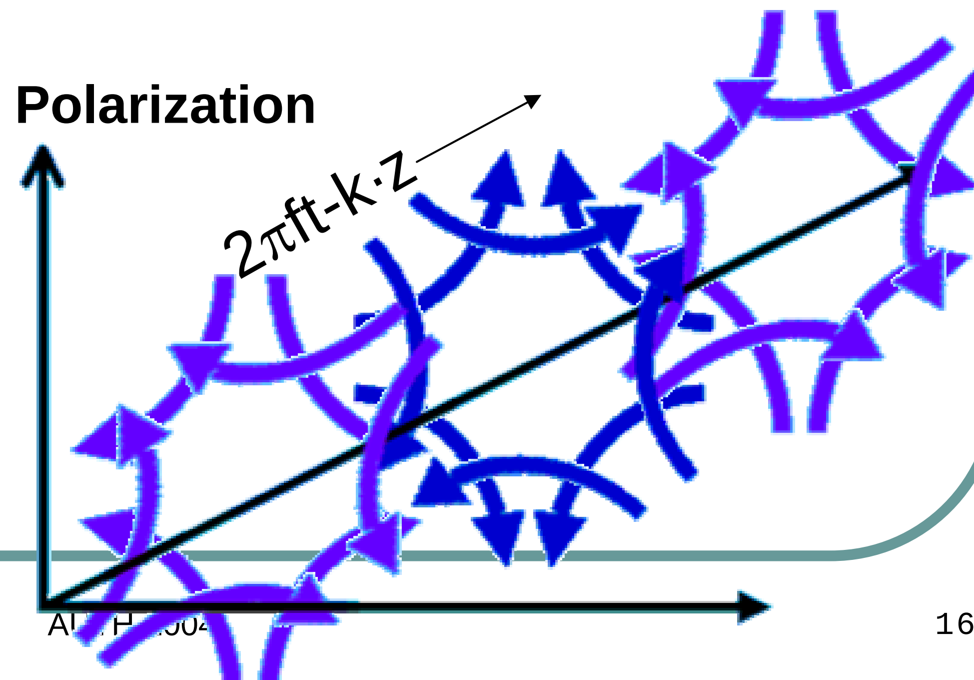
$$\Delta y = \frac{1}{2} h_{+} \cos\left[\omega(t - z)\right] y$$





…in other words

$$\frac{\Delta\ell}{\ell} = h$$



Stress-Energy carried by GWs

GWs exert forces and do work, they must carry energy and momentum

- The energy-momentum tensor in an arbitrary gauge

$$t_{\mu\nu}^{(GW)} = \frac{1}{32\pi}\left\langle \tilde{h}_{\alpha\beta;\mu}\tilde{h}^{\alpha\beta}{}_{;\nu} - \frac{1}{2}\tilde{h}_{;\mu}\tilde{h}_{;\nu} - \tilde{h}^{\alpha\beta}{}_{;\beta}\tilde{h}_{\alpha\mu;\nu} - \tilde{h}^{\alpha\beta}{}_{;\beta}\tilde{h}_{\alpha\nu;\mu}\right\rangle$$

- …in the TT-gauge:

$$t_{\mu\nu}^{(GW)} = \frac{1}{32\pi}\left\langle \tilde{h}^{jk\ TT}_{;\mu}\cdot\tilde{h}_{jk;\nu}^{TT}\right\rangle$$

- …it is divergence free

$$t^{\nu\ (GW)}_{\mu;\nu} = 0$$

- For waves propagating in the z-direction

$$t_{00}^{(GW)} = -\frac{1}{c}t_{0z}^{(GW)} = \frac{1}{c^2}t_{zz}^{(GW)} = \frac{1}{16\pi}\frac{c^2}{G}\left\langle \dot{h}_+^2 + \dot{h}_\times^2\right\rangle$$

- for a SN exploding in Virgo cluster the energy flux on Earth

$$t_{0z}^{(GW)} \approx \frac{\pi}{4}\frac{c^3}{G}f^2\left\langle h_+^2 + h_\times^2\right\rangle = 320\times\left(\frac{f}{1kHz}\right)^2\left(\frac{h}{10^{-21}}\right)^2\frac{\text{ergs}}{\text{cm}^2\text{sec}}$$

- The corresponding EM energy flux is:

$$\sim 10^{-9}\,\text{erg}\cdot\text{cm}^{-2}\cdot\text{sec}^{-1}$$

Wave-Propagation Effects

GWs affected by the large scale structure of the spacetime exactly as the EM waves

