Pythagoras Program
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Numerical studies on the excitation of magnetosonic waves by a gravitational wave

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Collaborators

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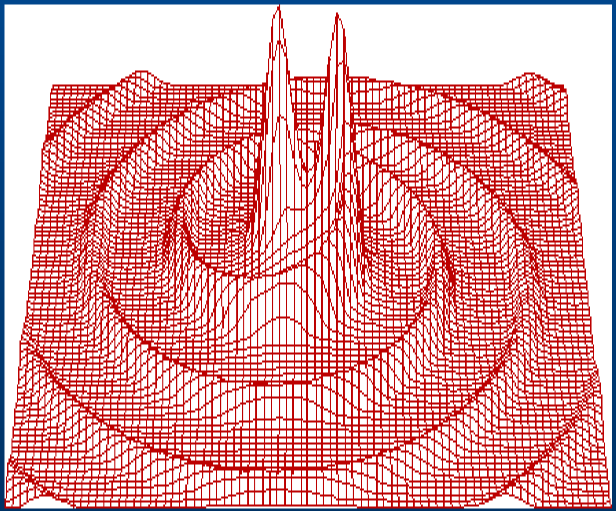
Outline

- **Motivation**
- **Background**
- **Model of equations**
- **Linear limit**
- **Numerical results**
- **Conclusions**
- **Future Plans**

Motivation

- Investigate numerically plasma wave phenomena driven by GW
- Diagnostic tool for indirect detections of GW...



Basic Assumptions

- 1+3 orthonormal frame
- TT gauge
- Orthogonal frame
- Strong magnetic field $\boldsymbol{B_0} = B_0 \, \boldsymbol{e_x}$
- + polarized GW in Minkowski space-time
- constant GW amplitude

RELATIVISTIC MHD EQS

COVARIANT FORM

Geometry

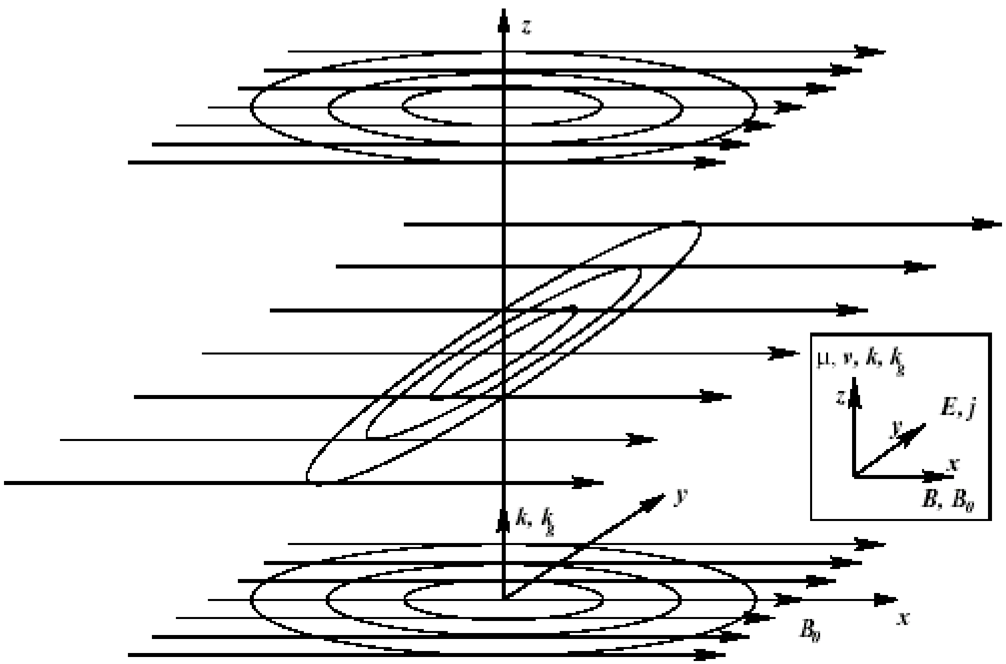


Fig. 1. A GW propagating in the positive z-direction across an ambient magnetic field (in the x-direction) excites a MSW. The orientations of the MSW components are indicated in the inset.

Travelling GW

↓

Deforms rings of plasma particles

↓

modifies magnetic field $\boldsymbol{B_1}$

↓

$\boldsymbol{B_1}$ compressions propagate

Model of Equations

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} - \vec{J}_B$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J}_m + \vec{J}_E + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\vec{J}_E = -\frac{B_0}{2}\frac{\partial h}{\partial z}\hat{e}_y$$

$$\vec{J}_B = -\frac{B_0}{2c}\frac{\partial h}{\partial t}\vec{e}_x$$

Model of Equations II

$$\rho\left(\frac{\partial\vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}\right)=-\nabla P+\frac{1}{c}(\vec{J}_m\times\vec{B})+\nu\nabla^2\vec{u}$$

