Kinetic aspects on the GW plasma interaction

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Gravitational wave as charge particle accelerator

- The motion of a charged particle in curved space is described by the Hamiltonian

$$H(x^a, p_a) = \frac{1}{2} g^{\mu\nu} (p_\mu - eA_\mu)(p_\nu - eA_\nu) = \frac{1}{2}$$

- $g^{\mu\nu}$ are the covariant components of the metric tensor $A_\mu = (0, 0, B_0 x, 0)$ the vector potential

$$\frac{dx^a}{d\tau} = \frac{\partial H}{\partial p_a}, \quad \frac{dp_a}{d\tau} = -\frac{\partial H}{\partial x^a}$$

Monochromatic GW

- Metric tensor up to order O(*a*) :

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu} + \mathrm{O}(a^2)$$

$$\eta^{\mu\nu} = diag\{1,-1,-1,-1,-1\} \quad , \quad h^{\mu\nu} = a\,\Theta_{\mu\nu}\cos\Phi,$$

$$\Theta_{\mu\nu} = \Theta_{\mu\nu}(\theta), \quad \Phi = \Phi(\nu,\theta,x^j)$$

$$\Theta_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -c_\theta^2 & 0 & c_\theta s_\theta \\ 0 & 0 & 1 & 0 \\ 0 & c_\theta s_\theta & 0 & -s_\theta \end{bmatrix} \quad , \quad c_\theta = \cos\theta, s_\theta = \sin\theta$$

Gravitational wave as charge particle accelerator

- We assume that the ambient magnetic field is along the z axis $\vec{B} = B_0 \hat{e}_z$
- The gravitational wave with amplitude α propagates in a direction $\vec{k}$ of angle θ with respect to the magnetic field
- The Parameter ν is equal to the ratio

$$\nu = \frac{\omega}{\Omega}$$

- GW frequency ω and the gyrofrequency Ω

Gravitational wave as charge particle accelerator

- Changing variables

$$(x^0, x^1, x^3, p_0, p_1, p_3) \Rightarrow (\chi, q, \phi, I, p, J)$$

$$H = \frac{1}{2}\left(I^2 - 2I\nu J - 2\sin\theta\nu Jp - \frac{1-a\sin^2\theta\cos\phi}{1-a\cos\phi} - \frac{q^2}{1+a\cos\phi} \right)$$

- χ is ignorable and I is a constant of the motion $\Rightarrow$ 2 degrees of freedom

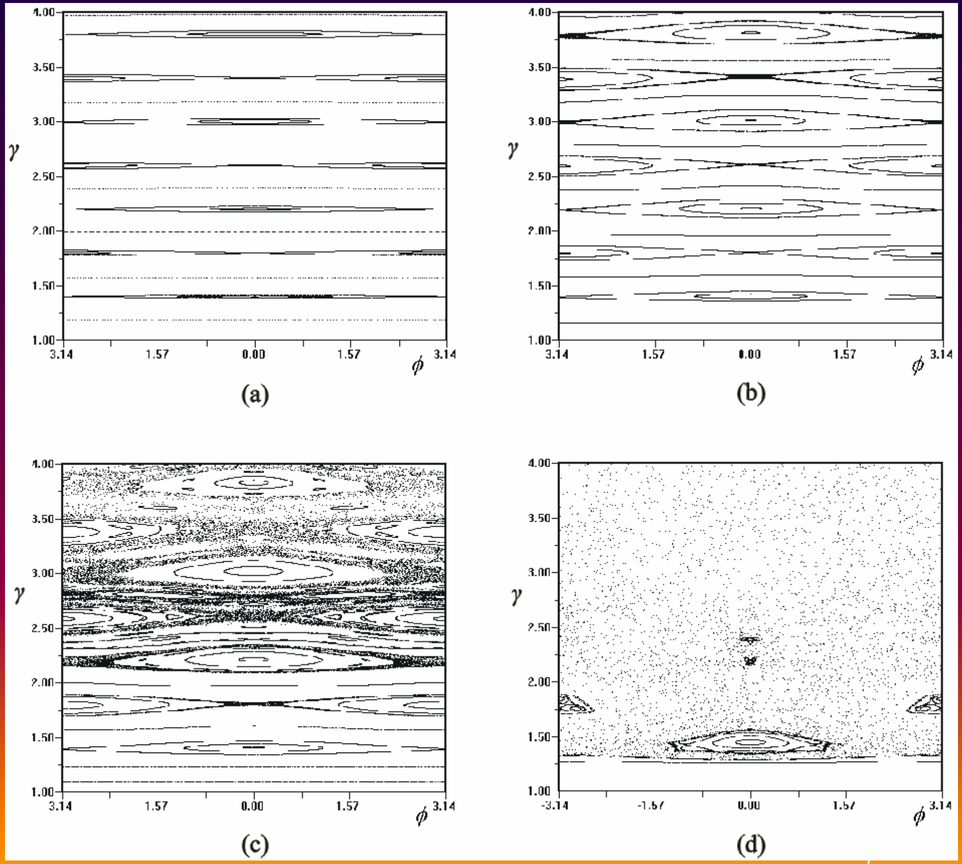
Gravitational wave as charge particle accelerator

