Nonlinear wave-wave interactions involving gravitational waves

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Outline

- Orthonormal frames.

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    - Tetrad bases.

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  - Examples.

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- Nonlinear coupled Alfvén and gravitational waves.

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  - Example of application

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- Nonlinearly coupled electromagnetic and gravitational waves in vacuum.

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  - Possible applications

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- Nonlinearly coupled electromagnetic and gravitational waves in vacuum.
- Future work.

Tetrads

- Why?

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    Easier to distinguish coordinate effects from physical processes.
  - Spacetime split into space+time, and metric locally Minkowski everywhere.$\Longrightarrow$

    Greatly simplifies the algebra. (Need not distinguish between co- and contravariant quantities.)

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  - Tensorial quantities expressed in either basis, e.g.:

Vector field: $\mathbf{A} = A^\mu \partial_\mu = A^a \mathbf{e}_a = A^a X_a{}^\mu \partial_\mu$

Metric: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{ab} \omega^a \omega^b = g_{ab} \omega^a{}_\mu \omega^b{}_\nu dx^\mu dx^\nu$

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  - Tetrad can be chosen so that $g_{ab} = \eta_{ab} = \text{diag}(-1, 1, 1, 1)$ (orthonormality).
  - Connection given by Ricci rotation coefficients:

$$\Gamma_{abc} = \frac{1}{2}(g_{ad}C^d{}_{cb} - g_{bd}C^d{}_{ca} + g_{cd}C^d{}_{ab})$$

    where $C^a{}_{bc}(x^\mu)$ are the commutation functions for the basis $\{\mathbf{e}_a\}$, (i.e. $[\mathbf{e}_a, \mathbf{e}_b] = C^c{}_{ab}\mathbf{e}_c$).

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    where $C^a{}_{bc}(x^\mu)$ are the commutation functions for the basis $\{\mathbf{e}_a\}$, (i.e. $[\mathbf{e}_a, \mathbf{e}_b] = C^c{}_{ab}\mathbf{e}_c$).
  - E.g. covariant derivative: $\nabla_b A_a = \mathbf{e}_b A_a - \Gamma^c{}_{ab} A_c$

Tetrad equations

- Introduce observer four-velocity V^a, $\Longrightarrow$ EM-field can be decomposed relative to this into electric and magnetic part:

$$E_a = F_{ab}V^b \quad , \quad B_a = \frac{1}{2}\epsilon_{abc}F^{bc}$$

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- Choose tetrad so that $\mathbf{e}_0 = V^a\mathbf{e}_a$ (i.e. $V^a = \delta^a_0$).
- Introduce three vector notation $\mathbf{E} = (E^1, E^2, E^3)$ etc. and $\nabla = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$

$\Longrightarrow$

Tetrad equations

Maxwell field equations: $\nabla_a F^{ab} = j^b$, $\nabla_a F_{bc} + \nabla_b F_{ca} + \nabla_c F_{ab} = 0$, and fluid evolution equations: $\nabla_b T^{ab} = F^{ab} j_b$, can be written

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \rho + \rho_E \\
\nabla \cdot \mathbf{B} &= \rho_B \\
\mathbf{e}_0 \mathbf{E} - \nabla \times \mathbf{B} &= -\mathbf{j} - \mathbf{j}_E \\
\mathbf{e}_0 \mathbf{B} + \nabla \times \mathbf{E} &= -\mathbf{j}_B \\
\mathbf{e}_0(\gamma n) + \nabla \cdot (\gamma n \mathbf{v}) &= \Delta n \\
(\mu + p)(\mathbf{e}_0 + \mathbf{v} \cdot \nabla)\gamma \mathbf{v} &= -\gamma^{-1} \nabla p - \gamma \mathbf{v}(\mathbf{e}_0 + \mathbf{v} \cdot \nabla) p \\
&\quad + q n (\mathbf{E} + \mathbf{v} \times \mathbf{B}) + (\mu + p)\mathbf{g}
\end{aligned}
$$

where $\gamma = 1/\sqrt{1 - v_i v^i}$, $i = 1, 2, 3$

Nonlinear coupled Alfvén and gravitational waves

(A. Källberg, G. Brodin and M. Bradley, PRD 2004)