- The magnitude of h_{jk}^{TT} falls of as $1/r$
- The polarization, like that of light in vacuum, is parallel transported radially from source to earth
- The time dependence of the waveform is unchanged by propagation except for a frequency-independent **redshift**

$$\frac{f_{\text{received}}}{f_{\text{emitted}}} = \frac{1}{1+z}$$

We expect

- Absorption, scattering and dispersion
- Scattering by the background curvature and tails
- Gravitational focusing
- Diffraction
- Parametric amplification
- Non-linear coupling of the GWs (frequency doubling)
- Generation of background curvature by the waves

The emission of grav. radiation

If the energy-momentum tensor is varying with time, GWs will be emitted

- The retarded solution for the linear field equation

$$\left(-\frac{\partial^2}{\partial t^2}+\nabla^2\right)\tilde{h}^{\mu\nu}=kT^{\mu\nu}_{(\text{matter})}$$

$$h^{\mu\nu}=2\int\frac{T^{\mu\nu}(t-|\vec{x}-\vec{x}'|,\vec{x}')}{|\vec{x}-\vec{x}'|}d^3x'$$

- For a point in the radiation zone in the slow-motion approximation

$$h^{\mu\nu}\approx\frac{2}{r}\int T^{\mu\nu}(t-r,\vec{x}')d^3x'\sim\frac{2}{r}\frac{\partial^2}{\partial t^2}\left[Q^{jk}(t-r)\right]^{TT}$$

- Where Q_{kl} is the quardupole moment tensor

$$Q^{kl}\equiv\int\rho(t,x^k)\left(x^kx^l-\frac{1}{3}r^2\delta^{kl}\right)d^3x$$

- **Power emitted in GWs**

$$L_{GW}=-\frac{dE}{dt}=\frac{1}{5}\frac{G}{c^5}\sum_{ij}\left\langle\dddot{Q}_{ij}\cdot\dddot{Q}_{ij}\right\rangle$$

Linearized GR vs Maxwell

	Einstein	Maxwell
Potentials	$h_{\alpha\beta}(x)$	$\left(\Phi(x), \vec{A}(x)\right)$
Sources	$T_{\alpha\beta}$	$\left(\rho_{\text{elect}}, \vec{J}\right)$
Lorentz gauge	$\tilde{h}^{\alpha\beta}_{;a}=0$	$\frac{\partial\Phi}{\partial t}+\vec{\nabla}\cdot\vec{A}=0$
Wave equation	$\Box\tilde{h}_{ij}=-8\pi T_{ij}$	$\Box\vec{A}=-4\pi\vec{J}$
Solution	$\tilde{h}^{ij}=2\int d^3x'\frac{[T^{ij}]_{ret}}{\lvert\vec{x}-\vec{x}'\rvert}$	$\vec{A}=\int d^3x'\frac{[\vec{J}]_{ret}}{\lvert\vec{x}-\vec{x}'\rvert}$
Solution (asymp)	$\tilde{h}^{ij}=2\frac{\left[\ddot{Q}^{ij}\right]_{ret}}{r}$	$\vec{A}=\frac{\left[\dot{\vec{p}}\right]_{ret}}{r}$
Radiated Power	$\frac{dE}{dt}=\frac{1}{5}\left\langle\dddot{Q}_{ij}\cdot\dddot{Q}^{ij}\right\rangle$	$\frac{dE}{dt}=\frac{2}{3}\left\langle\ddot{\vec{p}}^{\,2}\right\rangle$

Back of the envelope calculations!

- **Characteristic time-scale** for a mass element to move from one side of the system to another is:

$$T \sim \frac{R}{\upsilon} \sim \frac{R}{(M/R)^{1/2}} = \left(\frac{R^3}{M}\right)^{1/2}$$

- The **quadrupole moment** is approximately:

$$\dddot{Q}_{ij} \sim \frac{MR^2}{T^3} \sim \frac{M\upsilon^2}{T} \sim \frac{E_{ns}}{T} \sim \left(\frac{M}{R}\right)^{5/2}$$

- **Luminosity**

$$L_{GW} \sim \frac{G}{c^5}\left(\frac{M}{R}\right)^5 \sim \frac{G}{c^5}\left(\frac{M}{R}\right)^2 \upsilon^6 \sim \frac{c^5}{G}\left(\frac{R_{Sch}}{R}\right)^2\left(\frac{\upsilon}{c}\right)^6$$

$$\frac{c^5}{G} = 3.63\times10^{59}\, erg/s = 2.03\times10^5 M_{\odot} c^2/s$$

- The **amplitude** of GWs at a distance r ($R \sim R_{Schw} \sim 10Km$ and $r \sim 10Mpc \sim 3x10^{19}km$):

$$h \sim \frac{\ddot{Q}}{r} \sim \frac{1}{r}\left(\frac{MR^2}{T^2}\right) \sim \frac{1}{r}\frac{M^2}{R} \sim \ldots \sim 10^{-19}$$

