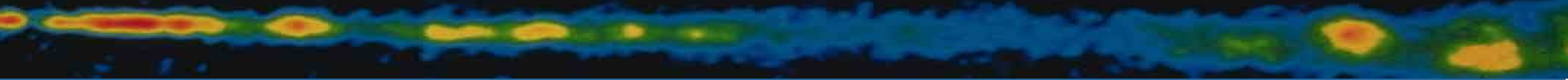

Astrophysical Applications of the GW – plasma interactions

Qualitative & Quantitative

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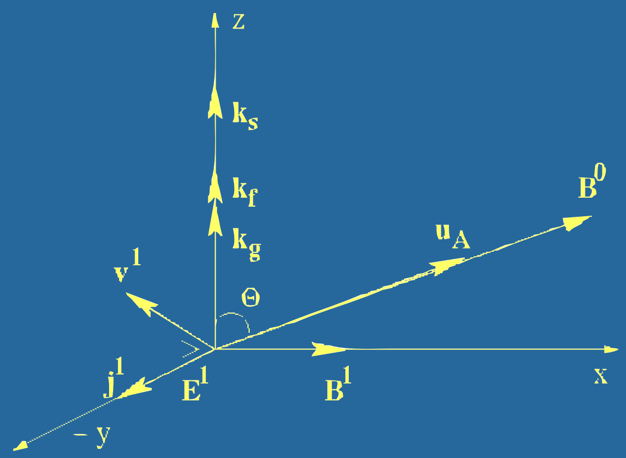
Outline

- Qualitative
  - ⋆ Explicit plasma wave solutions in Fourier – Laplace space,
  - ⋆ Explicit space-time solutions,
  - ⋆ Origin of the linear growth,
  - ⋆ Lorentz boost relativistic wind $\leftrightarrow$ observer frame.
- Quantitative
  - ⋆ Astrophysical Applications,
  - ⋆ Magnetars,
  - ⋆ Coalescing compact binaries,
  - ⋆ Numerical Estimates,
  - ⋆ Conclusions.

Explicit solutions in Fourier – Laplace space

Slow & fast magneto-acoustic



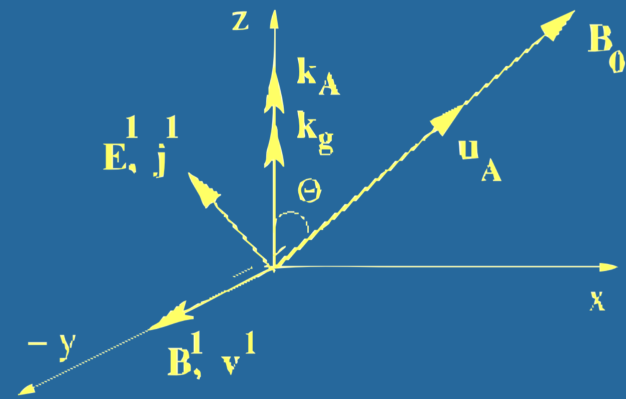
$$v_z^1 = \frac{i}{2} \frac{h_+ \omega^3 u_{\mathrm{A}\perp}^2}{(\omega^2 - k^2 u_{\mathrm{f}}^2)(\omega^2 - k^2 u_{\mathrm{s}}^2)} \frac{\omega + k}{\omega - k},$$

$$v_x^1 = -\frac{v_z(k,\omega)}{\tan\theta} \left(1 - \frac{k^2 c_{\mathrm{s}}^2}{\omega^2}\right),$$

$$p^1 = \frac{k}{\omega} \gamma p^0 v_z(k,\omega).$$

$$\frac{B_x^1}{B_x^0} = v_z^1 - \frac{v_x^1}{\tan\theta} \left[1 + \frac{\omega}{\omega + k} \frac{1 - u_{\mathrm{A}}^2}{u_{\mathrm{A}\parallel}^2}\right].$$

Alfvén



$$v_y^1 = -\frac{i}{2} \frac{h_\times \omega u_{\mathrm{A}\parallel} u_{\mathrm{A}\perp}}{\omega^2 - k^2 u_{\mathrm{A}}^2} \frac{\omega + k}{\omega - k},$$

$$B_y^1 = -v_y^1(k,\omega) \frac{B_x^0}{u_{\mathrm{A}\parallel} u_{\mathrm{A}\perp}} \frac{\omega + k u_{\mathrm{A}\parallel}^2}{\omega + k},$$