$$\frac{\partial\rho}{\partial t}+\nabla(\rho\vec{u})=0$$

$$\vec{E}=-\frac{1}{c}(\vec{u}\times\vec{B})+\eta\vec{J}_m$$

$$\nabla P=c_s^2\nabla\rho,\ \text{where}\ c_s^2\equiv\gamma P_0/\rho_0$$

One dimensional case

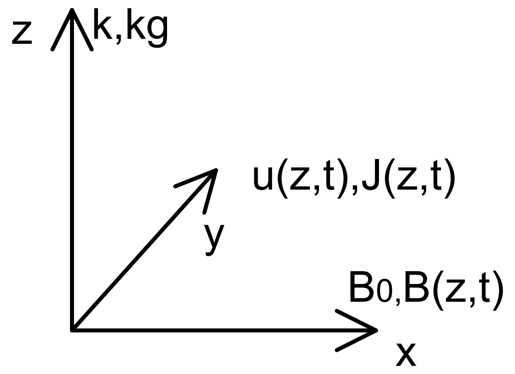
$$\rho(z,t)\left(\partial_t u_z(z,t)+\frac{1}{2}\partial_z u_z^2(z,t)\right)=-c_s^2\partial_z\rho(z,t)-\frac{1}{c}J_{my}(z,t)B+\nu\partial_{zz}u_z(z,t$$

$$E_y(z,t)+\frac{B}{c}u_z(z,t)=\eta J_{my}(z,t),$$

$$\partial_t\rho(z,t)+\partial_z(\rho(z,t)u_z(z,t))=0.$$

$$\partial_t E_y(z,t)=c\partial_z B_x(z,t)-4\pi J_{my}-cJ_E$$

$$\partial_t B_x(z,t)=c\partial_z E_y(z,t)-cJ_B$$



Linear limit

Linearize and keep First Order perturbed quantities *A=A₀ + A₁*exp(kz-ωt)*

$$\left[\omega^2\left(1+\frac{v_A^2}{c^2}\right)-k^2(c_s^2+v_A^2)+i\frac{\eta\omega c^2}{4\pi}\left(k^2-\frac{\omega^2}{c^2}\right)\left(1-\frac{c_s^2k^2}{\omega^2}+i\frac{k^2\nu}{\rho_0\omega}\right)+i\frac{k^2\nu\omega}{\rho_0}\right]E_y=$$

$$\left[-B_0k^2v_A^2h_0+i\eta\frac{B_0k^2h_0c^2\omega}{4\pi}\left(1-\frac{c_s^2k^2}{\omega^2}+i\frac{k^2\nu}{\rho_0\omega}\right)\right]\delta(k-k_g)\delta(\omega-\omega_g)$$

where $v_A=B_0/\sqrt{4\pi\rho_0}$.

$$|E_y|^2=(B_0k_g^2u_A^2h_0)^2\frac{1+\left(\eta\frac{\omega c^2}{4\pi v_A^2}\right)^2}{[\omega^2-k^2u_A^2]^2+\left[\frac{\eta\omega c^2}{4\pi}(k^2-\omega^2/c^2)+\frac{k^2\nu\omega}{\rho_0}\right]^2\left(1+\frac{v_A^2}{c^2}\right)^{-2}}$$

where $u_A^2=v_A^2/[1+v_A^2/c^2]$ is the relativistic Alfvén velocity.

Numerical solution

- Pseudo-spectral method
- Periodic boundary conditions
- Advance in time in fourier space (RK4)
- Non—linear part calculated in real space by using a FFT algorithm
- 256 grid points → 128 modes → 84 modes!