- The new Hamiltonian can be written as a perturbed Hamiltonian

$$H = H_0 + aH_1 + a^2 H_2 + \ldots$$

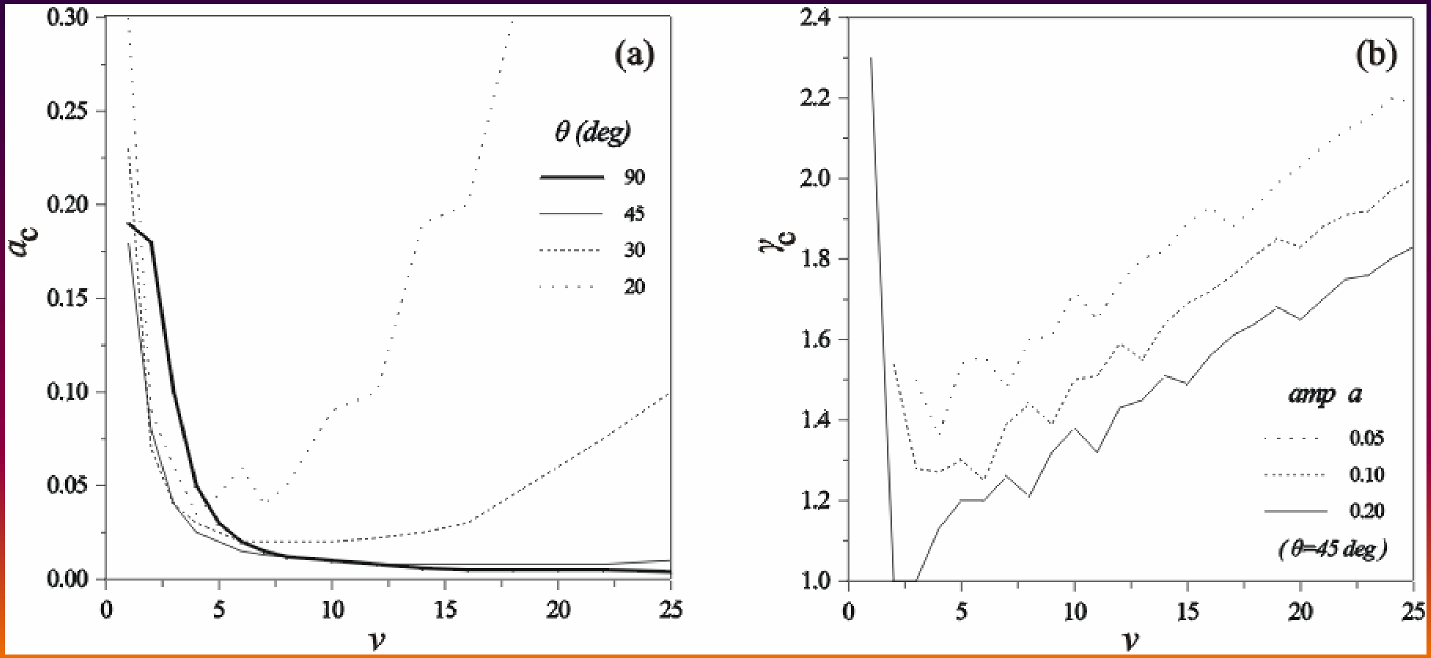
Gravitational wave as charge particle accelerator

- The dynamics presented by the Poincare map



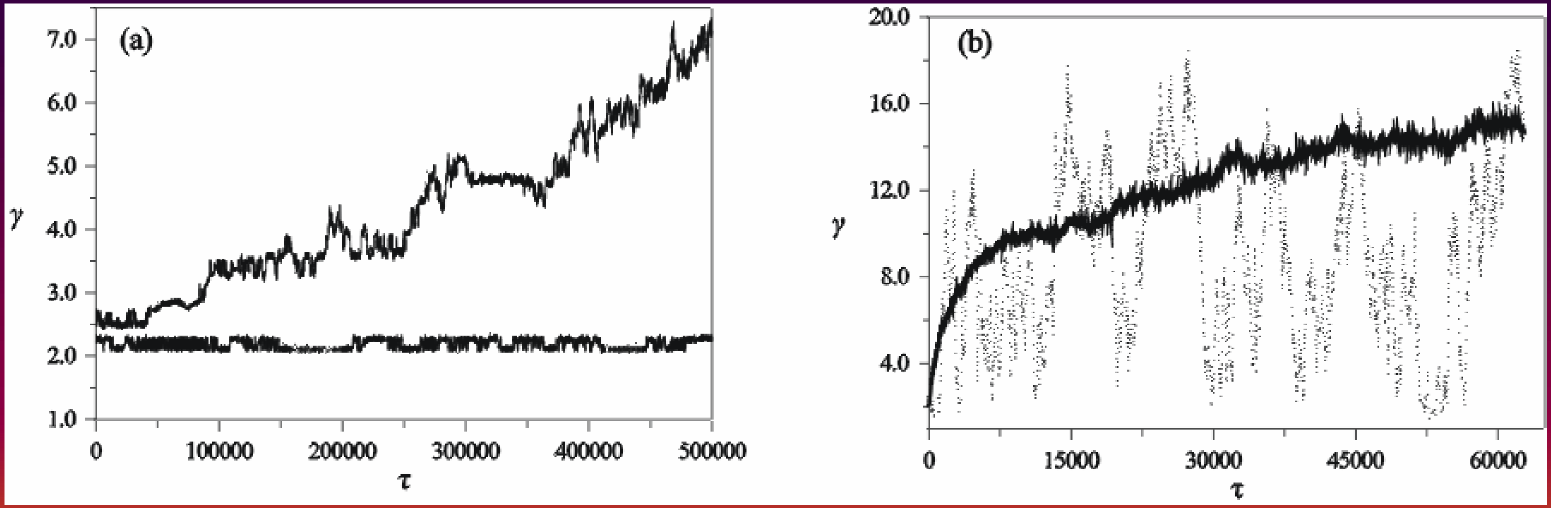
Gravitational wave as charge particle accelerator