- **Radiation damping**

$$\tau_{react} = \frac{E_{kin}}{L_{GW}} \sim \left(\frac{R}{M}\right)^{5/2} T \sim \left(\frac{\upsilon}{c}\right)\left(\frac{R}{R_{Schw}}\right)^3 T$$

3 relations that we should remember…

- **Length variation**

$$\frac{\Delta \ell}{\ell} = h$$

- **Amplitude**

$$h^{jk} \approx \frac{2}{r} \ddot{Q}^{jk}$$

- **Power emitted**

$$L_{GW} = -\frac{dE}{dt} = \frac{1}{5}\frac{G}{c^5}\sum_{ij}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle$$

Vibrating Quadrupole

- The position of the two masses

$$x = \pm[x_0 + \xi \sin(\omega t)] \quad , \quad x_0 \ll \xi$$

- The quadrupole moment of the system is

$$Q^{kl}(t-r) \simeq \left[1 + \frac{2\xi}{x_0}\sin\omega(t-r)\right] Q_0{}^{kl}$$

$$Q_0{}^{kl} = \begin{pmatrix} -2mx_0{}^2 & 0 & 0 \\ 0 & -2mx_0{}^2 & 0 \\ 0 & 0 & 4mx_0{}^2 \end{pmatrix}$$

- The radiated gravitational field is

$$h^{kl} = \frac{2}{3}\left(\frac{\xi}{x_0}\right)\frac{\omega^2}{r}\sin[\omega(t-r)]Q_0{}^{kl}$$

- The emitted power

$$-\frac{dE}{dt} = \frac{G}{45c^5}\left\langle \dddot{Q}_{kl}\dddot{Q}_{kl}\right\rangle = \frac{16}{15}\frac{G}{c^5}(mx_0\xi)^2\omega^6$$

- And the damping rate of the oscillator is

$$\gamma_{rad} = -\frac{1}{E}\left\langle\frac{dE}{dt}\right\rangle = \frac{16}{15}\frac{G}{c^5}mx_0{}^2\omega^4$$

Two-body collision

- The radiated power

$$-\frac{dE}{dt}=\frac{8}{15}\frac{G}{c^5}m^2(3\dot{z}\,\ddot{z}+z\,\dddot{z})^2$$

- The energy radiated during the plunge from $z=\infty$ to $z=-R$

$$-\Delta E=\frac{4}{105}\frac{1}{R^{7/2}}\frac{G}{c^5}m^2(2GM)^{5/2}$$

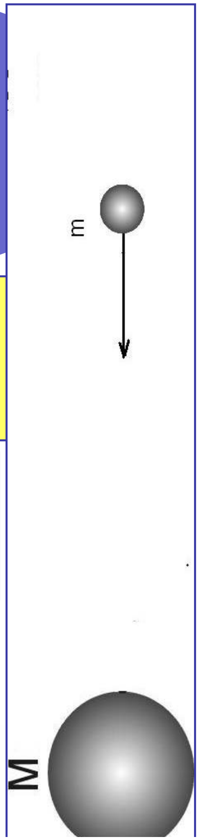
- If $R=R_{Schw}$ ($M=10M_\odot$ & $m=1M_\odot$)

