PROPAGATION of GWs

Through

MAGNETIZED PLASMA

in

CURVED SPACETIME

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MOTIVATION

- Our motivation is to study the evolution of Magnetized Plasma when a Gravitational Wave (GW) passes through. We want to examine this scenario in a curved space-time background, as for example under the conditions prevailing in the Early Universe.

- One of the desired outcomes of this approach, would be the transfer of energy from the GW to the plasma through the excitation of higher modes. In order for this transfer to be significant, the problem should be posed essentially in a non-linear manner.

- So we want to write a self-consistent, closed set of equations, where the background space-time will evolve dynamically, due to the simultaneous propagation of the GW and the evolution of the Plasma.

- Our approach differs from others up to now, because the background space-time evolves. Also the interaction of the GW with the Plasma is non-linear with respect to the GW amplitude. It is as follows:

- We consider the propagation of a GW through a magnetized plasma in a background close to a flat FRW Universe

$$ds^2 = -dt^2 + R^2(t)[dx^2 + dy^2 + dz^2] \qquad (1)$$

  and seek a *closed* set of equations describing its evolution.

- The FRW metric describes the evolution of the Early Universe, where we have roughly two periods:

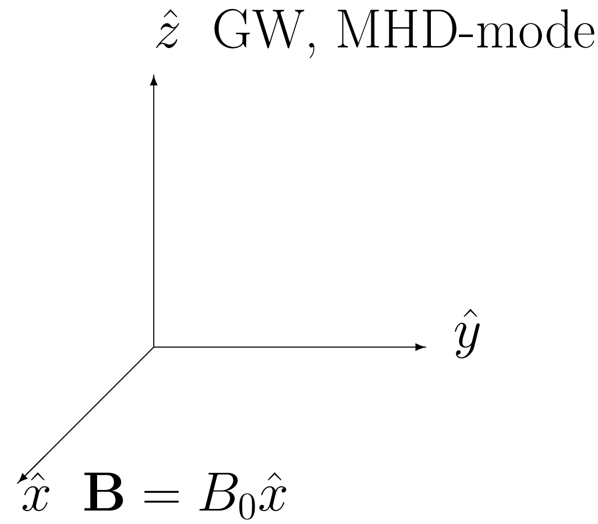
RADIATION	MATTER
$R(t) \propto t^{1/2}$	$R(t) \propto t^{2/3}$
$p = \frac{1}{3}\rho$	$p = 0$
$\rho \propto \frac{1}{R^4}$	$\rho \propto \frac{1}{R^3}$

t

$$R_{dec} \sim 10^{-3} R_0$$
$$t_{dec} \sim 10^{13} sec \simeq 3 \times 10^5 yrs$$

- The space-time is isotropic and homogeneous on any spatial hypersurface. We will use however an anisotropic space-time model.

- The geometry is given by



- We consider that the space-time metric splits into the background metric $g_{\mu\nu}^{(B)}$ and the GW perturbation $h_{\mu\nu}$ so that

$$g_{\mu\nu} = g_{\mu\nu}^{(B)} + h_{\mu\nu} \tag{2}$$

- The background metric is of the form

$$ds^2 = -dt^2 + R^2(t,z)[dx^2 + dy^2] + S^2(t,z)dz^2 \tag{3}$$

- This is anisotropic in the z-direction and we assume that it must be *expanding*, i.e., $\dot{R},\ \dot{S} > 0$.

- The main argument of our Proposal is to focus on the background space-time, which must evolve according to the Einstein's equations
([1] C. W. Misner et.al., *Gravitation*, Freeman 1973)
(Units $c = G = 1$)

$$
\begin{aligned}
G_{\mu\nu}^{(B)} &:= R_{\mu\nu}^{(B)} - \frac{1}{2} R^{(B)} g_{\mu\nu}^{(B)} = \\
&= 8\pi T_{\mu\nu}^{(GW)} + 8\pi T_{\mu\nu}^{(NG)} \qquad (4)
\end{aligned}
$$

- By construction $G^{\mu\nu}_{(B)|\nu} = 0$ is conserved with respect to the background!