Nonlinear coupled Alfvén and gravitational waves

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- Self-consistent weakly nonlinear analysis of Einstein-Maxwell system.
- EMW and GW propagating in strongly magnetized plasma described by multifluid description.
- Resonant wave coupling $\Longrightarrow$ direct interaction with matter magnified compared to other nonlinearities, (e.g. coupling to background curvature).
- WKB-approximation $\Longrightarrow$ Nonlinear Schrödinger equation (NLS).
- Weak 3D-dependence $\Longrightarrow$ Self-focusing and collapse of pulse.

Nonlinear coupled Alfvén and gravitational waves

• Focus on the direct interaction with matter $\Longrightarrow$ consider linearized gravitational wave (nonlinearity comes from resonant response from matter) in basis

$$\mathbf{e}_0 = \partial_t \ , \ \mathbf{e}_1 = (1 - \tfrac{1}{2}h_+)\partial_x \ , \ \mathbf{e}_2 = (1 + \tfrac{1}{2}h_+)\partial_y \ , \ \mathbf{e}_3 = \partial_z$$

• Linearized Einstein field equations

$$\left(\partial_t^2 - \partial_z^2\right) h_+ = \kappa \left(\delta T_{11} - \delta T_{22}\right)$$

• Background magnetic field, B_0, in 1-direction, wave propagation in 3-direction. Introduce perturbations $n = n_0 + \delta n$, $\mathbf{B} = (B_0 + B_x)\mathbf{e}_1$, $\mathbf{E} = E_y\mathbf{e}_2 + E_z\mathbf{e}_3$ and $\mathbf{v} = v_y\mathbf{e}_2 + v_z\mathbf{e}_3$.

$\Longrightarrow$

Nonlinear coupled Alfvén and gravitational waves

- Maxwell and fluid equations reduces to

$$(\partial_t + \mathcal{V}(B_x)\partial_z)B_x = \frac{1}{2}B_0\partial_t h_+ \quad (1)$$

$$(\partial_t^2 - \partial_z^2)h_+ = -2\kappa B_0 B_x \quad (2)$$

$\mathcal{V}(B_x) = 1 - (1/2C_A^2)\left(B_0/\left(B_0 + 2B_x\right)\right)^{3/2}$ and we have introduced the Alfvén velocity $C_A \equiv (1/\sum_s \omega_p^2/\omega_c^2)^{1/2}$.

- LHS of (1-2) are wave operators for compressional Alfvén and gravitational wave respectively. RHS are mutual interaction terms for the wave modes.

- May be combined to single wave equation:

$$(\partial_t + \partial_z)(\partial_t + \mathcal{V}(B_x)\partial_z)B_x = -\frac{\kappa B_0^2}{2}B_x \quad (3)$$

Nonlinear coupled Alfvén and gravitational waves

• Linearizing $\Longrightarrow$ dispersion relation:

$$(\omega - k)\left(\omega - k + \frac{k}{2C_A^2}\right) = \frac{\kappa B_0^2}{2}$$

• For large k we have two distinct modes: "fast" mode, $\omega \approx k$, where most of the energy is gravitational, and "slow" mode, $\omega \approx k(1 - \frac{1}{2C_A^2})$, where energy is mainly electromagnetic.

• For longer wavelengths, $k \lesssim \sqrt{\kappa B_0^2} C_A^2$, modes are not clearly separated, and energy is divided equally between electromagnetic and gravitational form.

Nonlinear coupled Alfvén and gravitational waves

- Include terms up to 3rd order in amplitude expansion of wave equation $\Longrightarrow$ higher harmonic generation. Apply WKB-approximation.
- Coordinate transformations, $z, t \rightarrow \xi, \tau$.
- (1) used to relate B and h_+

$$\Downarrow$$

Standard NLS-equation for rescaled GW amplitude:

$$\left(i\partial_\tau \pm \partial_\xi^2\right) \tilde{h}_+ = \pm|\tilde{h}_+|^2 \tilde{h}_+$$

$\pm$ refers to fast and slow mode respectively.