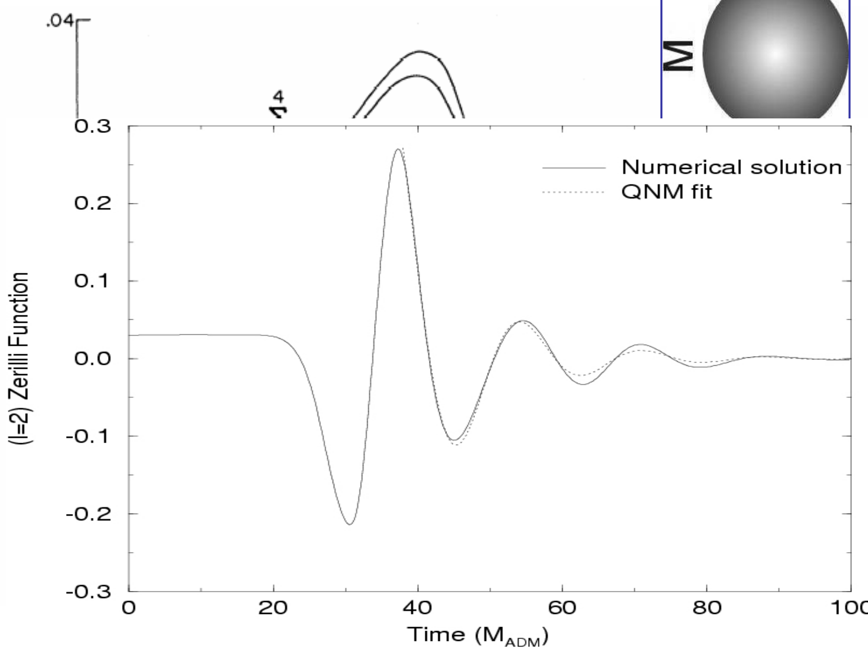
$$-\Delta E=0.019mc^2\frac{m}{M}$$

$$-\Delta E_{true}=0.0104mc^2\frac{m}{M}\rightarrow 2\times10^{51}erg$$

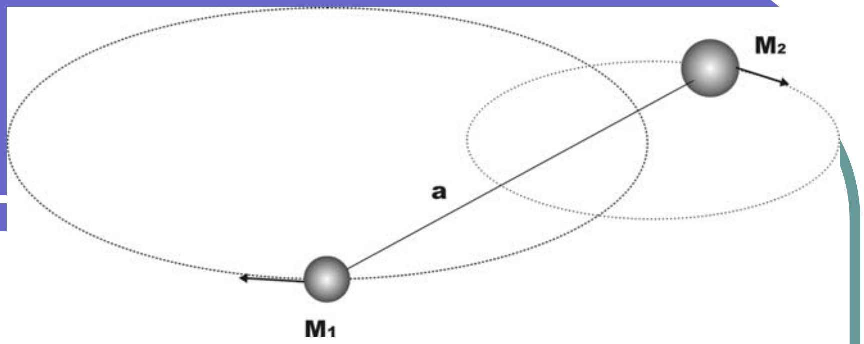
- Most radiation during $2R\text{-}{>}R$ phase

$$\Delta t\simeq R/\upsilon\simeq R/c\simeq 30km/c\approx 10^{-4}s$$





Rotating Quadrupole (a binary system)



THE BEST SOURCE FOR GWs

- **Radiated power**

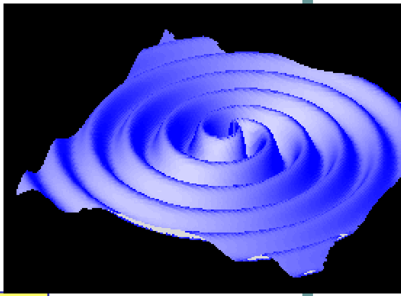
$$-\frac{dE}{dt}=\frac{32}{5}\frac{G}{c^5}\mu^2 a^4\omega^6=\frac{32}{5}\frac{G}{c^5}\frac{M^3\mu^2}{a^5}$$

- Energy loss leads to **shrinking of their orbital separation**

$$\frac{da}{dt}=-\frac{64}{5}\frac{G^3}{c^5}\frac{\mu M^2}{a^3}$$

- **Period changes** with rate

$$\frac{\dot{P}}{P}=-\frac{96}{5}\frac{G^3}{c^5}\frac{\mu M^2}{a^4}$$



- ...and the system **will coalesce** after

$$T_{inspiral}=\frac{5}{256}\frac{c^5}{G^3}\frac{a_0^4}{\mu M^2}$$

- The **total energy loss** is

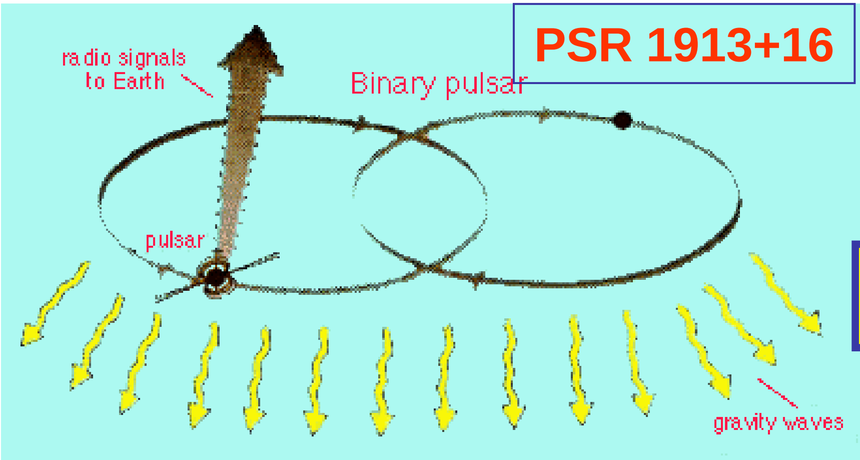
$$\Delta E_{rad}=\frac{G}{2}\mu M\left(\frac{1}{a_0}-\frac{1}{a}\right)$$

