KINETIC FORMULATION OF TRANSPORT OF CHARGED PARTICLES INTERACTING WITH ELECTROMAGNETIC WAVES IN MAGNETIZED PLASMAS

Abhay K. Ram
Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge, MA 02139. U.S.A.

Yannis Kominis, Kyriakos Hizanidis
National Technical University of Athens
Association EURATOM-Hellenic Republic
Zografou, Athens 15773, Greece.

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STOCHASTIC ACCELERATION OF LARGE M/Q IONS BY HYDROGEN CYCLOTRON WAVES IN THE MAGNETOSPHERE

K. Papadopoulos*

Science Applications, Inc., McLean, Virginia 22102

J. D. Gaffey, Jr. and P. J. Palmadesso

Geophysical & Plasma Dynamics Branch, Plasma Physics Division, Naval Research Laboratory

Washington, D. C. 20375

Abstract. It is shown that in hydrogen dominated multi-ion plasmas supporting coherent hydrogen cyclotron waves, the minority ion species with large M/Q are preferentially accelerated and the maximum energy achieved scales as $(M/M_{H^+})^{5/3}$. The importance of this scaling to $O^+$ acceleration in the auroral zones and to other high energy heavy ion observations in the earth's and Jupiter's magnetospheres is discussed.
STANDARD (CHIRIKOV-TAYLOR) MAP

Interaction of a charged particle with an infinite set of plane waves

\[ \frac{dx}{dt} = v \]

\[ \frac{dv}{dt} = \frac{qE}{m} \sum_{n=-\infty}^{\infty} \sin(kx - n\omega t) \]
STANDARD MAPPING EQUATIONS

The change in particle velocity and wave phase after every time step \( T = \frac{2\pi}{\omega} \) is

\[
I_{n+1} = I_n + K \sin \theta_n \quad \text{mod} \ 2\pi
\]

\[
\theta_{n+1} = \theta_n + I_{n+1} \quad \text{mod} \ 2\pi
\]

where \( kx = \theta \), \( kTv = I \), \( \left( \frac{2\pi}{\omega} \right)^2 \frac{qkE}{m} = K \).
SURFACE-OF-SECTION PLOT

\[ K = 1.5 \]
SURFACE-OF-SECTION PLOT

\[ K = 4.5 \]
Fick’s second law, or Fokker-Planck equation

$$\frac{\partial f(I)}{\partial n} = \frac{\partial}{\partial I}\left(D(I) \frac{\partial}{\partial I} f(I)\right)$$

$f(I)$ is the distribution function

$D(I)$ is the diffusion coefficient

What is to be substituted for $n$ and $D(I)$?
DIFFUSION IN VELOCITY SPACE

Evaluation of the diffusion coefficient

- Single step jump in velocity ($I_0 \rightarrow I_1$)
  - Markovian assumption;
  - random walk (or Brownian motion).

- Multiple step jump in velocity ($I_0 \rightarrow I_n$)
  - $n \gg n_c$
  - $n_c$ is the number of steps for phase randomization.
DIFFUSION IN VELOCITY SPACE

Quasilinear diffusion coefficient \((n=1)\)

\[
D_{QL} = \frac{\langle (\Delta I_1)^2 \rangle_{\theta_0}}{2} = \frac{1}{4\pi} \int_0^{2\pi} d\theta_0 (I_1 - I_0)^2 = \frac{K^2}{4}
\]

➤ Independent of \(I\)
Define the correlation function:

\[ C_n = \langle (I_n^p - I_{n-1}^p) (I_1^p - I_0^p) \rangle_p \]

where \( \langle \ldots \rangle_p \) is an ensemble average for a set of randomly distributed particles.

The correlation “time” \( n_c \) is such that

\[ \text{for } n > n_c, \quad C_n \approx 0. \]
Diffusion coefficient for $n > n_c$:

$$D_n = \lim_{n>n_c} \frac{\langle (I_n - I_0)^2 \rangle}{2n}$$

$\langle \ldots \rangle$ is the ensemble average.

$$\frac{\partial f(I)}{\partial n} = \frac{\partial}{\partial I} \left( D_n(I) \frac{\partial}{\partial I} f(I) \right)$$
DIFFUSION COEFFICIENT FOR THE STANDARD MAP

\[ \frac{D_n}{D_{QL}} \]

\[ n = 50 \]

\[ \Rightarrow D \text{ is independent of } I \]

In the standard map, the entire particle distribution function is affected.
PARTICLE INTERACTION WITH A SPATIALLY LOCALIZED FIELD
SURFACE-OF-SECTION PLOT

\[ \theta / 2\pi \]
DIFFUSION COEFFICIENT FOR LOCALIZED CHAOS

\[ \frac{D_n}{D_{QL}} \]
DIFFUSION COEFFICIENT FOR LOCALIZED CHAOS

\[ \frac{D_n}{D_{QL}} \]
STICKINESS OF ORBITS

$K = 1.5$
PREVIOUS APPROACHES TO QUASILINEAR DIFFUSION EQUATION

- Linearize the Vlasov equation and obtain an equation for the perturbed distribution function.

- Assume that the underlying particle dynamics is chaotic
  - Brownian motion (random walk);
  - no structure to phase space.

- Long time evolution is the same as for short times
  - allows the limit \( t \to \infty \) in evaluating \( D \).

- Obtain time-independent, singular diffusion operator
  \[
  \delta (\omega - n\omega_c - k||v_t)
  \]
A different approach is needed to describe the evolution of a distribution function of particles interacting with plasma waves.
Hamiltonian approach to particle dynamics and wave particle interactions:

\[ H(J, \theta) = H_0(J) + \epsilon H_1(J, \theta, t) \]

\( H_0(J) \) describes the motion of the particle in the absence of plasma waves.

\( H_1(J, \theta, t) \) includes the interaction with waves.
There exists an operator $O_L$ (Lie operator) such that

$$O_L : (J, \theta)_t \rightarrow (J, \theta)_{t+\Delta t}$$

An advantage of the Lie operator is that

$$O_L^{-1} f(J, \theta) = f(O_L \{J, \theta\})$$
EVOLUTION EQUATION FOR THE DISTRIBUTION FUNCTION

\[
f(J, \theta)_{t+\Delta t} - f(J, \theta)_t = (O_L^{-1} - I) \cdot f(J, \theta)_t
\]

Dividing by \( \Delta t \) and taking the limit \( \Delta t \to 0 \)

\[
\frac{\partial}{\partial t} f(J, \theta, t) = \left[ \frac{\partial}{\partial t} (O_L^{-1} - I) \right] \cdot f(J, \theta, t)
\]
EVOLUTION EQUATION FOR THE DISTRIBUTION FUNCTION

By averaging over the angles $\Theta$

$$\frac{\partial}{\partial t} f(J, t) = \left[ \frac{\partial}{\partial J} \cdot D(J, t) \cdot \frac{\partial}{\partial J} \right] f(J, t)$$
\[
\lim_{t \to \infty} D(J, t) \rightarrow \delta(m \cdot \omega_0 - \omega_m)
\]
CONCLUSIONS

- Dynamical studies of wave-particle interactions show a mixed phase space.

- The evolution of a distribution function requires proper accounting of this phase space.

- The Markovian assumption for evaluating the diffusion coefficient is invalid.

- Recent studies provide a detailed description for the evolution of the distribution function due to wave-particle interactions.