Nonlinear coupled Alfvén and gravitational waves

- In reality we will not have exact plane wave solutions to linearized Einstein and Maxwell equations.
- Assume deviation from plane waves small, and apply perturbative treatment.
- Keeping only lowest order terms, the wave equation (3) is modified to:

$$\left(\partial_t + \partial_z - \frac{1}{2}\partial_t^{-1}\nabla_\perp^2\right)\left(\partial_t + \mathcal{V}(B_x)\partial_z - \frac{1}{2}\partial_t^{-1}\nabla_\perp^2\right)B_x = -\frac{\kappa B_0^2}{2}B_x$$

where $\nabla_\perp^2 \equiv \partial_x^2 + \partial_y^2$.

Nonlinear coupled Alfvén and gravitational waves

• Including x- and y-dependence in WKB-ansatz, and keeping terms up to 3rd order in amplitude, one obtains:

$$\left(i\partial_\tau \pm \partial_\xi^2 + \Upsilon\nabla_\perp^2\right)\tilde{h}_+ = \pm|\tilde{h}_+|^2\tilde{h}_+ \tag{4}$$

• Same equation as before with a small correction to the linear wave operator. $\Longrightarrow$ allows self-focusing of pulse.

Nonlinear coupled Alfvén and gravitational waves

- Consider long 3D-pulses with shape depending only on (normalized) cylindrical radius, thus neglecting the dispersive term in (4).
- (4) can be written as cylindrically symmetric NLS equation:

$$\left(i\partial_\tau + \frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\left(\tilde{r}\frac{\partial}{\partial\tilde{r}}\right)\right)\tilde{h}_+ = \pm|\tilde{h}_+|^2\tilde{h}_+$$

- Consider case with minus sign (slow, electromagnetically dominated mode) $\Longrightarrow$ nonlinearity of focusing type.
- No (physically relevant) exact solutions known, but approximate variational techniques and numerical work has been done.

Nonlinear coupled Alfvén and gravitational waves

- Strong enough nonlinearity $\longrightarrow$ pulse radius, $\tilde{r}_p \to 0$ in finite time.
- Estimated condition of collapse: $\dfrac{c^2 k^2 r_{dist}^2}{C_A^2} \gtrsim 1$, can be fulfilled within reasonable parameter range $\Longrightarrow$ possibility of structure formation of electromagnetic radiation pattern
- Note that the collapse condition does not contain gravitational parameters, which reflects the EM dominance of the slow mode. However, the process is induced by GW-EMW coupling and thus still has gravitational origin.
- Systems of interest for this effect are: binary pulsars, quaking stars surrounded by accreting matter, supernovæ etc.

Four wave coupling of EMWs and GWs in vacuum

Background

- Parallel EMWs and GWs do not interact in vacuum.
- Parallel EMWs and GWs may interact and exchange energy through a medium (EM field, matter, background gravitational field etc.)
- Propagation on background leads to scattering and wave tail formation.
- Antiparallelly propagating waves may interact weakly in vacuum, causing polarization rotation, frequency shifting, energy exchange etc.
- What about four wave coupling in flat spacetime?

Four wave coupling of EMWs and GWs in vacuum

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Four wave coupling of EMWs and GWs in vacuum

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- Resonant wave coupling involving two GWs and two EMWs.
- Perturbative treatment up to 3rd order in amplitudes.
- Calculations performed in flat background, but results also valid in the high frequency approximation.
- Surprisingly simple result for coupling equations.
- Preliminary solutions to coupling equations and cross-section for incoherent process presented.