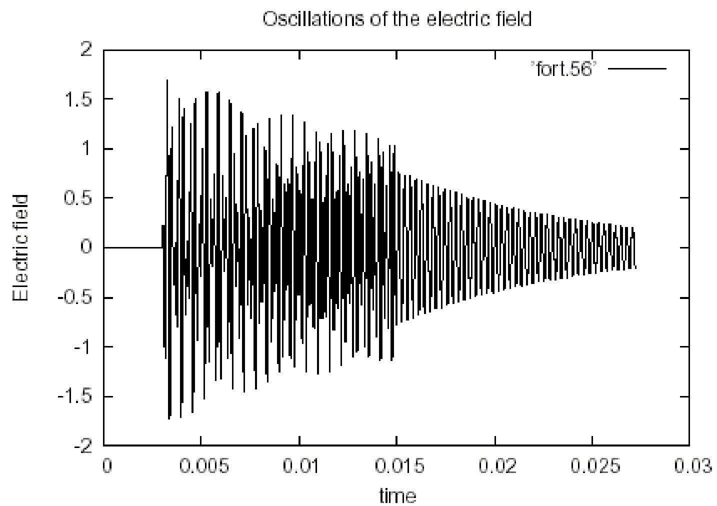
Typical values of the parameters

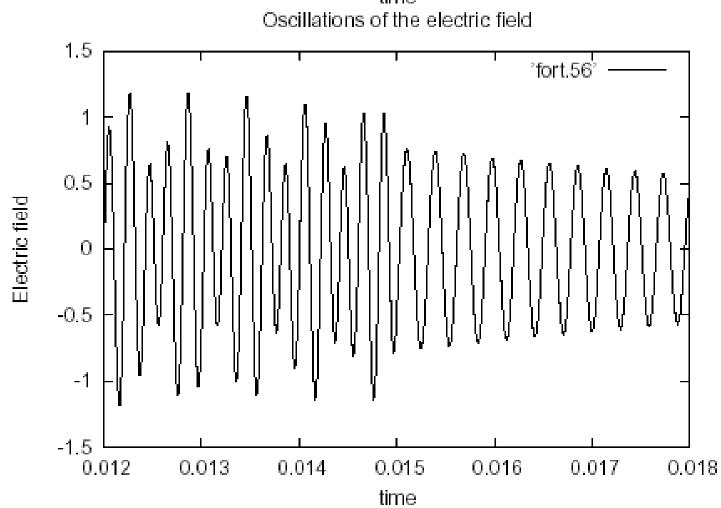
- k_0=1.16 x 10**(-7) 1/cm
- ΔZ=5.4 x 10**7 cm
- T=1159420 K (=100 eV)
- ρ0=10**(-14) gr/cm**3
- Spitzer resistivity
- $\nu = m\, u_{th} / \pi\, \lambda_D$**2
- k_g=9k_0

