Particle pressure radial profile in the dayside magnetosphere of Saturn during near-radial parts of Cassini’s trajectory.


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The Cassini spacecraft

Characteristics:
6.7m x 4.0m
5.7 tons

12 instruments onboard
Cassini trajectory during the First 5 years of orbiting Saturn (July 2004-May 2009)

top view

side view
Questions to be addressed

1. Can we obtain a representative radial (total) pressure profile for the dayside magnetosphere of Saturn?

2. How are plasma, suprathermal (keV) and magnetic pressure compared in the Saturnian ring current region?

3. Is the Saturnian ring current inertial or pressure-gradient driven?

4. How well can the ring current density be reproduced by models?
Assuming that all ion components have the same bulk velocity in the steady-state, the force balance equation can be written as:

\[ \rho V \nabla V + \nabla \cdot P - J \times B = 0 \]

- \( \rho \): plasma mass density
- \( V \): plasma bulk velocity
- \( P \): total plasma pressure
- \( J \): current density

Assuming that the pressure is isotropic and that \( V = \Omega x r \) (strict corotation), the radial component of the equation will be:

\[ -\rho \Omega^2 r + \frac{\partial P}{\partial r} - (J_\theta B_\phi - J_\phi B_\theta) = 0 \]

- \( \Omega \): Saturn’s rotational angular velocity
- \( J_\phi \): azimuthal current density

and as long as \( B_\theta >> B_\phi \) (equatorial plane orbits)

\[ J_\phi \approx \frac{1}{B_\theta} (\rho \Omega^2 r - \frac{\partial P}{\partial r}) \]

- Inertial contribution
- Pressure gradient contribution
**Instrumentation**

**Plasma:** CAPS instrument (IMS, ELS) energy range 1 eV to few keV.

**Energetic particles:** MIMI instrument (CHEMS, LEMMS and INCA). E>3 keV coverage (ions), E>20 keV (e⁻), composition, directional intensities (pitch angle measurements).

**Magnetic field:**
Cassini magnetometer (MAG)
High resolution magnetic field
Vector measurements.
(4 sec sampling, pT level)
All dayside Cassini passes were examined

- Equatorial plane orbits \(|z|<1\ R_S\)
- Minimum local time change \(\frac{d(LT)}{dr}\)
- Available CAPS electron moments

Less than 5 cases

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\[ \frac{d(LT)}{dr} \approx 0.20 - 0.25 \text{ hr}/R_S \]

Magnetosheath or near-magnetosheath data have been excluded.
Particle and magnetic radial pressure profiles
DOY 322/2007

Plasma: \( P = 3nkT \) (lower limit), \( \langle m \rangle = 12m_p \) (25% protons - 75% O+).

Results
This is a typical example!

\[ \frac{dP}{dr} \text{ turned out to be } \times 2 \text{ to } \times 5 \text{ compared to } \rho \Omega^2 r \]
Existing models fit the magnetic field data using 4 free parameters:

- Inner radius $R_{in}$
- Outer radius $R_{out}$
- Half thickness $D$
- Current strength $\mu_0 I_0$

Assuming square cross-section and a $J_\phi \sim 1/r$ dependence.
When the pressure gradient dominates over the inertial term, $J_\varphi$ develops a clear maximum and cannot be reproduced by a disk current model.
Statistical approach. What happens more often?

Energetic (E>3 keV) particle pressure in the equatorial plane
All available Cassini data 2004-2008

Sergis et al., JGR, 2009
Plasma (E<3 keV) pressure in the equatorial plane
All available Cassini data 2004-2008

25% p⁺, 75% O⁺
Te ≈ Tp  P=3nkT

log[P(dyne/cm²)]

Probability of occurrence

Range (Rₛ)
Inertial (centrifugal) body force radial profile
All available Cassini data 2004-2008

$\rho \Omega^2 r$ (N/m$^3$)

range ($R_S$)

$m=12m_p$
70% corotation
After 5 years in orbit we have sufficient data to look into the statistical behavior of the system. However, the dynamic nature of the Saturnian magnetosphere appears almost overwhelming!
We can compare the average \(-\frac{\partial P}{\partial r}\) to the average \(\rho \Omega^2 r\) in the context of the radial force balance equation

\[
\rho \Omega^2 r - \frac{\partial P}{\partial r} \approx J_\phi B_\theta
\]

(under the adopted assumptions)

Both terms are of the same order of magnitude and comparable within their uncertainty range.

However...

outside 10 Rs, the pressure gradient is almost always higher (x2 or x3) than the inertial term.

The shape of the profile is determined by \(\frac{\partial P}{\partial r}\)
The Saturnian ring current has been captured by MIMI/INCA

Sergis et al., JGR, 2009
To sum up...

A dynamic, O\(^+\)-rich, typically pressure-driven ring current at keV energies in Saturn.