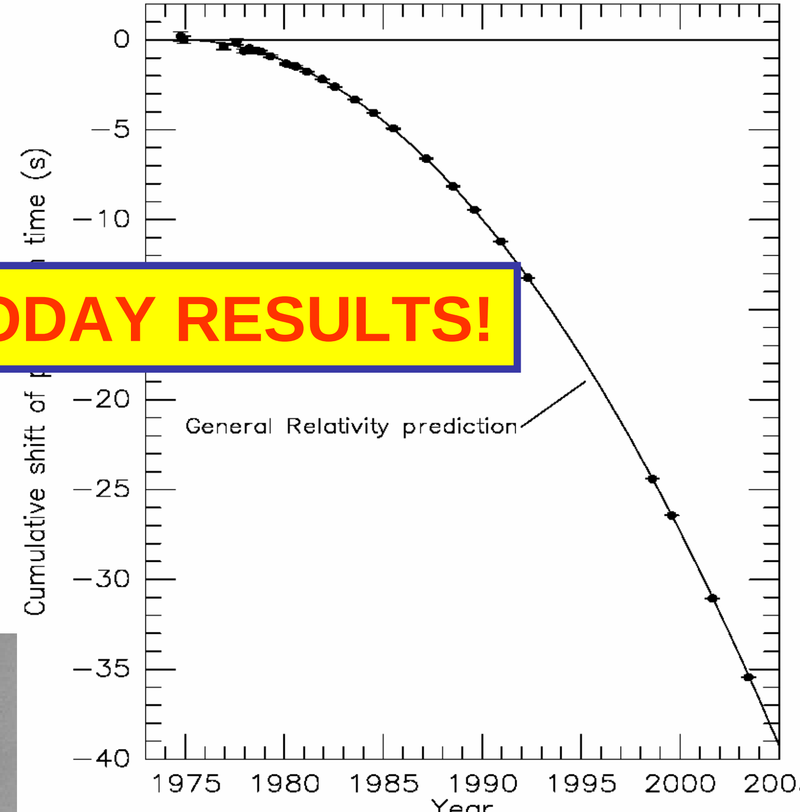
- Typical **amplitude** of GWs

$$h\approx 5\times 10^{-22}\left(\frac{M}{2.8M_{\odot}}\right)^{2/3}\left(\frac{\mu}{0.7M_{\odot}}\right)\left(\frac{f}{100Hz}\right)^{2/3}\left(\frac{15Mpc}{r}\right)$$

First verification of GWs

PSR 1913+16





TODAY RESULTS!

Nobel 1993

Hulse & Taylor

$$\frac{\dot{P}_{b,corrected}}{\dot{P}_{b,GR}} = 1.0013 \pm 0.0021$$

Discovery of a new binary pulsar

Burgay et al Nature 2003

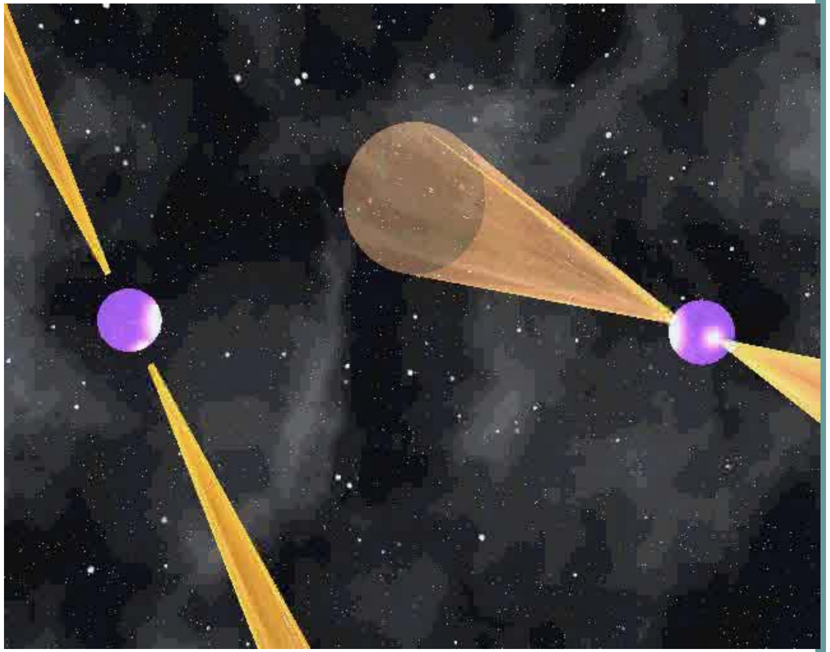
Fastest known binary pulsar J0737-3039

- **Burgay et al** (December 2003) discovered a new pulsar in a binary **J0737-3039** that is expected to open a new area of astrophysics/astronomy
- **Strongly relativistic** (period 2.5 Hrs), **mildly eccentric** (0.088), **highly inclined** (i > 87 deg)
- Faster than PSR 1913+16, J7037-3039 is **the most relativistic neutron star binary**
- **Greatest periastron advance**: $d\omega/dt$ **16.8 degrees per year** (thought to be fully general relativistic) – very large compared to relativistic part of Mercury's perihelion advance of 42" per century