- Threshold for stochasticity and endless acceleration

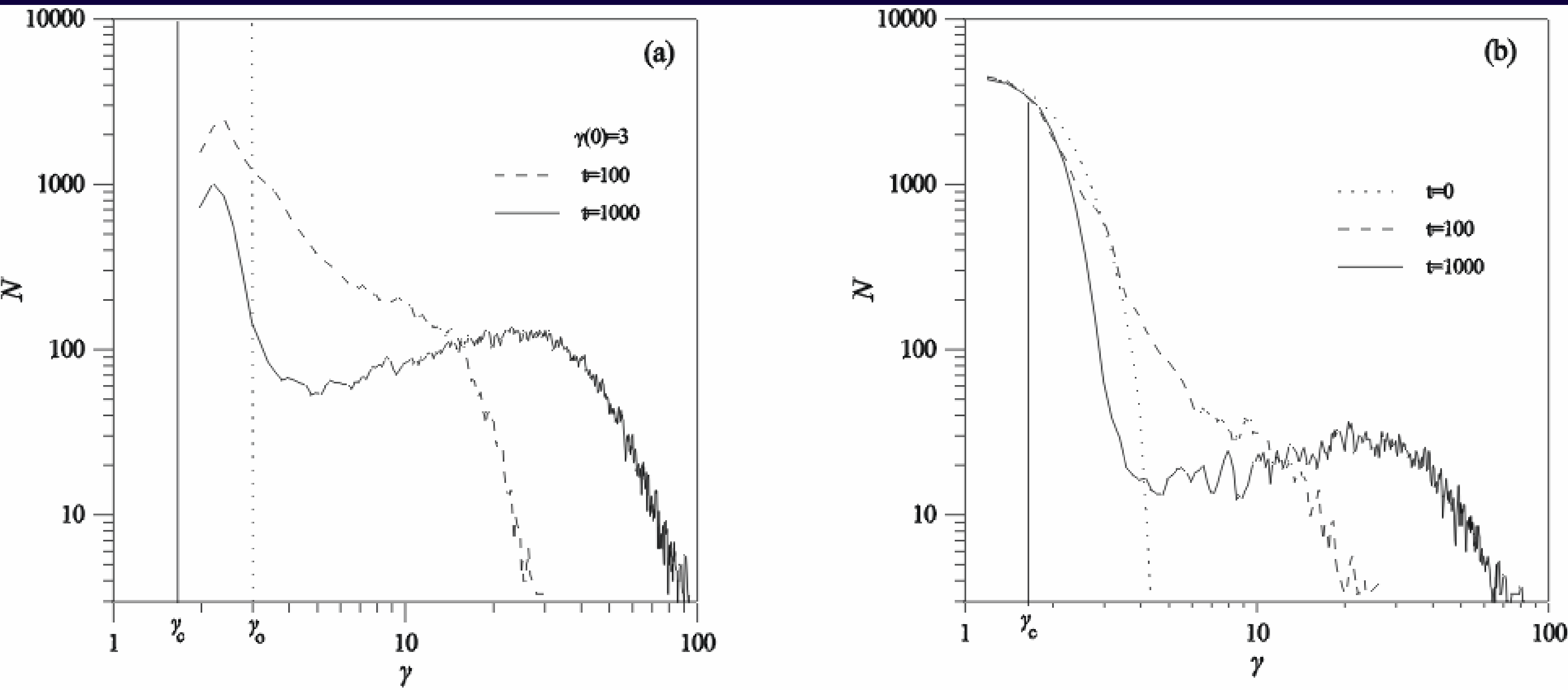


Gravitational wave as charge particle accelerator

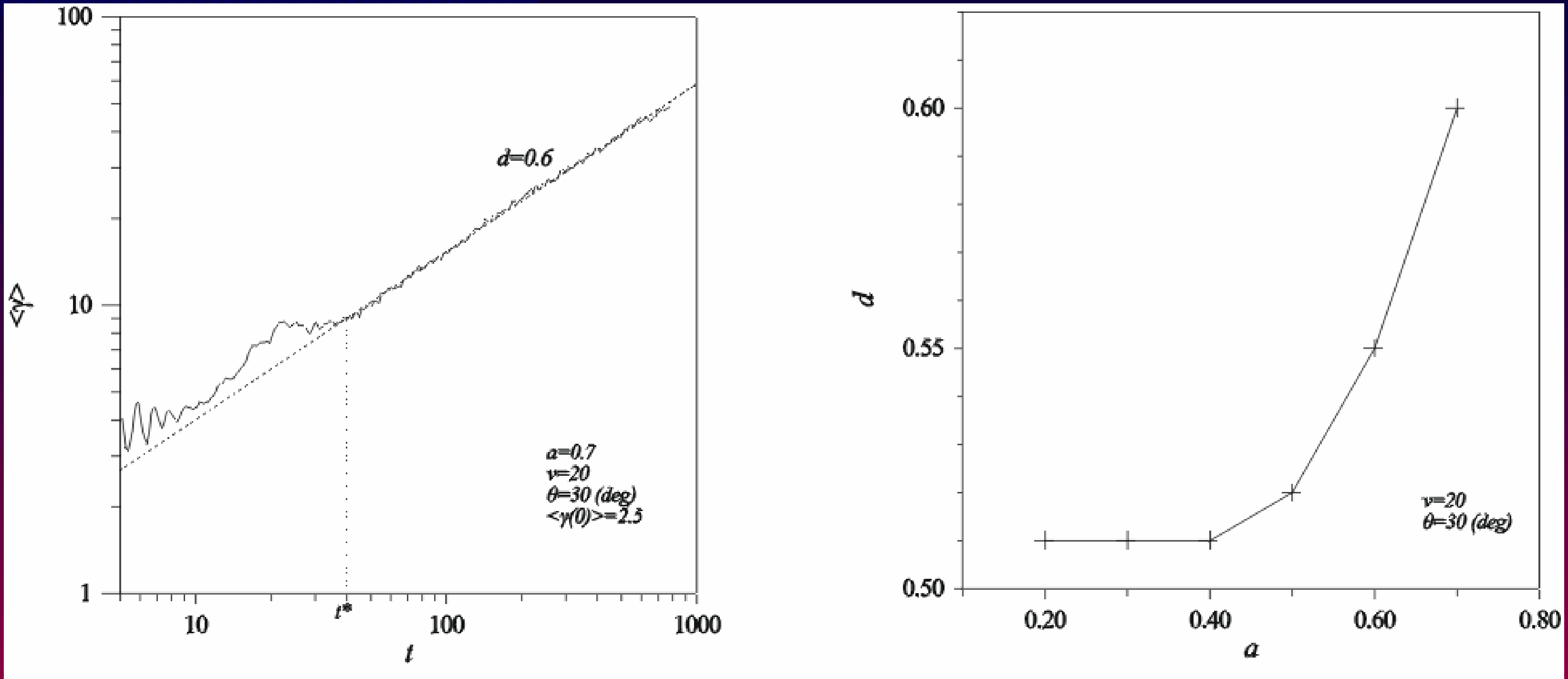
- Chaotic diffusion



GW interacting with ensemble of particles



Anomalous diffusion



(a) (b)