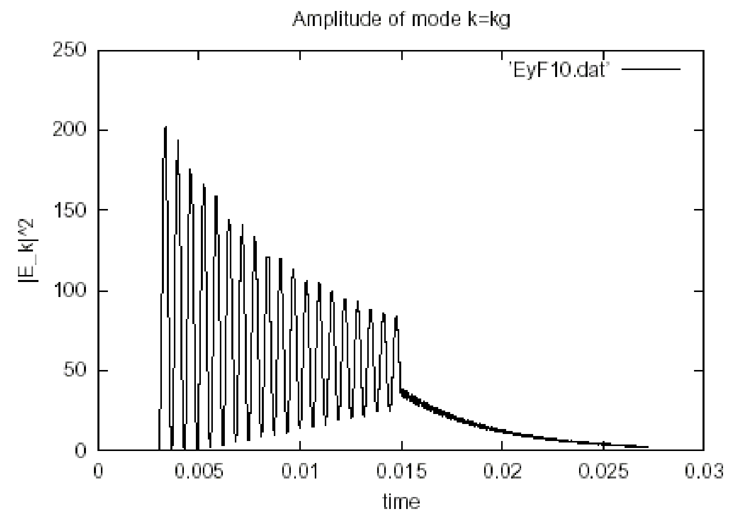
$$T_{gw}(k_g) = 0.0002 \text{ sec}$$

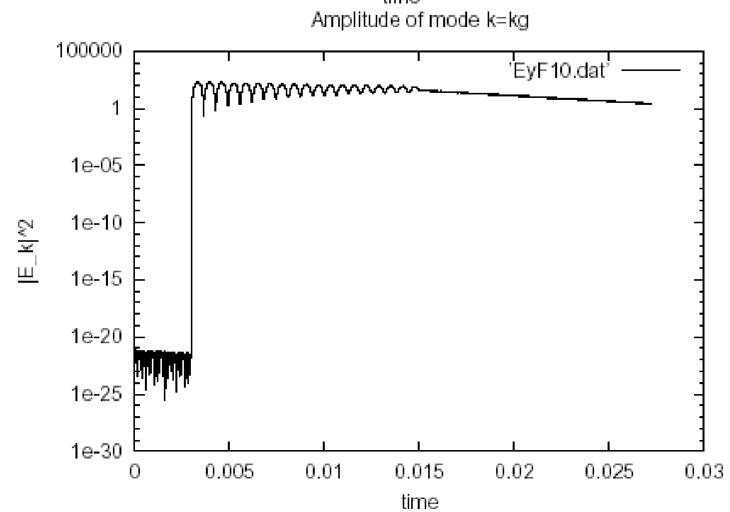
$$T(k_g) = 0.00029 \text{ sec}$$

Numerical Experiment

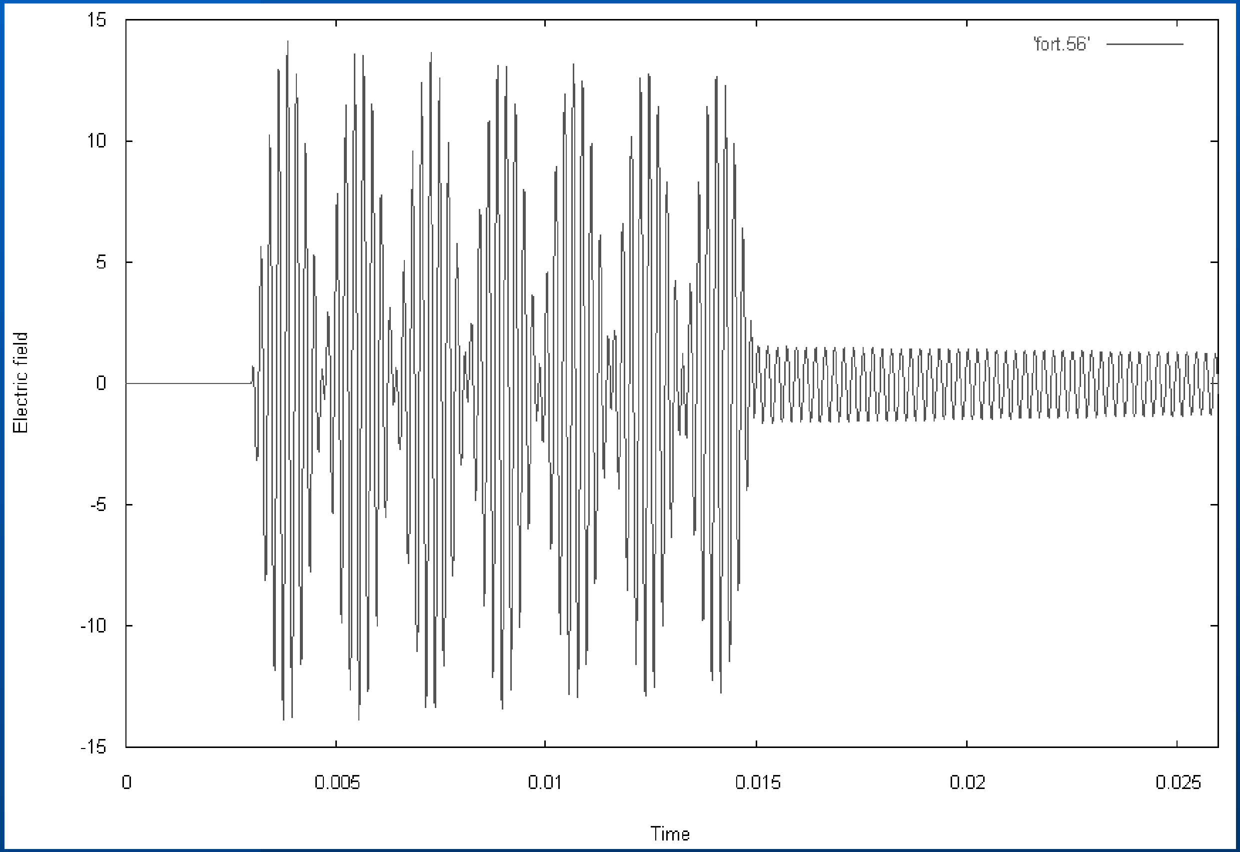








Numerical Results

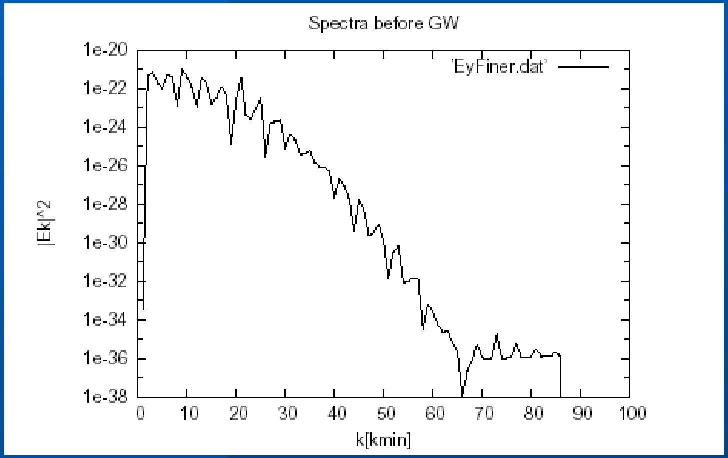


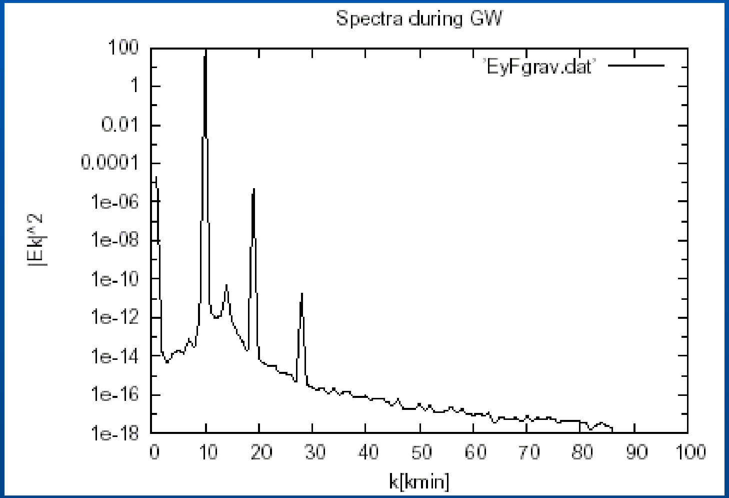
$$\omega_{gw} = k_g c$$