Implications for NS coalescence rates

Lyne et al Science 2004

- Coalescence rate of Galactic binary neutron stars revised upwards by about 7
- Current re-estimate of NS-NS coalescences in initial LIGO interferometers
  - Once per few years to once per 25 years
  - BH-BH rates must be revised too: Probably a few events per year in LIGO-VIRGO-GEO network
- Soon the companion was detected directly and confirmed to be a pulsar
- B has a spin period much larger: 2.5 s as opposed to 2.25 ms of A



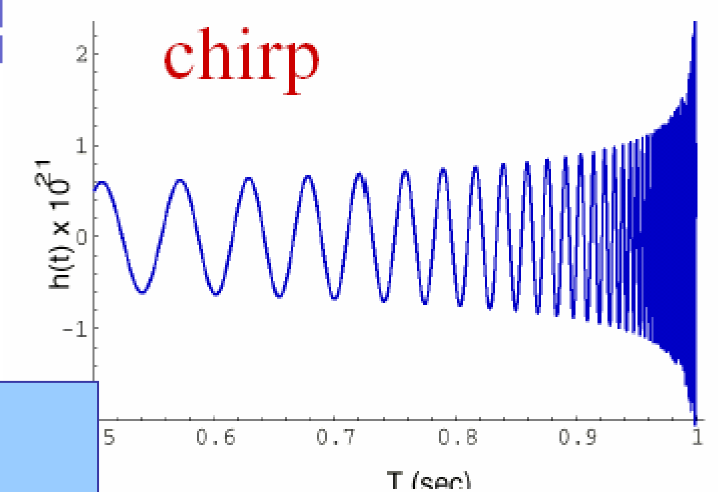
Binary systems (examples)

PSR 1913+16

$M_1=M_2\sim1.4M_\odot$, P=7h 45m 7s, r=5kpc,

$h_{earth}\sim10^{-20}$, $f\sim10^{-4}$Hz, $T_{insp}\sim3\times10^8$yr

$dP_{theo}/dt=-7.2\times10^{-12}$s/yr $dP_{obs}/dt=-(6.9\pm06)\times10^{-12}$s/yr



The LIGO/VIRGO binary (10-1000Hz)

$M_1=M_2\sim1.4M_\odot$, f_0=10Hz, f_{final}=1000Hz,

$T_{insp}\sim$15min, after ~15000 cycles (inspiral/merging 300Mpc)

$M_1=50M_\odot$, $M_2\sim50M_\odot$, f_0=10Hz,

f_{final}=100Hz, (inspiral/merging 400Mpc)

$$f_{BH}\sim 12kHz\left(\frac{1M_\odot}{M}\right)$$

The LISA binary (10^{-5}-10^{-2}Hz)

$M_1=M_2\sim10^6M_\odot$, $f_0=10^{-4}$Hz, f_{final}=0.01Hz, (inspiral/merging at r~3Gpc)

$M_1=M_2\sim10^5M_\odot$, $f_0=10^{-4}$Hz, f_{final}=0.1Hz, (inspiral/merging at r~3Gpc)

$M_1=M_2\sim10^4M_\odot$, $f_0=10^{-3}$Hz, f_{final}=1Hz, (inspiral at r~3Gpc)

Smaller Stars/BHs plunging into super-massive ones

An interesting observation

- The observed frequency change will be:
- The corresponding amplitude will be :

$$\dot{f} \sim f^{11/3} \mathcal{M}_{chirp}^{5/3}$$

$$\mathcal{M}_{chirp}^{5/3} = \mu M^{2/3}$$

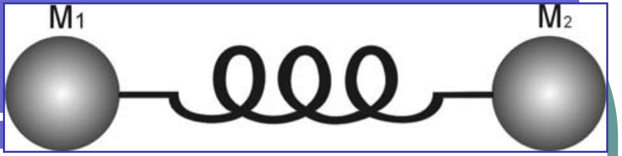
$$h \sim \frac{\mathcal{M}_{chirp}^{5/3} f^{2/3}}{r} = \frac{\dot{f}}{f^{3} r}$$

•Since both frequency and its rate of change are measurable quantities, we can immediately compute the chirp mass.