Explicit Space-time solutions

- Slow & fast magneto-acoustic ($\phi_{\rm A}^{\pm} = \pm k_{\rm A} z - \omega t$ etc.):

$$v_z^1(z,t) = \frac{h_+}{4} \frac{u_{\rm s}^2 u_{\rm A\perp}^2}{u_{\rm f}^2 - u_{\rm s}^2} \left[\frac{1+u_{\rm s}}{1-u_{\rm s}} \frac{{\rm e}^{i\phi_{\rm s}^+}}{u_{\rm s}} - \frac{1-u_{\rm s}}{1+u_{\rm s}} \frac{{\rm e}^{i\phi_{\rm s}^-}}{u_{\rm s}} - \frac{4{\rm e}^{i\phi_{\rm g}}}{1-u_{\rm s}^2} \right]$$

$$- \frac{h_+}{4} \frac{u_{\rm f}^2 u_{\rm A\perp}^2}{u_{\rm f}^2 - u_{\rm s}^2} \left[\frac{1+u_{\rm f}}{1-u_{\rm f}} \frac{{\rm e}^{i\phi_{\rm f}^+}}{u_{\rm f}} - \frac{1-u_{\rm f}}{1+u_{\rm f}} \frac{{\rm e}^{i\phi_{\rm f}^-}}{u_{\rm f}} - \frac{4{\rm e}^{i\phi_{\rm g}}}{1-u_{\rm f}^2} \right]$$

$$v_x^1(z,t)\tan\theta = \frac{h_+}{4} \frac{c_{\rm s}^2 u_{\rm A\perp}^2}{u_{\rm f}^2 - u_{\rm s}^2} \left[\frac{1+u_{\rm f}}{1-u_{\rm f}} \frac{{\rm e}^{i\phi_{\rm f}^+}}{u_{\rm f}} - \frac{1-u_{\rm f}}{1+u_{\rm f}} \frac{{\rm e}^{i\phi_{\rm f}^-}}{u_{\rm f}} - \frac{4u_{\rm f}^2{\rm e}^{i\phi_{\rm g}}}{1-u_{\rm f}^2} \right.$$

$$\left. - \frac{1+u_{\rm s}}{1-u_{\rm s}} \frac{{\rm e}^{i\phi_{\rm s}^+}}{u_{\rm s}} + \frac{1-u_{\rm s}}{1+u_{\rm s}} \frac{{\rm e}^{i\phi_{\rm s}^-}}{u_{\rm s}} + \frac{4u_{\rm s}^2{\rm e}^{i\phi_{\rm g}}}{1-u_{\rm s}^2} \right] - v_z^1$$

- Alfvén:

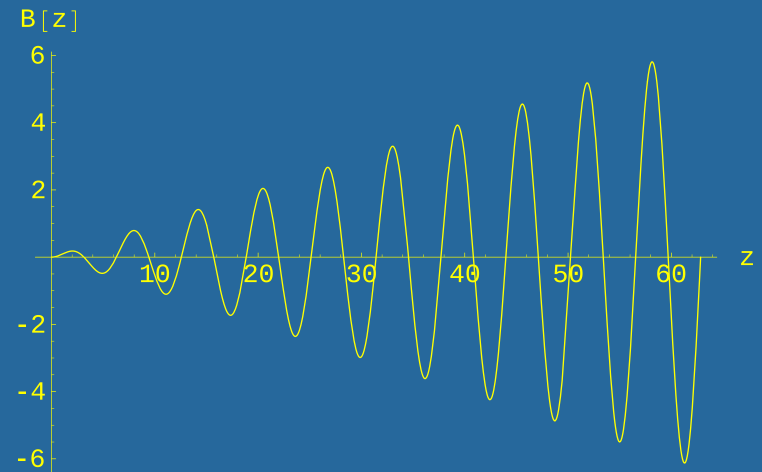
$$v_y^1(z,t) = \frac{h_\times u_{\rm A\perp}}{4} \left[\frac{1+u_{\rm A\|}}{1-u_{\rm A\|}} {\rm e}^{i\phi_{\rm A}^+} - \frac{1-u_{\rm A\|}}{1+u_{\rm A\|}} {\rm e}^{i\phi_{\rm A}^-} - \frac{4u_{\rm A\|}^2}{1-u_{\rm A\|}^2} 2{\rm e}^{i\phi_{\rm g}} \right]$$