$$< \gamma > \sim t^d$$

Main points from the GW-charged particle interaction

1. The GW can accelerate electrons from the tail of the ambient velocity distribution to very high energies provided that

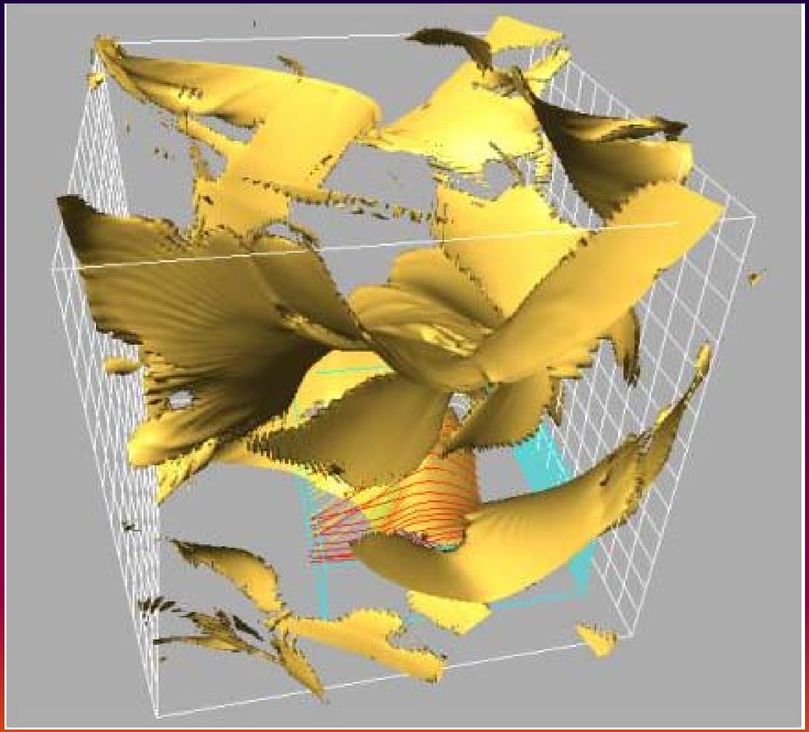
$$0.005 < a < 0.5 \quad 5 < \omega / \Omega < 20$$