Four wave coupling of EMWs and GWs in vacuum

- Consider Maxwell's equations, keeping only the effective gravitational sources.
- Can derive the generalized wave equations

$$\tilde{\Box} E^\alpha = -\mathbf{e}_0 j_E^\alpha - \varepsilon^{\alpha\beta\gamma}\mathbf{e}_\beta j_{B\gamma} - \delta^{\alpha\gamma}\mathbf{e}_\gamma \rho_E - \varepsilon^{\alpha\beta\gamma} C^a_{\beta 0}\mathbf{e}_a B_\gamma - \delta^{\alpha\gamma} C^a_{\beta\gamma}\mathbf{e}_a E^\beta \quad (5)$$

$$\tilde{\Box} B^\alpha = -\mathbf{e}_0 j_B^\alpha + \varepsilon^{\alpha\beta\gamma}\mathbf{e}_\beta j_{E\gamma} - \delta^{\alpha\gamma}\mathbf{e}_\gamma \rho_B + \varepsilon^{\alpha\beta\gamma} C^a_{\beta 0}\mathbf{e}_a E_\gamma - \delta^{\alpha\gamma} C^a_{\beta\gamma}\mathbf{e}_a B^\beta \quad (6)$$

where $\tilde{\Box} \equiv \mathbf{e}_0\mathbf{e}_0 - \nabla\cdot\nabla$

- Consider waves of the form $E = E(x^\mu)e^{ik_\mu x^\mu} + c.c.$, amplitude variations are slow compared to exponential part.

Four wave coupling of EMWs and GWs in vacuum

- No resonant three-wave coupling: Matching conditions $\Longrightarrow$ parallel propagation $\Longrightarrow$ no interaction.
- Matching conditions for resonant four-wave interaction: $k^\mu_{E_A} + k^\mu_{E_B} = k^\mu_{h_A} + k^\mu_{h_B}$.
- Use center of mass system $\Longrightarrow \omega_{E_A} = \omega_{E_B} = \omega_{h_A} = \omega_{h_B} = \omega$ and $\mathbf{k}_{h_B} = -\mathbf{k}_{h_A}$, $\mathbf{k}_{E_B} = -\mathbf{k}_{E_A}$.
- Interaction equations of the form

$$
\begin{aligned}
\Box E_A &= C_{E_A} h_A h_B E_B^\star \\
\Box E_B &= C_{E_B} h_A h_B E_A^\star \\
\Box h_A &= C_{h_A} E_A E_B h_B^\star \\
\Box h_B &= C_{h_B} E_A E_B h_A^\star
\end{aligned}
$$

Four wave coupling of EMWs and GWs in vacuum

- Nonlinear gravitational response to GW calculated from metric ansatz: $g_{\mu\nu} = \eta_{\mu\nu} + h^{TT}_{\mu\nu} + h^{(2)}_{\mu\nu}$, (TT-gauge: $h^{TT}_{11} = -h^{TT}_{22} \equiv h_+$, $h^{TT}_{12} = h^{TT}_{21} \equiv h_\times$) $\Longrightarrow$ Vacuum Einstein's equations on the form:

$$R^{(1)}_{ab} + R^{(2)}_{ab} = 0$$

- Nonlinear response terms connected to wave perturbations, h_+, $h_\times$, through $R^{(2)}_{ab} = 0$.
- From form of evolution equations we see that we need only solve for terms $\propto e^{-2i\omega t} \Longrightarrow$

$$h^{(2)}_{11} = h^{(2)}_{22} = -h^{(2)}_{33} = (h^2_+ + h^2_\times)/4$$

Four wave coupling of EMWs and GWs in vacuum

- Largest nonlinear terms in (5) of the form Eh, Bh etc. $\Longrightarrow$ induction of nonresonant ($\omega_{nr} \neq k_{nr}$) EM fields.
- Total EM field of the form $E^{tot} = E_A + E_B + E_{nr}$; E_{nr} of one order higher in amplitude.
- Will combine with terms of appropriate frequency/wavenumber in (5) and produce terms resonant with original wave perturbation.
- Will also enter the energy momentum tensor and contribute to back reaction on GWs.
- Introduce linear polarization states, $E_+, E_\times$, of EMWs, $\Longrightarrow$