Coherent interaction in PFD wind

- All solutions of the same form i. t. o. phase velocity $u_{\rm ph}$ (A–D are constants):

$$A e^{-i\omega t}\left[\frac{B e^{+\frac{i\omega z}{u_{\rm ph}}}}{1-u_{\rm ph}} + \frac{C e^{-\frac{i\omega z}{u_{\rm ph}}}}{1+u_{\rm ph}} + \frac{D e^{+i\omega z}}{1-u_{\rm ph}^2}\right]$$

- Coherent interaction possible for $u_{\rm ph} \uparrow 1$ or $\Delta k = \omega\left[\frac{1}{u_{\rm ph}} - 1\right] \downarrow 0$.
- Retreating plasma wave (C term) negligible,
- $\mathcal{O}[\Delta k]$ leads to linear growth: $\frac{\omega\ e^{i\omega z}}{u_{\rm ph}\Delta k}\left[1 - e^{i\Delta k z}\right] = \frac{i\omega z \Delta k}{u_{\rm ph}\Delta k} e^{i\omega z}$.
- Slow: $u_{\rm s} < 1$; Fast $u_{\rm f} \uparrow 1$; Alfvén $u_{\rm A}\cos\theta \uparrow 1$ for $\theta \downarrow 0$, but $A \propto \sin\theta$.



Relativistic PFD wind or jet

Comoving fast magneto-acoustic waves:

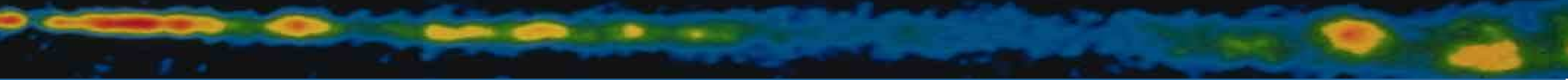
$$\frac{B^1_x(z,t)}{B_0} = \frac{v^1_z(z,t)}{\sin\theta} = -\frac{v^1_x(z,t)}{\cos\theta} = \frac{\mu^1(z,t)}{\mu^0\sin\theta} = -\frac{E^1_y(z,t)}{B_0} \simeq \frac{h_+}{2}\sin\theta\,\omega z\,\Im\left[\mathrm{e}^{i\phi_\mathrm{g}}\right],$$

$$\frac{B^0_x j_y(z,t)}{\mu^0\omega} \simeq \frac{h_+}{2}\sin^2\theta\,\omega z\,\Re\left[\mathrm{e}^{i\phi_\mathrm{g}}\right].$$

Comoving Alfvén waves: $\frac{B^1_y(z,t)}{B^0} \sim \frac{\theta h_\times}{2}\omega z\,\Im[\mathrm{e}^{i\phi_\mathrm{g}}] + \mathcal{O}[\theta^2]$ etc.

- Lorentz boost to observer frame, with $\Gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \phi_\mathrm{ph} = \phi'_\mathrm{ph}$; $u'_\mathrm{ph} = \frac{u_\mathrm{ph}-\beta}{1-\beta u_\mathrm{ph}}$; $\boldsymbol{B}^0 = (\Gamma B^{0\prime}_x, 0, B^{0\prime}_z)$; $\omega = \Gamma(\omega' + \beta k') \simeq 2\Gamma\omega'$; $L = \Gamma L'$; angles;
- Most importantly relativistic fast mode becomes $E_y = -B_x$ and:

$$\boxed{B_x \simeq \frac{h_+}{2\Gamma^2} B^0_x \omega L\,\Im\left[\mathrm{e}^{i\phi}\right]}.$$



Quantitative – Astrophysical Applications

Astrophysical Applications

Perturbations proportional to:

- ⋆ ambient magnetic field (perpendicular $B^0_\perp$),
- ⋆ GW amplitude ($h_{+,\times}$) & frequency ($\omega = kc$),
- ⋆ interaction length scale (L), or equivalently GW burst duration ($\Delta t = \frac{L}{c}$),
- ⋆ bulk flow velocity (Γ^{-2} in case of rel. wind/jet).

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Astrophysical applications are therefore:

- ▶ rapidly spinning NS with small ($< 10^{-5}$) eccentricity,
- ▶ LMXB (low-mass X-ray binaries $=$ accreting NS),
- ▶ NS with r-mode instabilities,
- ▶ asymmetric SN core collapse and bounce,
- ▶ newly born NS that boil and oscillate (magnetars),
- ▶ coalescing compact binaries, NS-BH, BH-BH & NS-NS $\Rightarrow$ short GRB.