2. The acceleration time depends in general on a but it is relatively short msecs and the distance 10^8cm
3. The mean energy diffusion of the electrons interacting with the GW follows a simple scalling

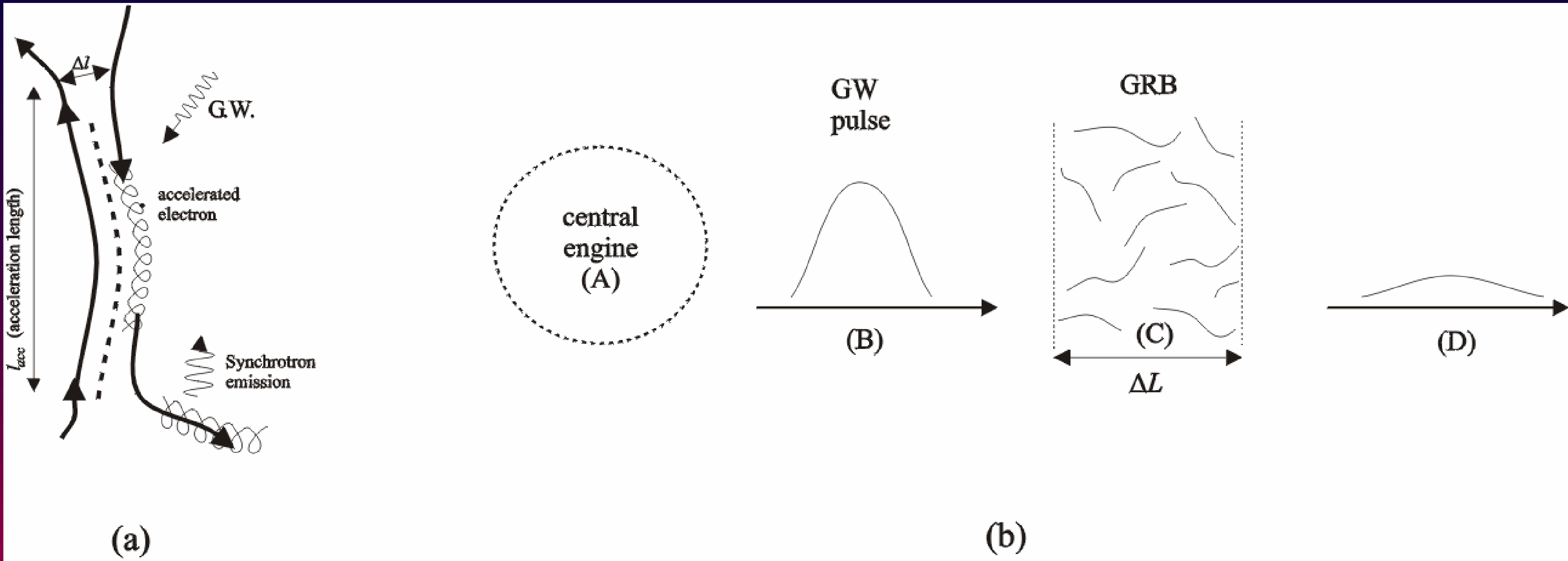
$$< \gamma > \sim t^d$$

A new model for GW-Plasma coupling

- Modern studies on MHD turbulence show that the magnetized plasma is highly inhomogeneous-full of null surfaces



A new model for GRB



Energetics

- Let us assume that the null sheets have a distribution

$$N(\ell) \sim \ell^{-b}$$

- The acceleration time $t_{acc} \sim \ell_{acc} / c$

$$N(t) \sim t^{-b}$$

Energetics

- We also know that $< \gamma > \sim t^{d}$
- N(γ)dγ=N(t)dt

$$N(\gamma) \sim \gamma^{(-b+1-d)/d}$$

- b=3/2, d=0.5 $N(\gamma) \sim \gamma^{-2}$

Energetics

- Acceleration in one Null Sheets

$$E_{ns} \sim \left(\frac{n_t}{n_0} \right) n_0 (\ell^2 \times \Delta\ell) \times \gamma mc^2$$

- Acceleration from many Null Sheets

$$W \sim \left[f \frac{(\Delta L)^3}{l^2 \Delta l} \right] E_{ns} \sim 10^{47-48}\, ergs$$

Linear combination of *K* gravitational waves

- Metric Tensor

$$g^{\mu\nu} = \eta^{\mu\nu} + \sum_{k=0}^{K-1} h^{\mu\nu}_{(k)} + \mathrm{O}(a^2)$$