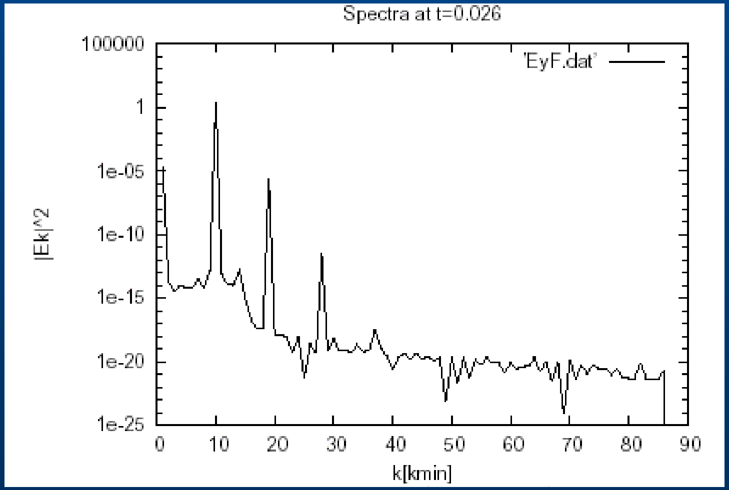
$$\omega = k_n u_A$$

$$\Omega = \omega_{gw} - \omega_k(k_{gw})$$

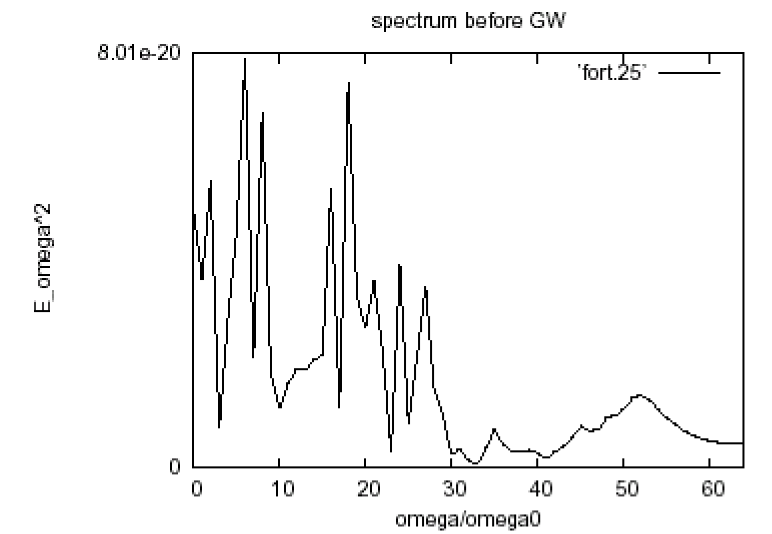
Spatial Spectrum

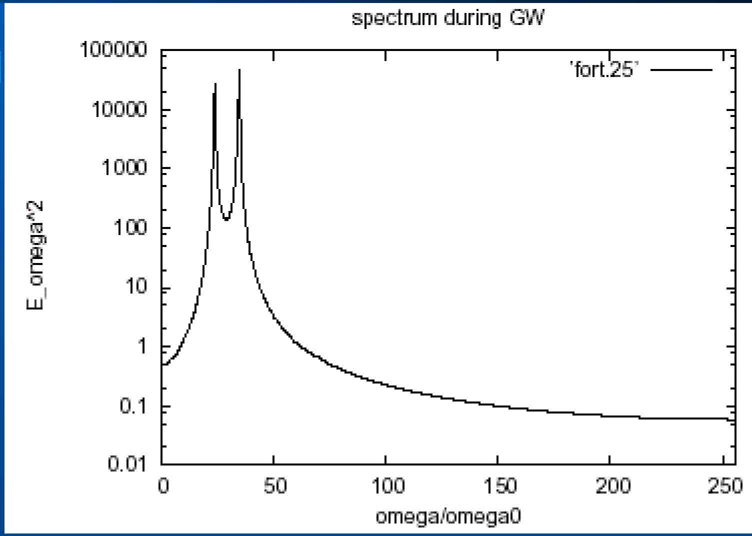


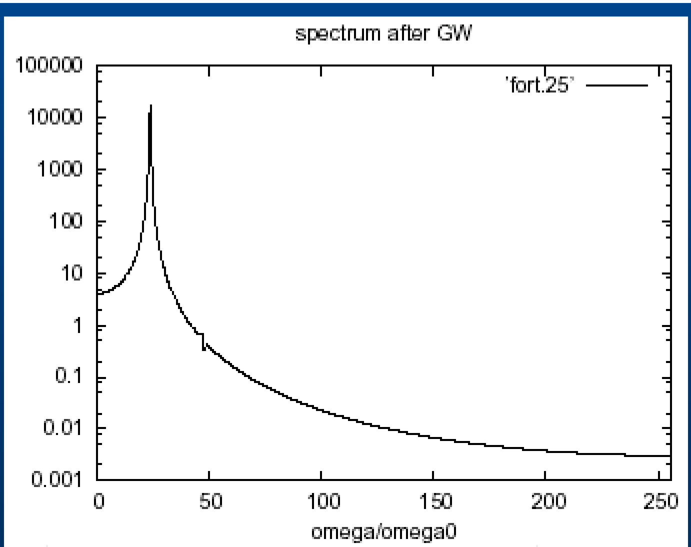




Temporal Spectrum







Dominant harmonics during GW

- @ $k=k_g$ $\omega=\omega_{GW}$ (ALSO @ $k=\omega_{GW}/u_a$)