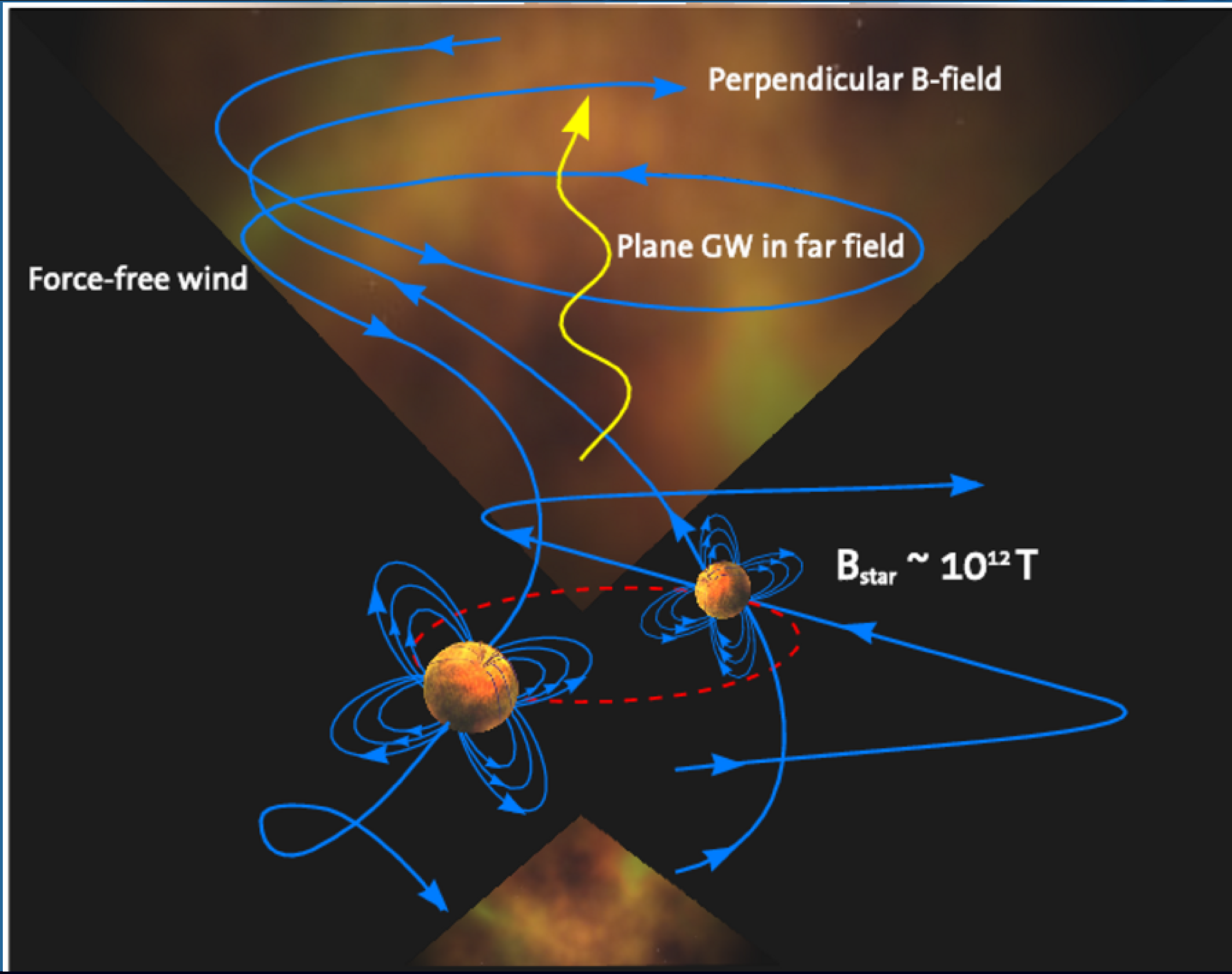
Magnetars

Probably most efficient:

- ⋆ Strongest magnetic fields in the universe: $B^0 \uparrow 10^{12}$ T,
- ⋆ highest GW frequencies due to quakes $\omega \uparrow 10^5$ rad/s ($>$ plasma freq. ISM),
- ⋆ Associated with AXP, SGR & DIN (dim isolated NS),
- ⋆ birth rate could be similar to NS / pulsars depending on selection criteria $\Rightarrow$ same # of sources for GW detectors.

However, relatively ill-understood.