•The third relation provides us with a direct estimate of the distance of the source

•Post-Newtonian relations can provide the individual masses

A Quadrupole Detector



Tidal force is the driving force of the oscillator

- Plane wave

$$h^{\mu\nu} = h_{+}\varepsilon^{\mu\nu} e^{i(\omega t - kz)}$$

- Displacement & Tidal force

$$\xi_x \approx L h_{+} e^{i\omega t} \qquad f_x \approx m L h_{+} \omega^2 e^{i\omega t}$$

- Equation of motion

$$\ddot{\xi} + \dot{\xi}/\tau + \omega_0^{\ 2}\xi = -\frac{1}{2}\omega^2 L h_{+} e^{i\omega t}$$

- Solution

$$\xi = \frac{\omega^2 L h_{+} e^{i\omega t}/2}{\omega_0^2 - \omega^2 + i\omega/\tau}$$

$$\xi_{\max} = \omega_0 \tau \cdot L \cdot h_{+}/2 = Q \cdot L \cdot h/2$$

- **Cross section**

$$\sigma = \frac{32\pi}{15}\frac{G}{c^3}\omega_0 \cdot Q \cdot M \cdot L^2$$

- Weber's detector:
  $M=1410kg$, $L=1.5m$,
  $d=66cm$, $\omega_0=1660Hz$,
  $Q=2x10^5$.

$$\sigma_{Weber} \simeq 3\times 10^{-19}\, cm^2$$

Quadrupole Detector Limitations

Problems

- Very small cross section $\sim 3x10^{-19} cm^2$.
- Sensitive to periodic GWs **tuned** in the right frequency of the detector
- Sensitive to bursts **only** if the pulse has a substantial component at the resonant frequency
- The **width of the resonance** is:

$$\Delta\nu \sim \gamma / 2\pi \sim 10^{-2}\, Hz$$

•Thermal noise limits our ability to detect the energy of GWs.

•The excitation energy has to be greater than the thermal fluctuations $E \gtrsim kT$

$$h_{\min} \geq \frac{1}{\omega_0 L Q}\sqrt{\frac{15kT}{M}} \sim 10^{-20}$$

BURSTS

•Periodic signals which match the resonant frequency of the detector are extremely rare.

•A great number of events produces short pulses which spread radiation over a wide range of frequencies.

•The minimum detectable amplitude is

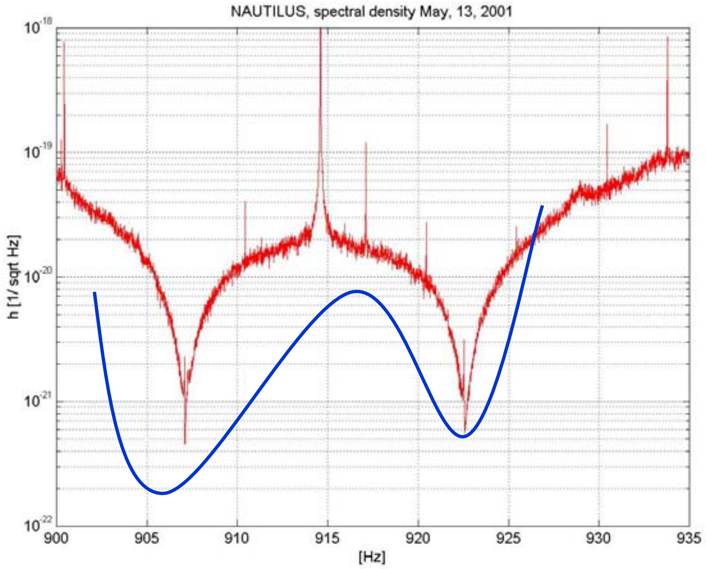
$$h_{\min} \geq \frac{1}{\omega_0 L}\sqrt{\frac{30kT_{eff}}{\pi M}} \sim 10^{-16}$$

The total energy of a pulse from the Galactic center (r=10kpc) which will provide an amplitude of $h \sim 10^{-16}$ or energy flux $\sim 10^9$ erg/cm^2.

$$4\pi r^2 \times 10^9\, erg / cm^2 = 10^{55}\, erg$$
$$\approx 10 M_{\odot} c^2 \;!!!$$

Modern Bar Detectors

	WEBER	NAUTILUS
mass(kg)	1410	2270
Length(m)	1.53	2.97
ω_0(Hz)	1660	910
$Q = \omega/\gamma$	2×10^5	2.3×10^6
$\sigma\ (\omega_0)_{\rm abs}\ ({\rm cm}^2)$	2×10^{-19}	70×10^{-19}
Typical pulse sensitivity h	10^{-16}	9×10^{-19}