  $\omega=\omega_{MS}$

- @ $k=2k_g$ $\omega=2\omega_{MS}$

  $\omega=2\omega_{MS}+\Omega$

  $\omega=2\omega_{MS}+2\Omega$

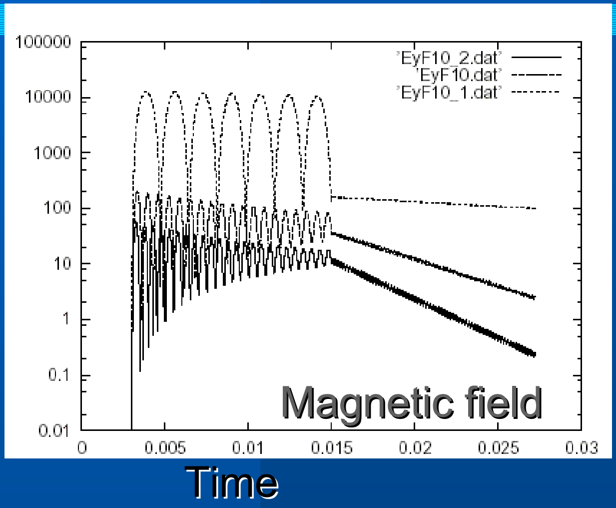
- @ $k=3k_g$ $\omega=3\omega_{MS}$

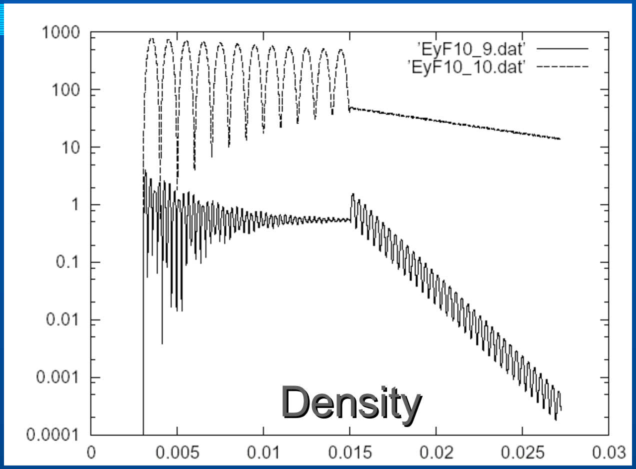
  $\omega=3\omega_{MS}+\Omega$

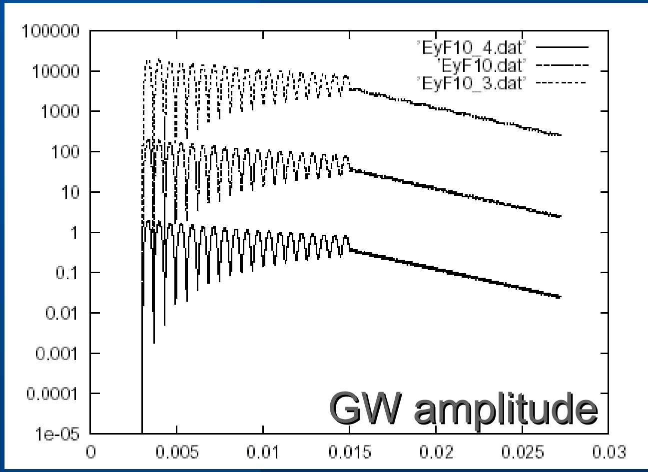
  $\omega=3\omega_{MS}+2\Omega$

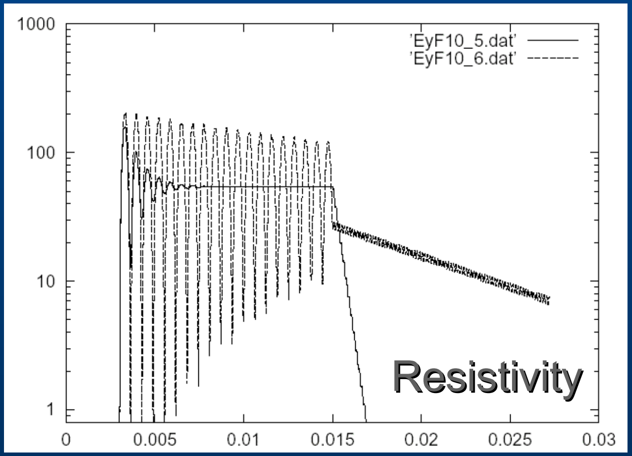
$$\Omega = \omega_{gw} - \omega_k(k_{gw})$$

