Primordial magnetic fields and their super-adiabatic amplification

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Magnetic Fields in the Universe

Magnetic fields appear everywhere in the universe.

Large scale magnetic fields, between 10^{-7} and 10^{-5} G, have been observed in galaxies, galaxy clusters and in high redshift objects.

Despite their widespread presence, however, the origin of cosmic magnetic fields is still a mystery.

Even if the galactic dynamo works fine, it still requires a small "seed" magnetic field to begin with.

A number of suggestions have been put forward for producing these seed fields.

The scenarios vary from eddies and density fluctuations in the early plasma, to cosmological phase-transitions, inflationary and superstring inspired scenarios.

The Galactic Dynamo

The dynamo amplifies weak magnetic fields, by combining the turbulent motion of the gas with the differential rotation of the galaxy.

The origin of these seed fields is still elusive.

If the dynamo is efficient, the initial magnetic fields can be as low as $\sim 10^{-23}$ G (at present).

This limit is relaxed down to $\sim 10^{-34}$ G if the universe is dominated by "dark energy".

Without dynamo magnetic seeds of 10^{-12} G, or even 10^{-8} G, are required.

The coherence scale of the seed field is also an issue.

Typically, the coherence length must be comparable to the largest turbulent eddy ($\sim$ 100 pc collapsed, or $\sim$ 10 kpc comoving).

These magnetic seed fields can result from local astrophysical processes (e.g. buttery effects), or be the remnants of a large-scale primordial field.

Primordial Magnetism

Primordial magnetic fields can explain both the galactic fields, as well as those detected in galaxy clusters and highly redshifted formations.

There have been many attempts to generate pre-recombination magnetic fields in the out of equilibrium epochs between inflation and decoupling.

However, causality means that the coherence scale of the seed fields is unacceptably small.

Magnetism and Inflation

Inflation has long been suggested as a solution to the causality problem, as it naturally achieves superhorizon correlations.

Nevertheless, the general perception is that due to the conformal invariance of electromagnetism any magnetic field present during inflation will be strongly diluted.

One can get around this obstacle by breaking away from standard electromagnetic theory.

More than one ways of doing that have been suggested in the literature, with a number of them introducing ad hoc new physics.

The success of these proposals, however, is usually achieved at the expense of simplicity.

Maxwell's Equations

Relative to an observer with 4-velocity u_a, Maxwell's equations decompose into two propagation equations

$$\dot{E}_{\langle a\rangle} = \left(\sigma_{ab} + \varepsilon_{abc}\omega^c - \tfrac{2}{3}\Theta h_{ab}\right) E^b + \varepsilon_{abc}\dot{u}^b B^c + \mathrm{curl} B_a - J_{\langle a\rangle}\,, \tag{1}$$

$$\dot{B}_{\langle a\rangle} = \left(\sigma_{ab} + \varepsilon_{abc}\omega^c - \tfrac{2}{3}\Theta h_{ab}\right) B^b - \varepsilon_{abc}\dot{u}^b E^c - \mathrm{curl} E_a\,, \tag{2}$$

and two constraints

$$\mathrm{D}^a E_a + 2\omega^a B_a = \rho_{\mathrm{e}}\,, \tag{3}$$

$$\mathrm{D}^a B_a - 2\omega^a E_a = 0\,. \tag{4}$$

Here E_a and B_a are the electric and magnetic fields, as measured by the observer, with $E_a u^a = 0 = B_a u^a$.

Also, J_a is the 4-current density and ρ_{e} is the charge density.