Four wave coupling of EMWs and GWs in vacuum

- Amplitude evolution equations

$$\Box E_{A+} = \frac{1}{2}\omega^2(1+\cos^2\theta)H_I E^\star_{B+} + \omega^2\cos\theta H_{II} E^\star_{B\times} \quad (7)$$

$$\Box E_{A\times} = -\omega^2\cos\theta H_{II} E^\star_{B+} + \frac{1}{2}\omega^2(1+\cos^2\theta)H_I E^\star_{B\times} \quad (8)$$

$$\Box E_{B+} = \frac{1}{2}\omega^2(1+\cos^2\theta)H_I E^\star_{A+} - \omega^2\cos\theta H_{II} E^\star_{A\times} \quad (9)$$

$$\Box E_{B\times} = \omega^2\cos\theta H_{II} E^\star_{A+} + \frac{1}{2}\omega^2(1+\cos^2\theta)H_I E^\star_{A\times} \quad (10)$$

where $H_I \equiv h_{A+}h_{B+} - h_{A\times}h_{B\times}$, $H_{II} \equiv h_{A+}h_{B\times} + h_{A\times}h_{B+}$

Four wave coupling of EMWs and GWs in vacuum

- Back reaction on GWs given by:

$$\begin{aligned} \delta G_{11} - \delta G_{22} &= \kappa(\delta T_{11} - \delta T_{22}) \\ \delta G_{12} + \delta G_{21} &= \kappa(\delta T_{12} + \delta T_{21}) \end{aligned}$$

- Largest terms in δT_{ab} of the form $E_A E_B$ with oscillating part $\propto e^{-2i\omega t} \Longrightarrow$ induction of nonresonant GW fields with same temporal variation.

- Nr fields will combine with GW fields of appropriate frequency/wavenumber through nonlinearities in EE, and form terms resonant with original perturbation.

- Can separate terms in EE describing energy momentum pseudotensor from metric response to EMW energy momentum tensor $\Longrightarrow$ nonresonant GW fields calculated separately.

Four wave coupling of EMWs and GWs in vacuum

- Expanded metric ansatz: $g_{\mu\nu} = \eta_{\mu\nu} + h^{TT}_{\mu\nu} + h^{(2)}_{\mu\nu} + h^{(nr)}_{\mu\nu}$ and corresponding tetrad basis in EE $\Longrightarrow$

$$\Box h_{A+} = \kappa(1+\cos^2\theta)E_I h^{\star}_{B+} + 2\kappa\cos\theta E_{II} h^{\star}_{B\times} \quad (11)$$

$$\Box h_{A\times} = 2\kappa\cos\theta E_{II} h^{\star}_{B+} - \kappa(1+\cos^2\theta)E_I h^{\star}_{B\times} \quad (12)$$

$$\Box h_{B+} = \kappa(1+\cos^2\theta)E_I h^{\star}_{A+} + 2\kappa\cos\theta E_{II} h^{\star}_{A\times} \quad (13)$$

$$\Box h_{B\times} = 2\kappa\cos\theta E_{II} h^{\star}_{A+} - \kappa(1+\cos^2\theta)E_I h^{\star}_{A\times} \quad (14)$$

where $E_I \equiv E_{A+}E_{B+} + E_{A\times}E_{B\times}$, $E_{II} \equiv E_{A+}E_{B\times} - E_{A\times}E_{B+}$

Four wave coupling of EMWs and GWs in vacuum

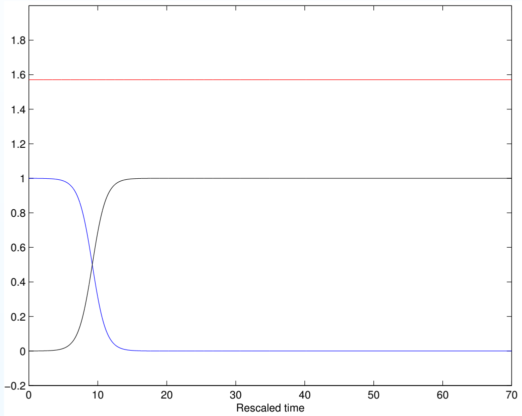
- Considering long pulses, so that we may let $\Box \to -2\mathrm{i}\omega\partial_t$ energy conservation is easily verified.