Parametric Studies: electric field







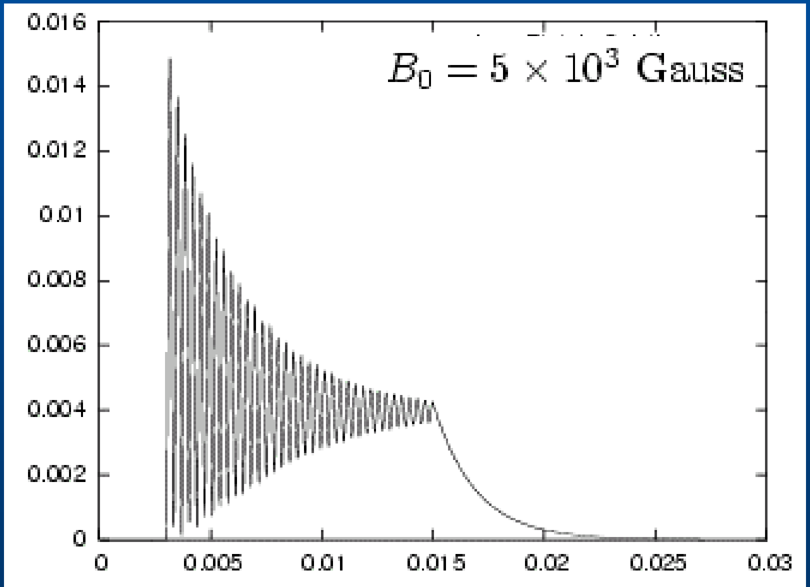


Energy deposited in the plasma

$$E_{total}(t) = \frac{1}{8\pi}\int E_y(z,t)^2\,dz + \frac{1}{8\pi}\int (B_x(z,t)^2 - B_0^2)\,dz + \frac{1}{2}\int \rho(z,t)u_z(z,t)^2\,dz$$