The Magnetic Wave Equation

Taking the time derivative of (1) and eliminating the electric field we arrive at the following wave-like equation

$$
\begin{aligned}
\ddot{B}_{\langle a\rangle} = \;& \mathrm{D}^2 B_a + \tfrac{1}{3}\mu(1+3w)B_a \\
& + \left(\sigma_{ab} - \varepsilon_{abc}\omega^c - \tfrac{5}{3}\Theta h_{ab}\right)\dot{B}^b \\
& + \tfrac{1}{3}\Theta\left(\sigma_{ab} + \varepsilon_{abc}\omega^c - \tfrac{4}{3}\Theta h_{ab}\right)B^b \\
& - \sigma_{\langle a}{}^c\sigma_{b\rangle c}B^b + \varepsilon_{abc}B^b\sigma^{cd}\omega_d + \tfrac{4}{3}\left(\sigma^2 - \tfrac{2}{3}\omega^2\right)B_a \\
& + \tfrac{1}{3}\omega_{\langle a}\omega_{b\rangle}B^b + \dot{u}^b\dot{u}_b B_a - \tfrac{5}{2}\varepsilon_{abc}\dot{u}^b \mathrm{curl} B^c \\
& + \mathrm{D}_{\langle a}B_{b\rangle}\dot{u}^b - \tfrac{2}{3}\varepsilon_{abc}E^b\mathrm{D}^c\Theta - \varepsilon_{abc}E_d\mathrm{D}^b\sigma^{cd} \\
& - \mathrm{D}_{\langle a}\omega_{b\rangle}E^b - \tfrac{3}{2}\varepsilon_{abc}E^b\mathrm{curl}\,\omega^c - 2\mathrm{D}_{\langle a}E_{b\rangle}\omega^b \\
& + 2\varepsilon_{abc}\sigma^b{}_d\mathrm{D}^{\langle c}E^{d\rangle} - \varepsilon_{abc}\ddot{u}^bE^c - \tfrac{7}{3}\dot{u}^b\omega_bE_a \\
& - \tfrac{4}{3}E^b\omega_b\dot{u}_a - 3\dot{u}^bE_b\omega_a - 3\varepsilon_{abc}\dot{u}^b\sigma^{cd}E_d \\
& - \tfrac{2}{3}\rho_{\mathrm{e}}\omega_a - 2\varepsilon_{abc}\dot{u}^b\mathcal{J}^c + \mathrm{curl}\mathcal{J}_a \\
& - \mathcal{R}_{ab}B^b - E_{ab}B^b - H_{ab}E^b\,. \qquad (5)
\end{aligned}
$$

where $w = p/\rho$. Similarly one obtains the wave equation for E_a.

The latter and the above isolate all the sources affecting the evolution of electromagnetic fields in a general spacetime (with a perfect fluid).

The case of a FRW background

On a curved FRW model the linear magnetic wave equation reads

$$\begin{aligned}\ddot{B}_a - \mathsf{D}^2 B_a &= \tfrac{5}{3}\Theta\dot{B}_a - \tfrac{4}{9}\Theta^2 B_a + \tfrac{1}{3}(\rho+p)B_a \\ &\quad -\mathcal{R}_{ab}B^b\,. \end{aligned} \tag{6}$$

The $\mathcal{R}_{ab}B^b$ term results from the general relativistic coupling between the electromagnetic field and the spacetime curvature.

Introducing conformal time (η, with $\dot{\eta}=1/a$), we have

$$\mathcal{B}_a'' - \mathsf{D}^2\mathcal{B}_a + 2k\mathcal{B}_a = 0\,, \tag{7}$$

where $\mathcal{B}_a = a^2 B_a$ and $k=\pm1$.

The above closely resembles the wave equation derived by Turner and Widrow.

The main difference is that here the magneto-curvature term is natural and not the result of an ad hoc coupling between the EM field and the spacetime curvature.

The Case of a $k = -1$ background

The decomposition $\mathcal{B}_a = \sum_n \mathcal{B}_n Q_a^n$ leads to

$$\mathcal{B}_n'' + n^2 \mathcal{B}_n + 2k\mathcal{B}_n = 0\,, \tag{8}$$

where $n^2 = \nu^2 + 1$ ($\nu \geq 0$) and $k = -1$. Then,

$$\mathcal{B}_\nu'' + (\nu^2 - 1)\mathcal{B}_\nu = 0\,. \tag{9}$$

So, for $\nu^2 \to 0$, which corresponds to the largest subcurvature modes, we obtain the solution

$$\mathcal{B}_\nu = \mathcal{C}_1 \cosh\eta + \mathcal{C}_2 \sinh\eta\,. \tag{10}$$

The above translates into

$$\mathcal{B}_\nu \propto a \Longrightarrow B_\nu \propto a^{-1}\,, \tag{11}$$

through most of the evolution of an open FRW model with $p = 0$, $p = \rho/3$ and $p = -\rho$ (provided that the electrical conductivity is low).