- Example: Let $E_A = E_{A+} \equiv E$, $E_B = E_{B+} \equiv E$, $h_A = h_{A+} \equiv h$, $h_B = h_{B+} \equiv h$ and put $E = \hat{E}e^{\mathrm{i}\varphi_E}$, $h = \hat{h}e^{\mathrm{i}\varphi_h}$ and rewrite in terms of normalized energy densities, $\tilde{\mathcal{E}}_{EM} \equiv \mathcal{E}_{EM}/\mathcal{E}_{tot}$, $\tilde{\mathcal{E}}_{GW} \equiv \mathcal{E}_{GW}/\mathcal{E}_{tot}$, for the waves $\Longrightarrow$

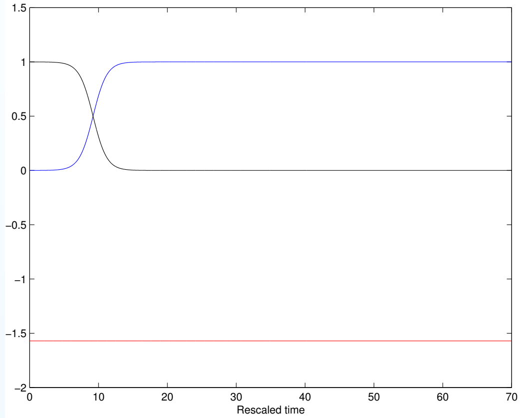
$$
\begin{aligned}
\partial_\tau \tilde{\mathcal{E}}_{EM} + \sin\Psi \tilde{\mathcal{E}}_{EM}\tilde{\mathcal{E}}_{GW} &= 0 \\
\partial_\tau \tilde{\mathcal{E}}_{GW} - \sin\Psi \tilde{\mathcal{E}}_{EM}\tilde{\mathcal{E}}_{GW} &= 0 \\
\partial_\tau \Psi - \cos\Psi (\tilde{\mathcal{E}}_{EM} - \tilde{\mathcal{E}}_{GW}) &= 0
\end{aligned}
$$

where $\tau \equiv \frac{(1+\cos^2\theta)\kappa\mathcal{E}_{tot}}{\omega} t$ and $\Psi \equiv 2\varphi_h - 2\varphi_E$

Four wave coupling of EMWs and GWs in vacuum



$\left(\frac{\mathcal{E}_{EM}}{\mathcal{E}_{GW}}\right)_{t=0} \approx 10^4 \, , \, \Psi_{t=0} = \frac{\pi}{2}$



$\left(\frac{\mathcal{E}_{GW}}{\mathcal{E}_{EM}}\right)_{t=0} \approx 10^4 \, , \, \Psi_{t=0} = -\frac{\pi}{2}$

$$\frac{\mathcal{E}_{EM}}{\mathcal{E}_{GW}}(t) = \frac{\mathcal{E}_{EM}}{\mathcal{E}_{GW}}(0) e^{-\frac{\kappa(1+\cos^2\theta)\mathcal{E}_{tot}}{\omega} t}$$

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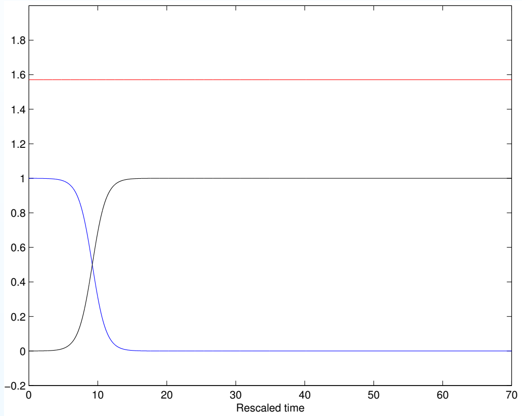