Sergis et al., JGR, 2009
Conclusions

1. Can we obtain a representative radial (total) pressure profile for the dayside magnetosphere of Saturn?
   
   Still very difficult. Cassini’s orbit, measurement uncertainties and intense dynamics are the basic problems.

2. How are plasma, suprathermal and magnetic pressure compared between 8 and 15 RS?
   
   Particle pressure and plasma $\beta$ are highly variable (one order of magnitude)
   Half the particle pressure in the keV energy range.
   Plasma beta $> 1$ outside of 9 RS.

3. How is the radial pressure gradient ($dP/dr$) compared to the centrifugal body-force ($\rho \Omega^2 r$)? Is the Saturnian ring current inertial or pressure-gradient driven?

   Outside of 10 RS, the pressure gradient is almost always higher than the inertial term resulting a pressure gradient driven ring current, with a vital role played by the energetic (keV) particles.

4. How well can the ring current density be reproduced by existing models?

   Disk current models cannot reproduce the ring current when $dP/dr$ is greater than or comparable to $\rho \Omega^2 r$, which is usually the case.
“I have been inspired by Tesla. The man thought big, he had revolutionary ideas. He was a risk taker, he had high risk, high payoff ideas. You expect, if you're lucky, to have one percent of these ideas be true, then you've made a tremendous contribution. Tesla had much more than one percent of his ideas being true. I would be lucky if I had one percent of my ideas being utilized, even one hundredth of what Tesla has succeeded.”

Dennis Papadopoulos, December 2000.

Thank you
Suprathermal pressure calculation

\[
P_i = \frac{8\pi}{3} \int_{E_{i-min}}^{E_{i-max}} dE \frac{E_i}{\nu_i} j_i(E) \Rightarrow P_{part.} = \frac{8\pi}{3} \sum_i \left[ \Delta E_i \left( \frac{E_i}{\nu_i} \right) j_i(E) \right]
\]

Pi: pressure supplied by the i-energy channel,
Ppart: partial particle pressure,
Pmag: magnetic pressure,
E_{i-min}, E_{i-max}: lower and upper limits of each i-energy channel of central energy E_i
\Delta E_i: channel energy width,
j_i: differential intensity,
\nu_i: velocity of either of the ion species (H^+ or O^+ in our case) in a particular i-channel.

Plasma pressure calculation

\[
T_p \approx T_e \quad T_w \approx \alpha T_e \quad (3 < \alpha < 5)
\]

\[
\begin{align*}
 n_p & \approx n_e \\
n_w & \approx 3n_e
\end{align*}
\]

\[
\begin{align*}
 & \iff \quad P \approx 4n_e kT_e
\end{align*}
\]
Assuming ion components of the same bulk velocity (V) in the steady state, the force (momentum) balance relation can follow from the first (velocity v) moment of the collisionless (source-free) Vlasov equation:

\[ \rho V \nabla V + \nabla \cdot P - J \times B = 0 \]

For non-relativistic particles (v<<c for ions and electrons) we can use velocity (rather than momentum) space distribution functions for each component.

\[ P = \int d^3v(v-V)(v-V)\sum m_i f_i(r,p,t) = \int d^3v(vv)\sum m_i f_i(r,p,t) - \rhoVV \]

\[ \rho V = \int d^3v(v)\sum m_i f_i(r,p,t) \quad \rho = \int d^3v\sum m_i f_i(r,p,t) \]

The radial component of this equation in the equatorial plane expressed in spherical polar coordinates (r,θ,φ) can be written as:

\[ -\rho \Omega^2 r + \frac{\partial P}{\partial r} - (J_\theta B_\phi - J_\phi B_\theta) = 0 \]

We assume that the pressure is isotropic (P) and that the plasma is corotating with a constant angular velocity, so that \( V=\Omega \times r \). If the plasma bulk velocity does not obey strict corotation, other terms will appear in the radial component of \( V \cdot \nabla V \).

In Saturn’s equatorial plane, \( J_\theta \) is field-aligned and small, \( |B_\phi/B_\theta|<<1 \) and \( B_\theta \approx B \), therefore:
\[- \rho \Omega^2 r + \frac{\partial P}{\partial r} + J_\phi B \approx 0\]

At the radius where the pressure is near its maximum ($\partial P/\partial r \approx 0$), the centrifugal body force must becomes comparable to the radial component of the $\mathbf{J} \times \mathbf{B}$ force. Elsewhere:

\[J_\phi \approx \frac{1}{B} \left( \rho \Omega^2 r - \frac{\partial P}{\partial r} \right)\]

Both $\rho \Omega^2 r$ and $\partial P/\partial r$ must be taken into account. Outside $r \approx 9 R_S$, $\partial P/\partial r < 0$, and both terms add together to contribute to the $J_\phi$. 