Thus, during de Sitter inflation, when the conductivity is effectively zero, the magnetic flux increases on sufficiently long wavelengths and the field no longer obeys the a^{-2}-law (i.e. superadiabatic magnetic amplification).

Are the magnetic scales and the strengths of astrophysical interest?

The Curvature Scale

In an open FRW universe (with $0 < \Omega < 1$) the Friedmann equation reads

$$1 - \Omega = \frac{1}{H^2 a^2} = \frac{\lambda_H^2}{\lambda_k^2}\,, \tag{12}$$

where $\Omega = \rho/3H^2$ is the density parameter, $\lambda_H = 1/H$ is the Hubble scale and $\lambda_k = a$ is the curvature scale.

This means that

$$\lambda_k = \frac{\lambda_H}{\sqrt{1-\Omega}}\,. \tag{13}$$

So, $\lambda_k > \lambda_H$ always and $\lambda_k \simeq \lambda_H$ when $\Omega \ll 1$.

Note that inflation makes the universe look flatter, by pushing the curvature scale further out, but does not change its geometry.

Magnetic Scales

Consider GUT scale inflation at $T \sim 10^{16}$ Gev and $H \sim 10^{13}$ Gev.

We want to redshift the curvature scale at the end of inflation to the present.

At the end of "reheating" in an open FRW model

$$1 - \Omega_{reh} \sim 10^{-23}(1 - \Omega_0)T_{reh}^{-2}\,, \tag{14}$$

with T_{reh} (in Gevs) and $1 - \Omega_0 \sim 10^{-2}$.

At the same time

$$\lambda_k = \frac{\lambda_H}{\sqrt{1 - \Omega_{reh}}} \gg \lambda_H\,. \tag{15}$$

Then,

$$(\lambda_k)_0 = (\lambda_k)_{reh}\left(\frac{a_0 = 1}{a_{reh}}\right) = (\lambda_k)_{reh}\left(\frac{T_{reh}}{T_0}\right)\,, \tag{16}$$

given that $\lambda \propto a \propto T^{-1}$.

Magnetic Strengths

At the end of inflation and on wavelengths close to the curvature scale, where $B_\nu \propto a^{-1}$, we have

$$r = \frac{\rho_B}{\rho_\gamma} \simeq 10^{-52}\lambda^{-2}\,, \tag{17}$$

where $\rho_B = B^2$, $\rho_\gamma \propto a^{-4}$ is the density of the radiation field and λ is the coherence scale of the field (in MPcs).

After inflation, the ratio r remains invariant because the conductivity of the cosmic medium increases and therefore $\rho_B \propto a^{-4}$.

So, today

$$(\rho_B)_0 \simeq 10^{-52}(\rho_\gamma)_0(\lambda_k)_0^2 \tag{18}$$

The above does not account for any potential magnetic amplification during reheating or during the protogalactic collapse.

Numerical Results

$H/m_{\rm Pl}$	$T_{\rm RH}$	λ_0	$\log r$	B_0
10^{-6}	10^{16} GeV	10^{6} Mpc	-64	10^{-37} G
10^{-6}	10^{14} GeV	10^{2} Mpc	-56	10^{-33} G
10^{-6}	10^{13} GeV	1 Mpc	-52	10^{-31} G
10^{-6}	10^{12} GeV	10^{-2} Mpc	-48	10^{-29} G
10^{-6}	10^{11} GeV	10^{-4} Mpc	-44	10^{-27} G

Numerical estimates for the present scale (λ_0), the invariant energy density ratio (r) and the current magnitude (B_0) of the super-adiabatically amplified magnetic field for GUT scale inflation ($H/m_{\rm Pl} \simeq 10^{-6}$) and representative reheating temperatures ($T_{\rm RH}$).