- where
- $h^{\mu\nu}_{(k)} = a_k\, \Theta^{(k)}_{\mu\nu} \cos\Phi_{(k)}, \quad \Theta^{(k)}_{\mu\nu} = \Theta^{(k)}_{\mu\nu}(\theta_k), \quad \Phi_{(k)} = \Phi(\nu_k, \theta_k, \varphi_{0k}, x^j)$
- (ϕ_{0k} : is the initial phase of the *k*-the GW)

Conditions - Simplifications

- Magnetic field

$$\vec{B} \,//\, z \quad \rightarrow \quad \vec{A} = (0, 0, B_0(x^1 + c), 0)$$

- GW Polarization

$$\Phi_{(k)} = \nu_k (s_{\theta_k} x^1 + c_{\theta_k} x^3 - x^0) + \varphi_{0k}$$

- Constant propagation angle

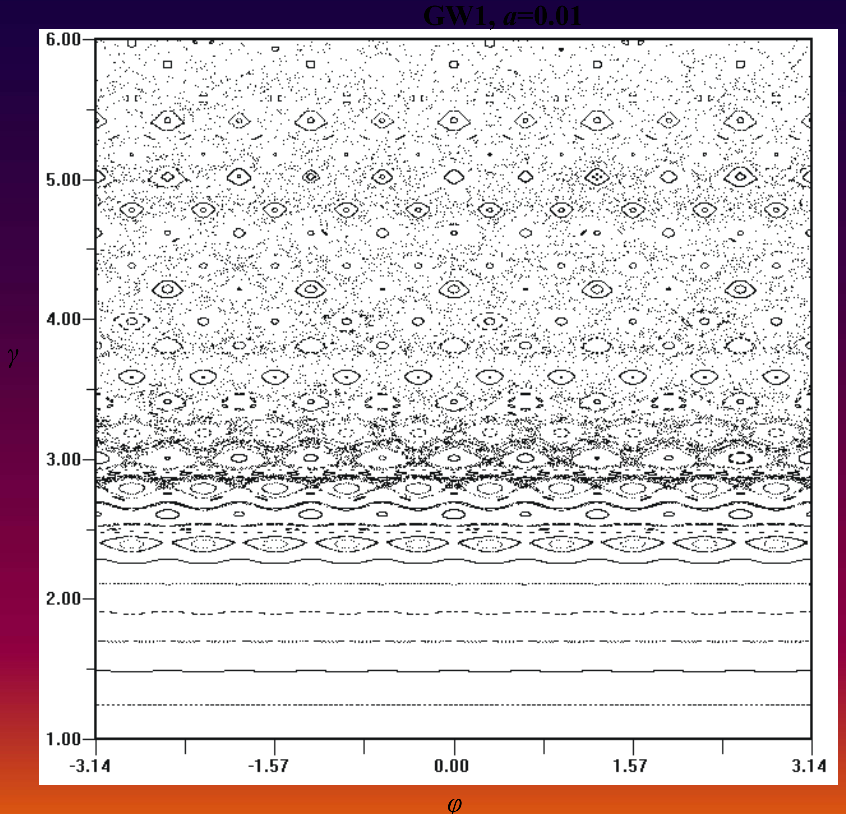
Linear combination of *K* gravitational waves

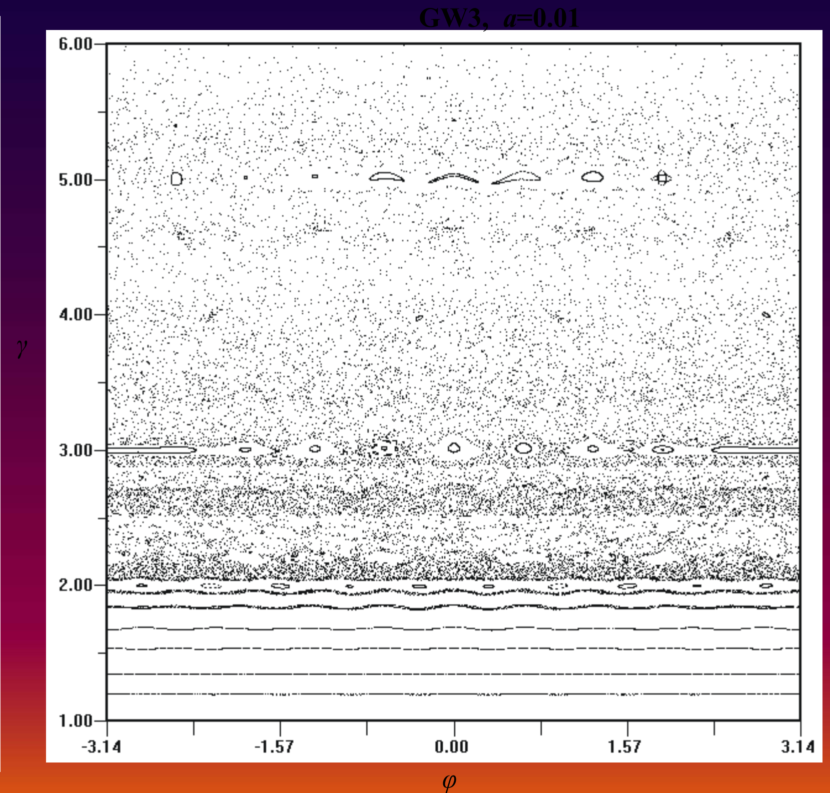
- The Hamiltonian

$$H = \frac{1}{2}(p_0^2 - p_1^2 - p_3^2 - (x^1)^2) + \frac{1}{2}\left[-c_\theta^2 p_1^2 + (x^1)^2 - s_\theta^2 p_3^2 + 2c_\theta s_\theta p_1 p_3\right]\sum_{k=0}^{K-1} a_k \cos\Phi_k$$