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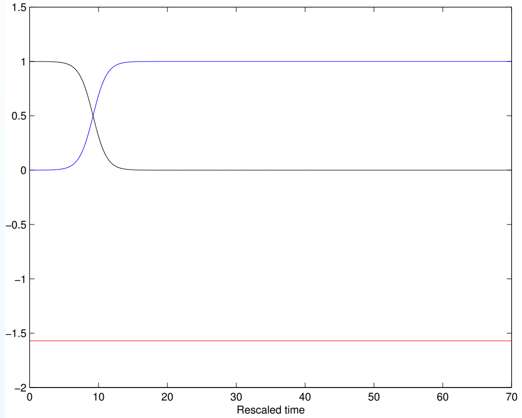
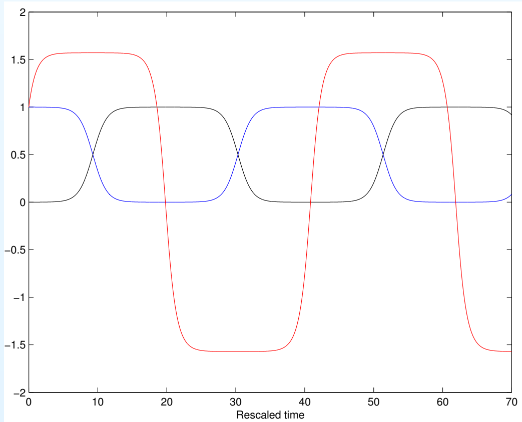
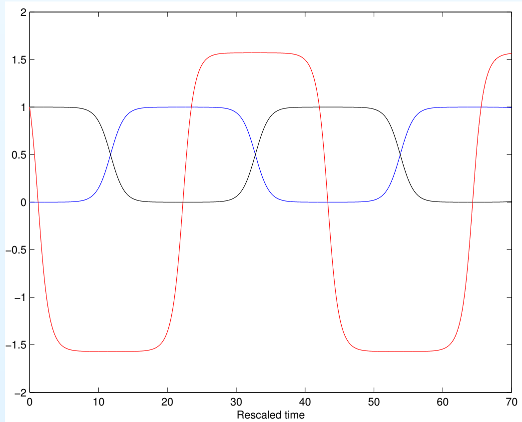


$$\left(\frac{\mathcal{E}_{GW}}{\mathcal{E}_{EM}}\right)_{t=0} \approx 10^4 \,,\ \Psi_{t=0} = -\frac{\pi}{2}$$



$$\left(\frac{\mathcal{E}_{EM}}{\mathcal{E}_{GW}}\right)_{t=0} \approx 10^4 \,,\ \Psi_{t=0} \neq \pm\frac{\pi}{2}$$



$$\left(\frac{\mathcal{E}_{GW}}{\mathcal{E}_{EM}}\right)_{t=0} \approx 10^4 \,,\ \Psi_{t=0} \neq \pm\frac{\pi}{2}$$

Four wave coupling of EMWs and GWs in vacuum

- Time scale for coherent interaction: $T_{coh} \sim h^{-2}\omega^{-1}$
- Cross-section for incoherent interaction: $\sigma \sim L_P^2 \omega^2 T_P^2$, $T_{inc} \sim T_{coh}/\omega^2 T_P^2$
- Collisional frequency: $\nu = \sigma n c$, n photon/graviton number density.
- Possible applications (to be worked out)
  - Processes in early Universe: thermalization of high frequency GW background, but perhaps not of GWs with longer wavelength.
  - Enables us to put boundaries on energy density of GWs in early universe.
  - Highly exotic astrophysical systems.
- Relevant process if $\nu > H$.

Future work

- Calculations for early universe

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- Find effective Lagrangian field theory

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- 4-wave coupling involving 4 gravitational waves

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- Coupling to other types of fields?

Thank You!