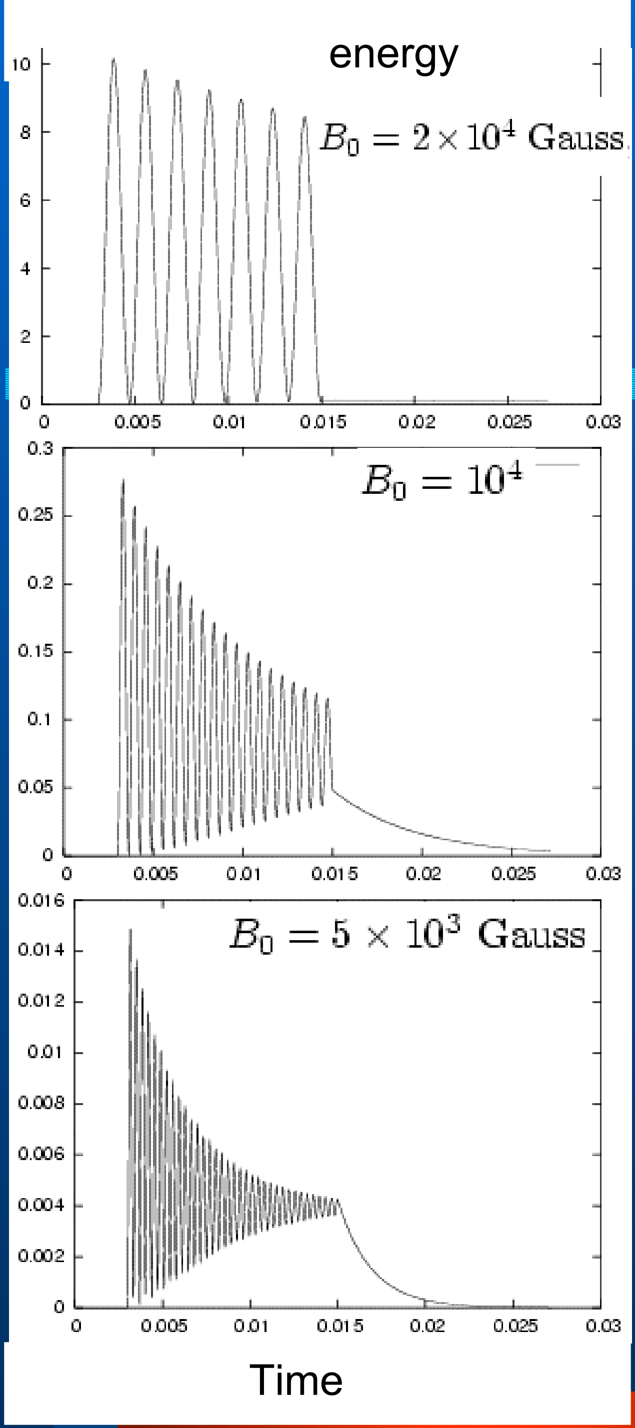
Energy

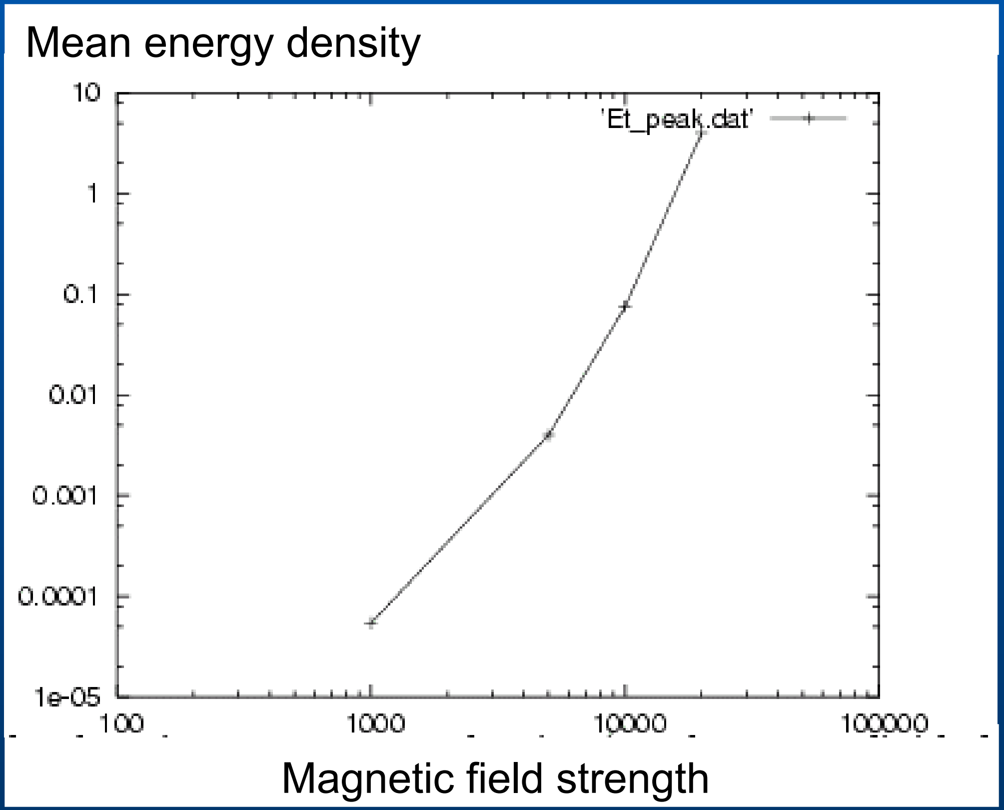
density (erg/cm)



Time

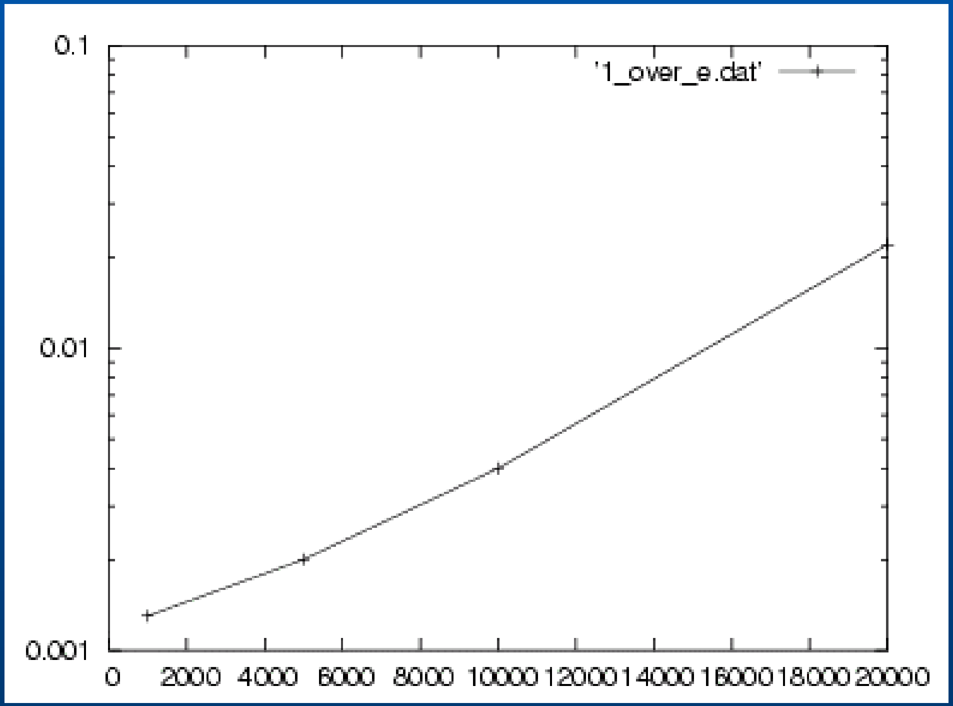
Plasma Energy





Decay time

Decay time of energy



Magnetic field strength

Future studies

- Treat numerically the 2D problem
- Include both GW polarities
- Include higher order GW effects
- Include inhomogeneous plasma and magnetic field profile

THANK YOU