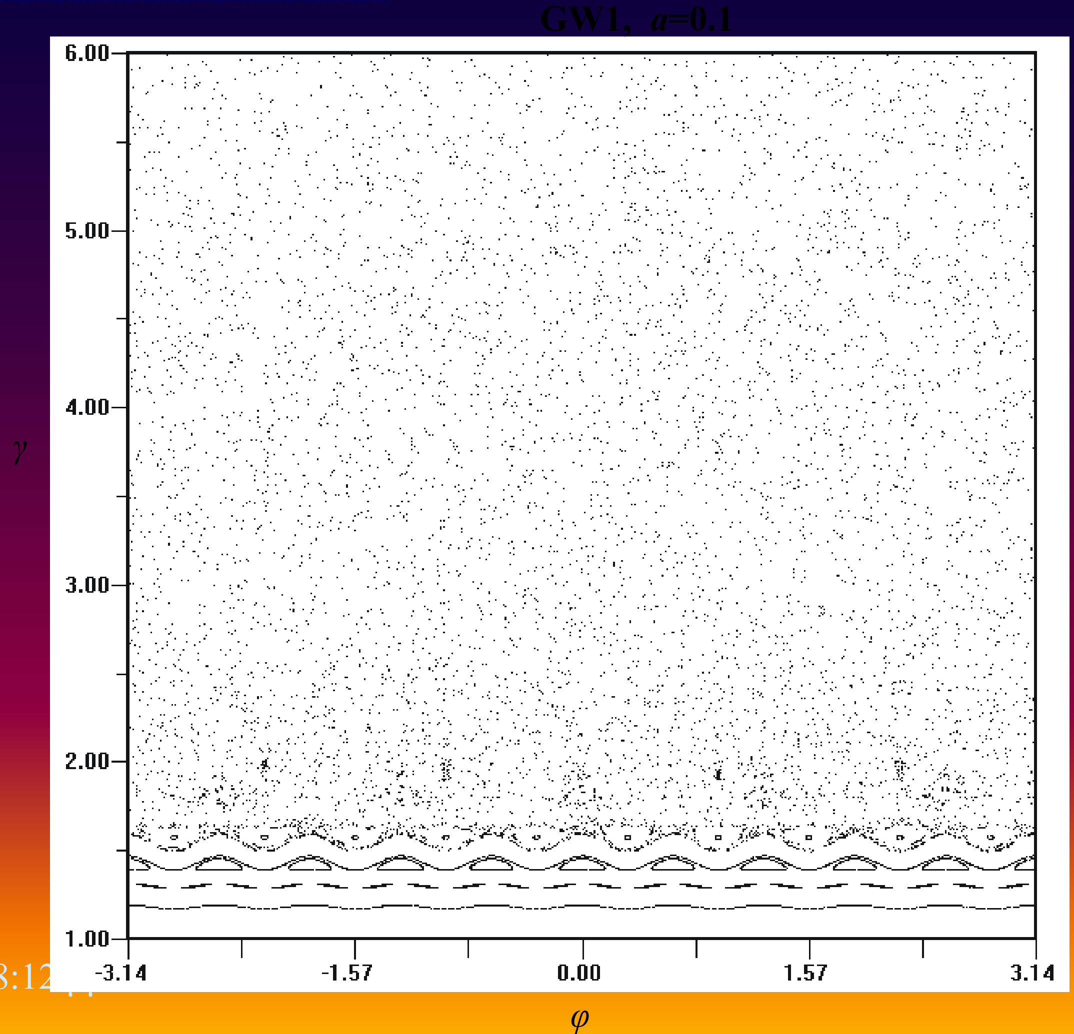
unperturbed part

Linear combination of *K* gravitational waves

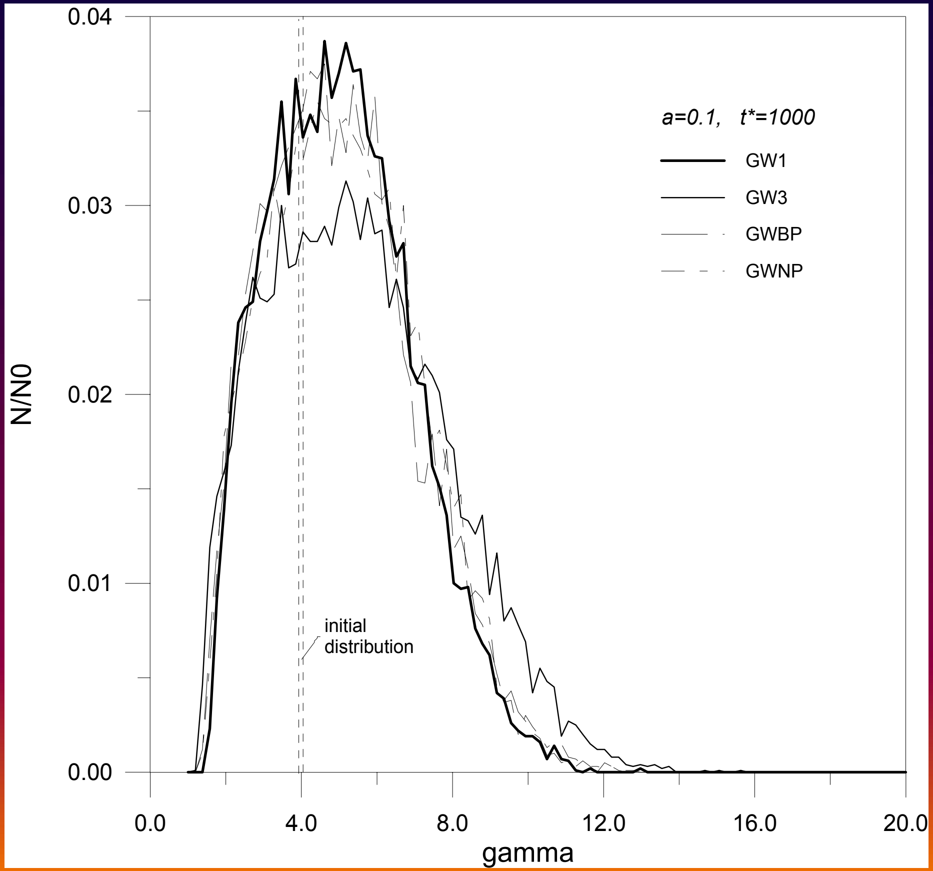




Increasing the amplitude of the GW



Linear combination of *K* gravitational waves



GW interacting exciting High frequency EM waves

Maxwell's equations in presence of GW

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} - \vec{J}_B \qquad \vec{J}_E = -\frac{B_0}{2}\frac{\partial h}{\partial z}\hat{e}_y$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J}_m + \vec{J}_E + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \qquad \vec{J}_B = -\frac{B_0}{2c}\frac{\partial h}{\partial t}\vec{e}_x$$

- Marlkund, Brodin, Dunsby (ApJ, 2000)

$$\vec{J}_m = nq\vec{v}$$

- The magnetic field is inhomogeneous

$$\vec{B} = B(z)\hat{e}_1$$

GW interacting exciting High frequency EM waves

- All perturbed quantities have the form

$$\vec{E}_1(z)\exp(-i\omega t)$$

$$h = \overline{h}\exp(ik(z-t)), \quad \omega = k$$

- The plasma wave excited is in resonance with the GW

GW interacting exciting High frequency EM waves

- The particles motion in the linear approximation

$$-i\omega m\vec{v} = q[\vec{E} + \vec{v} \times \vec{B}] \qquad \vec{J}_m = nq\vec{v}$$

$$\left(\frac{\partial^2}{\partial z^2} + k^2 \right) \overline{E}^2 + i\omega\mu_0 \overline{J}_m^2 = -S(z)$$

GW interacting exciting High frequency EM waves