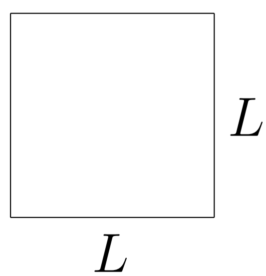
- So if we construct the GW energy-momentum tensor (which is $\propto h^2$) to satisfy
$$T^{\mu\nu}_{(GW)|\nu} = 0 \qquad (5)$$
then the non-gravitational degrees of the system

(Plasma $\oplus$ EM), must also satisfy
$$T^{\mu\nu}_{(NG)|\nu} = 0 \qquad (6)$$

- Following [1] we use

$$
\begin{aligned}
T^{(GW)}_{\mu\nu} &= \frac{1}{32\pi} < \bar{h}_{\alpha\beta|\mu}\bar{h}^{\alpha\beta}_{|\nu} >= \\
&= \frac{1}{32\pi}\frac{1}{V}\int_{L^4} d^4x\sqrt{-g^{(B)}}(\bar{h}_{\alpha\beta})_{|\mu}(\bar{h}^{\alpha\beta})_{|\nu}
\end{aligned}
\tag{7}
$$

with $V = \int_{L^4} d^4x\sqrt{-g^{(B)}}$ a volume under consideration which is a box of proper length L,



where the wavelength of the GW, λ must satisfy $\lambda \ll L \leq \mathcal{R}$ =curvature radius$\simeq (RS)^{1/2}$.

- The gauge conditions on the GW give only two (2) independent components for $h_{\mu\nu}$, and with

$$
\begin{aligned}
\bar{h}_{\mu\nu} &= h_{\mu\nu} - \frac{1}{2}hg^{(B)}_{\mu\nu}, \\
\bar{h} &:= g^{\mu\nu}_{(B)}\bar{h}_{\mu\nu} = 0
\end{aligned}
\tag{8}
$$

we can drop bars.

These components are

$$
\begin{aligned}
h_1 := h_{uu} = -h_{vv}, \\
h_2 := h_{uv} \\
h_{uu} = \frac{1}{4}(h_{xx} + h_{yy} - 2h_{xy}) \\
h_{vv} = \frac{1}{4}(h_{xx} + h_{yy} + 2h_{xy}) \\
h_{uv} = \frac{1}{4}(h_{xx} - h_{yy})
\end{aligned}
\tag{9}
$$

- Then we obtain

$$
\begin{aligned}
8\pi T_{00}^{(GW)} &= \frac{1}{V_2}\int_0^L dtdz \frac{S}{R^2}\left[2[(h_{1,t})^2 + (h_{2,t})^2] - \right. \\
&- 8\frac{\dot{R}}{R}[h_1 h_{1,t} + h_2 h_{2,t}] + \\
&+ \left. 8\left(\frac{\dot{R}}{R}\right)^2 [(h_1)^2 + (h_2)^2]\right] \\
8\pi T_{zz}^{(GW)} &= \frac{1}{V_2}\int_0^L dtdz \frac{S}{R^2}\left[2[(h_{1,z})^2 + (h_{2,z})^2] - \right. \\
&- 8\frac{R'}{R}[h_1 h_{1,z} + h_2 h_{2,z}] + \\
&+ \left. 8\left(\frac{R'}{R}\right)^2 [(h_1)^2 + (h_2)^2]\right]
\end{aligned}
\tag{10}
$$

$$8\pi T_{xx}^{(GW)} = \frac{1}{V_2} \int_0^L dtdz \left[-4S[\left(\frac{\dot{R}}{R}\right)^2 - \frac{1}{S^2}\left(\frac{R'}{R}\right)^2] \right.$$
$$\left. [(h_1)^2 + (h_2)^2] \right] \quad (11)$$

with $V_2 = \int_0^L dtdzR^2S$.

PROPAGATION OF GW

- The propagation equation for the GW must be linear in h and it can be derived along the lines of the sections 35.13-14 of [1]. However here we do *not* have an empty spacetime but it contains also non-gravitational fields. We must have

$$(\bar{h}_{\mu\nu})^{\alpha}_{|\alpha} + 2R^{(B)}_{\mu\alpha\nu\beta}\bar{h}^{\alpha\beta} - 2t_{\alpha(\mu}\bar{h}^{\alpha}_{\nu)} = 0 \quad (12)$$

where

$$\begin{aligned} t_{\alpha\mu} &:= 8\pi\left(T_{\alpha\mu}^{(NG)} - \frac{1}{2}T^{(NG)}g^{(B)}_{\alpha\mu}\right), \\ T^{(NG)} &= g^{\mu\nu}_{(B)}T^{(NG)}_{\mu\nu} \\ T^{(NG)}_{\mu\nu} &= (\rho+p)u_\mu u_\nu + pg^{(B)}_{\mu\nu} + \\ &+ F_{\mu\alpha}F_{\nu}{}^{\alpha} - \frac{1}{4}g^{(B)}_{\mu\nu}F^2 \end{aligned} \quad (13)$$

This is the correct generalization that gives linear in h evolution equation!