Merging NS-NS binary



Numerical Estimate I

- Energy in GW (where $h_{+,\times} \sim h_{\rm in} \frac{R_{\rm in}}{r}$):

$$\epsilon_{\rm GW} = \frac{c^2\omega^2}{32\pi G}(h_+^2 + h_\times^2) = F\omega^2 h_{+,\times}^2 \simeq 2 \cdot 10^{29} \frac{\rm J}{\rm m^3} \left[\frac{h_{+,\times}}{0.01}\right]^2 \left[\frac{\omega}{4\pi 10^3 \rm rad/s}\right]^2 .$$

- Energy in electromagnetic field ($E^2 + B^2 = 2B^2$):

$$\epsilon^1_{\rm EM} = 2\,\frac{|\boldsymbol{B}^1|^2}{2\mu_0} = \left[\frac{(B_x^0)^2}{2\mu_0}\right] \frac{\omega^2 h_+^2 \Delta t^2}{2\Gamma^4} \quad \Rightarrow \quad \boxed{\frac{\epsilon^1_{\rm EM}}{\epsilon_{\rm GW}} = \epsilon^0_{\rm EM}\left[\frac{\sin^2\theta \Delta t^2}{2\Gamma^4 F}\right]}.$$

- NB at $R_{\rm in}$ after $\Delta t \sim 0.1$ s. for $\Gamma < 4$: $\omega^2 h_+^2 \Delta t^2 = \mathcal{O}[1] \Rightarrow \boxed{\epsilon^1_{\rm EM} \uparrow \epsilon^0_{\rm EM}}$.

Numerical Estimate II

- NS force-free wind outside light-cylinder $R_{\rm lc} = \frac{c}{\Omega_{\rm bi}} \simeq 50$ km; $B_\star \sim 10^{12}$ T:

$$B_{\rm lc} = B_\star \left[\frac{R_\star}{R_{\rm lc}}\right]^3, \quad \boxed{B_{\rm t}(r > R_{\rm lc}) = B_{\rm lc}\left[\frac{R_{\rm lc}}{r}\right]}, \quad B_{\rm p}(r > R_{\rm lc}) = B_{\rm lc}\left[\frac{R_{\rm lc}}{r}\right]^2$$

- NS-NS merger $\Rightarrow$ GW burst of $\Delta t \sim 0.1$ s. in rel. plasma wind $\Gamma \sim 30$:

$$E^1_{\rm EM} = \int_{r_{\rm in}}^{r_{\rm in}+c\Delta t} \epsilon^1_{\rm EM} 4\pi r^2 {\rm d}r = \frac{\pi}{\mu_0}\left[\frac{r_{\rm in}}{\Gamma}\right]^4 (B^0_{\rm in} h_{\rm in} k)^2 c\Delta t$$

$$= 10^{34}\ {\rm J} \left[\frac{R_{\rm lc}}{5\cdot 10^4\ {\rm m}}\right]^4 \left[\frac{h_{\rm lc}}{0.01}\right]^2 \left[\frac{B_{\rm lc}}{10^{10}\ {\rm T}}\right]^2 \left[\frac{\Gamma}{30}\right]^{-4} \left[\frac{k}{4\cdot 10^{-5}\ {\rm m}^{-1}}\right]^2 \left[\frac{\Delta t}{0.1\ {\rm s}}\right]$$

- For magnetars $\omega \uparrow 2\pi\, 15\cdot 10^3$ rad/s; For mildly rel. $\Gamma \downarrow 3$: $E^1_{\rm EM} \uparrow 10^{40}$ J.
- For persistent GW source $\uparrow 10^7 \times (\frac{L}{c} = \Delta t)$ in force-free wind.
- Spinning NS: $h \downarrow$ & $\boldsymbol{B} \downarrow$,
- LMXB: $h \downarrow$ & $\boldsymbol{B} \downarrow$ & $\omega \downarrow$,

Conclusion

GW in magnetized plasma excite 3 fundamental plasma modes:

- Alfvén waves by $\times$ polarized GW,
- slow & fast magnetosonic waves by $+$ polarized GW,
- all amplitudes $\propto$ GW amplitude & frequency and *perpendicular* $B^0_\perp$,
- slow mode cannot interact coherently with GW ($u_\mathrm{s} \ll c$),
- Alfvén mode only when B^0 is almost *parallel* ($u_\mathrm{A} \cos\theta \simeq c$), amplitude $\downarrow$,
- most efficient is fast MSW ($u_\mathrm{f} = c$),
- GW energy $\Rightarrow$ energy in MHD waves. EM radiation mimics GW spectrum.

Even in the most extreme astrophysical sources, it is unlikely that the interaction efficiency is high enough to be detectable.

Thank you

[This tutorial & more info on: `http://moortgat.astro.kun.nl`,]