$$\left(\frac{\partial^2}{\partial z^2} + k^2 - \Delta k^2\right)\overline{E}^2 = -S(z)$$

$$\Delta k^2 = \frac{\omega^2{}_p(\omega^2 - \omega^2{}_p)}{\omega^2 - \omega^2{}_h}$$

$$E^2(z,t) = E_{out}\exp[i(kz - \omega t)]$$

$$E_{out} = -\frac{ik\overline{h}}{2}\int_0^a B(z)\exp\left(\frac{i\Delta k^2 z}{2k}\right)dz$$

$$L = \frac{2k}{\pi\Delta k^2}$$

Astrophysical application

- GW frequency 10^3Hz, the plasma frequency for typical interstellar medium is also 10^3Hz, magnetic field

$$B(r) = \frac{B_s r_s^{\,3}}{r^3}$$

$$B_s = 10^6 - 10^8\, T$$

$$\Delta k^2 = \frac{\omega^2{}_p(\omega^2 - \omega^2{}_p)}{\omega^2 - \omega^2{}_h} << k$$

$$E_{\max} \sim 1.5\times 10^6\, \bar{h}B \quad V/m$$

$$E_{\max} \sim 50 \quad MV/m$$

$$h = 0.001$$

$$B_s = 10^8\, T$$

Vlassov Equation for GW-plasma interaction

- The article of Macedo and Nelson (Phys rev D, 1983) no time to go in details at this point, but it is an excellent start to develop a theory for the direct excitation of high frequency EM waves
- They reach negative results but their study was directed to the interaction at the interstellar medium, which we know that its really weak. It is interesting to apply their article near the strong magnetic field sources….

Future plans

- Non linear interaction of particles and GW in a inhomogeneous plasma….e.g. Current sheets in the turbulent atmosphere