- The propagation equation looks like this!

$$\partial_{tt}(h_1) + \left(2\frac{\dot{R}}{R} + \frac{\dot{S}}{S}\right)\partial_t(h_1) - \frac{1}{S^2}\partial_{zz}(h_1) -$$
$$-\frac{1}{S^2}\left(2\frac{R'}{R} + \frac{S'}{S}\right)\partial_z(h_1) +$$
$$+\left[\left(\frac{\dot{R}}{R}\right)^2 - \frac{1}{S^2}\left(\frac{R'}{R}\right)^2\right](h_1) + 8\pi(\rho - p)(h_1) -$$
$$-8\pi R^2[-(E^y)^2 + S^2(B^x)^2](h_2) = 0 \tag{14}$$

and similar for $h_2 \leftrightarrow h_1$.

PLASMA EVOLUTION

- The non-gravitational degrees, namely plasma and EM from the conservation Eq. (6) give

$$\frac{D\rho}{ds} + 3\theta(\rho + p) = -\frac{1}{4\pi}u_\alpha F^\alpha_{\ c} F^{cb}_{\ |b} \tag{15}$$

$$(\rho + p)\frac{Du^\alpha}{ds} = H^{\alpha b}\nabla_b p - \frac{1}{4\pi}H^{\alpha b}F_{bc}F^{cd}_{\ |d} \tag{16}$$

where $u^\alpha = (u^0(t,z), u^z(t,z))$ is the four-velocity of the fluid and also assume the continuity equation $(\rho u^\alpha)_{|\alpha} = 0)$

$$\partial_t(\rho u^0) + \partial_z(\rho u^z) \; + \; \rho u^0 \left(2\frac{\dot{R}}{R} + \frac{\dot{S}}{S}\right) + \\ + \; \rho u^z \left(2\frac{R'}{R} + \frac{S'}{S}\right) = 0 \tag{17}$$

The EM-FIELD

Maxwell's Equations

$$F^{\alpha\beta}_{|\beta} = 4\pi J^\alpha \\ F_{\alpha\beta|\gamma} + F_{\beta\gamma|\alpha} + F_{\gamma\alpha|\beta} = 0 \tag{18}$$

give

$$-\partial_t E^y + \partial_z B^x \; - \; E^y \left(2\frac{\dot{R}}{R} + \frac{\dot{S}}{S}\right) + \\ + \; B^x \left(2\frac{R'}{R} + \frac{S'}{S}\right) = 4\pi J^y \tag{19}$$

$$- \; \partial_z E^y + S^2 \partial_t B^x = 0 \tag{20}$$

with the current

$$J^y = -\sigma E^y u^0 - \sigma B_0 S^2 u^z \tag{21}$$

The BACKGROUND EVOLUTION

- Using Eq. (4) we can write

$$G_{00} := -\frac{2}{S^2}\frac{R''}{R} + \left[\left(\frac{\dot{R}}{R}\right)^2 - \frac{1}{S^2}\left(\frac{R'}{R}\right)^2\right] + \\ +2\left[\frac{\dot{R}\dot{S}}{RS} + \frac{1}{S^2}\frac{R'S'}{RS}\right] = \\ = 8\pi T_{00}^{(GW)} + 8\pi T_{00}^{(NG)} \quad (22)$$