Laser Interferometers

- The output of the detector is

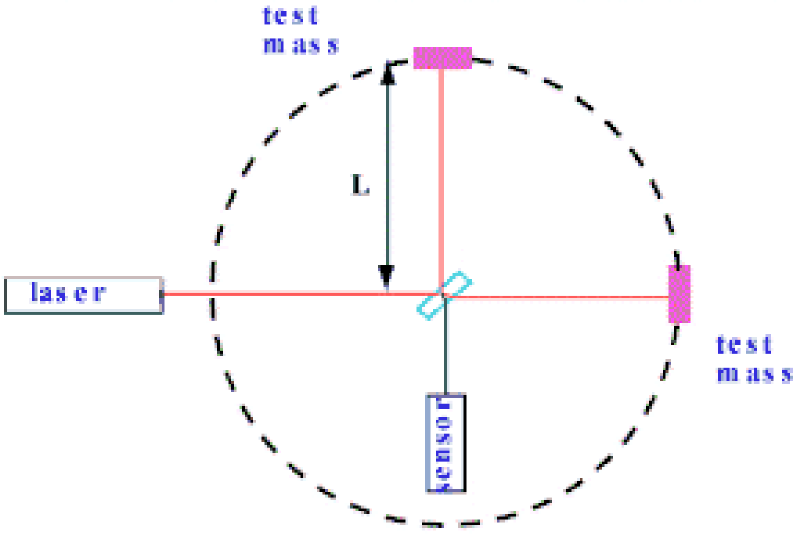
$$\frac{\Delta L}{L} = F_+ h_+(t) + F_\times h_\times(t) = h(t)$$

- Technology allows measurements $\Delta L \sim 10^{-16} cm$.
- For signals with $h \sim 10^{-21}$-10^{-22} we need arm lengths $L \sim 1$-$10 km$.
- Change in the arm length by ΔL corresponds to a phase change

$$\Delta\varphi = \frac{4\pi b \Delta L}{\lambda} \sim 10^{-9}\,\text{rad}$$

- The number of photons reaching the photo-detector is proportional to laser-beam's intensity $[\sim sin^2(\Delta\varphi/2)]$

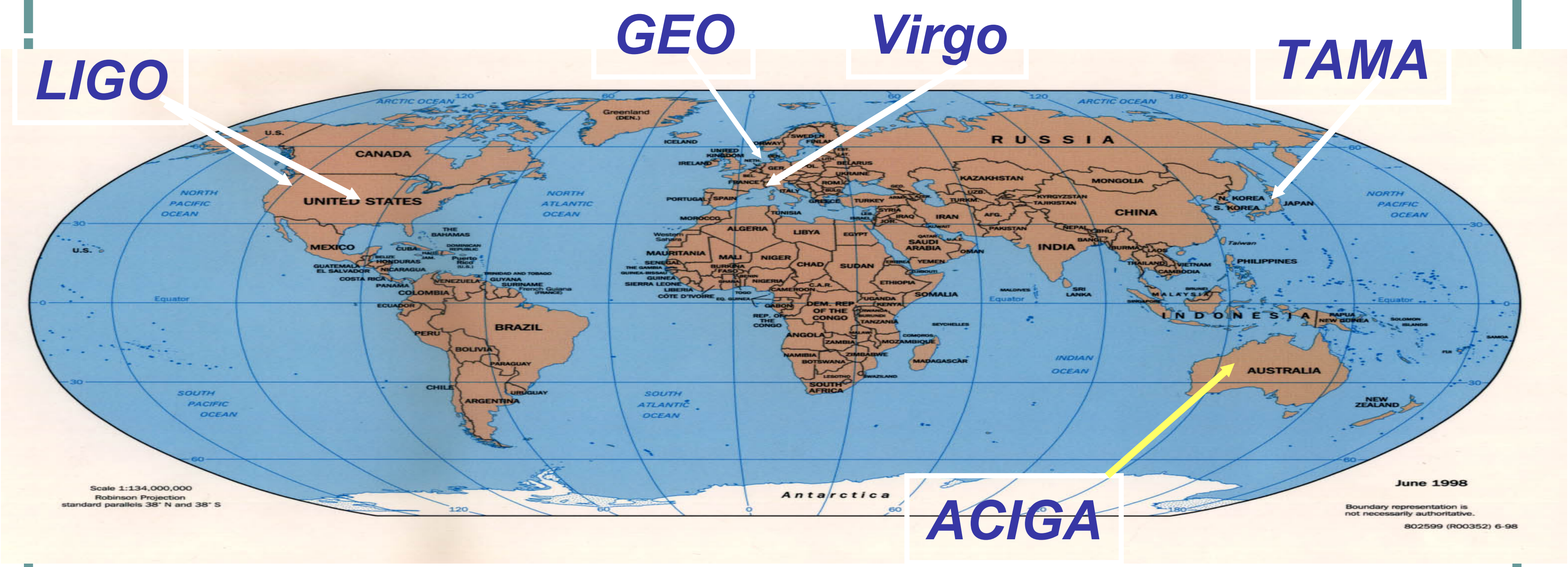
$$N_{\text{out}} = N_{\text{input}} \sin^2(\Delta\varphi/2)$$



OPTIMAL CONFIGURATION

- Long arm length L
- Large number of reflections b
- Large number of photons (but be aware of radiation pressure)
- Operate at interface minimum $cos(2\pi b\Delta L/\lambda)=1$.

International Network *interferometers*



Simultaneously detect signal (within msec)

detection confidence

- locate the sources
- decompose polarization of gravitational waves