with

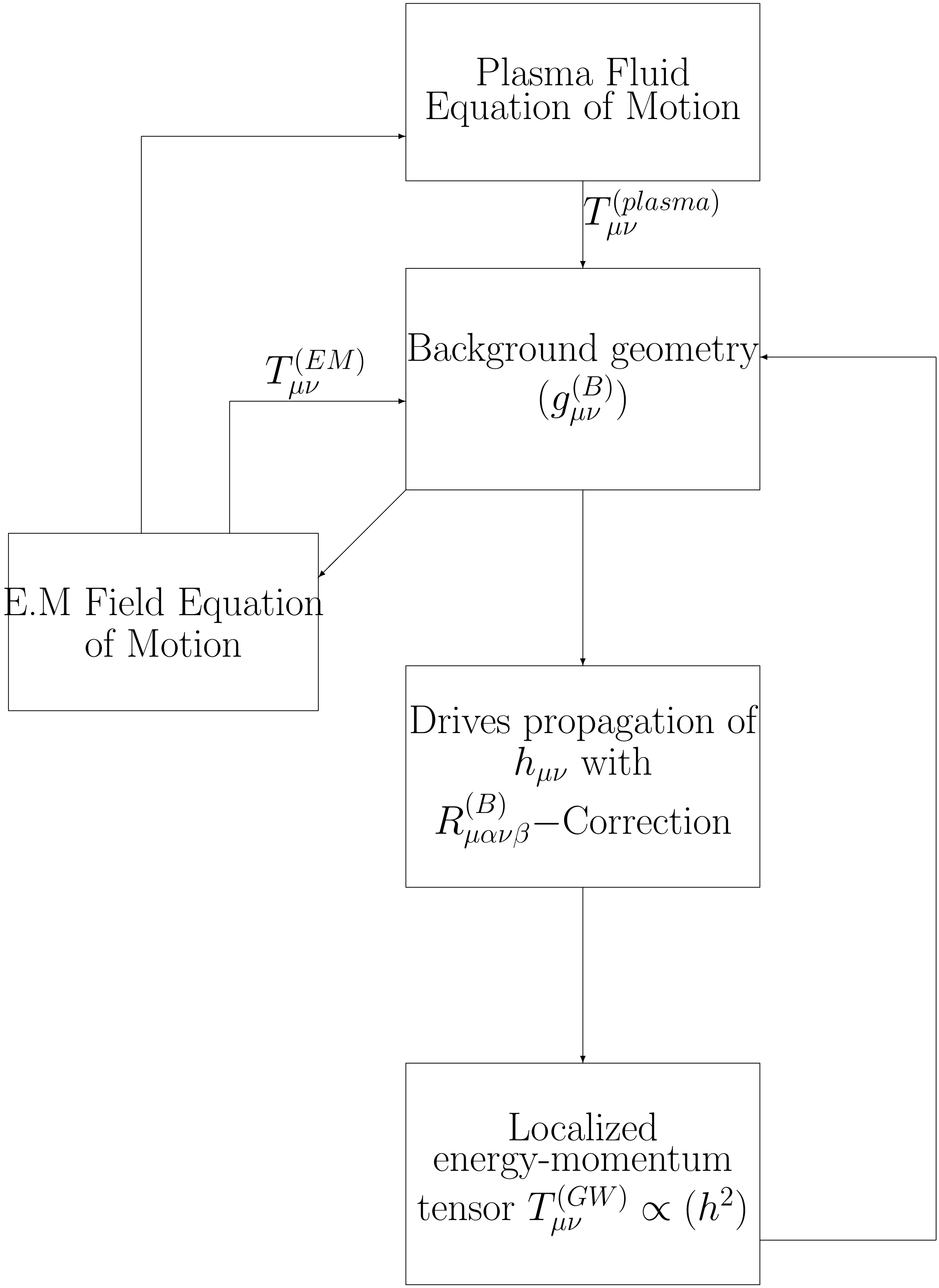
$$T_{00}^{(NG)} = \rho(u^0)^2 + pS^2(u^z)^2 + \\ \frac{1}{2}R^2[(E^y)^2 + S^2(B^x)^2] \quad (23)$$

- In the same way one obtains the rest of the field equations, as for example

$$G_{zz} := -S^2\left[2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 - \frac{1}{S^2}\left(\frac{R'}{R}\right)^2\right] = \\ = 8\pi T_{zz}^{(GW)} + 8\pi T_{zz}^{(NG)} \quad (24)$$

with

$$T_{zz}^{(NG)} = (\rho + p)S^4(u^z)^2 + S^2 p + \\ \frac{1}{2}R^2S^2[(E^y)^2 + S^2(B^x)^2] \quad (25)$$


Plasma Fluid Equation of Motion
$T_{\mu\nu}^{(plasma)}$
Background geometry $(g_{\mu\nu}^{(B)})$
$T_{\mu\nu}^{(EM)}$
E.M Field Equation of Motion
Drives propagation of $h_{\mu\nu}$ with $R_{\mu\alpha\nu\beta}^{(B)}-$Correction
Localized energy-momentum tensor $T_{\mu\nu}^{(GW)} \propto (h^2)$

DISCUSSION AND FUTURE PLANS

- We have presented a set of equations to study the evolution of plasma and the possible excitation of modes which gain energy.

- This energy comes from the GW, propagating through the plasma in a curved space-time, which also evolves dynamically.

- These equations apply for *strong* GW, in the Early Universe, in the Radiation era and/or earlier.

- We plan to study the system first numerically. Also we want to study the system analytically. This will be possible by choosing metric functions $R(t,z)$, $S(t,z)$ close to standard metrics describing Radiation Universes, or Universes with other equations of state. Also metrics describing Black-Hole spaces can be introduced.