Theory and simulations of nonlinear whistler-mode chorus waves in the magnetosphere

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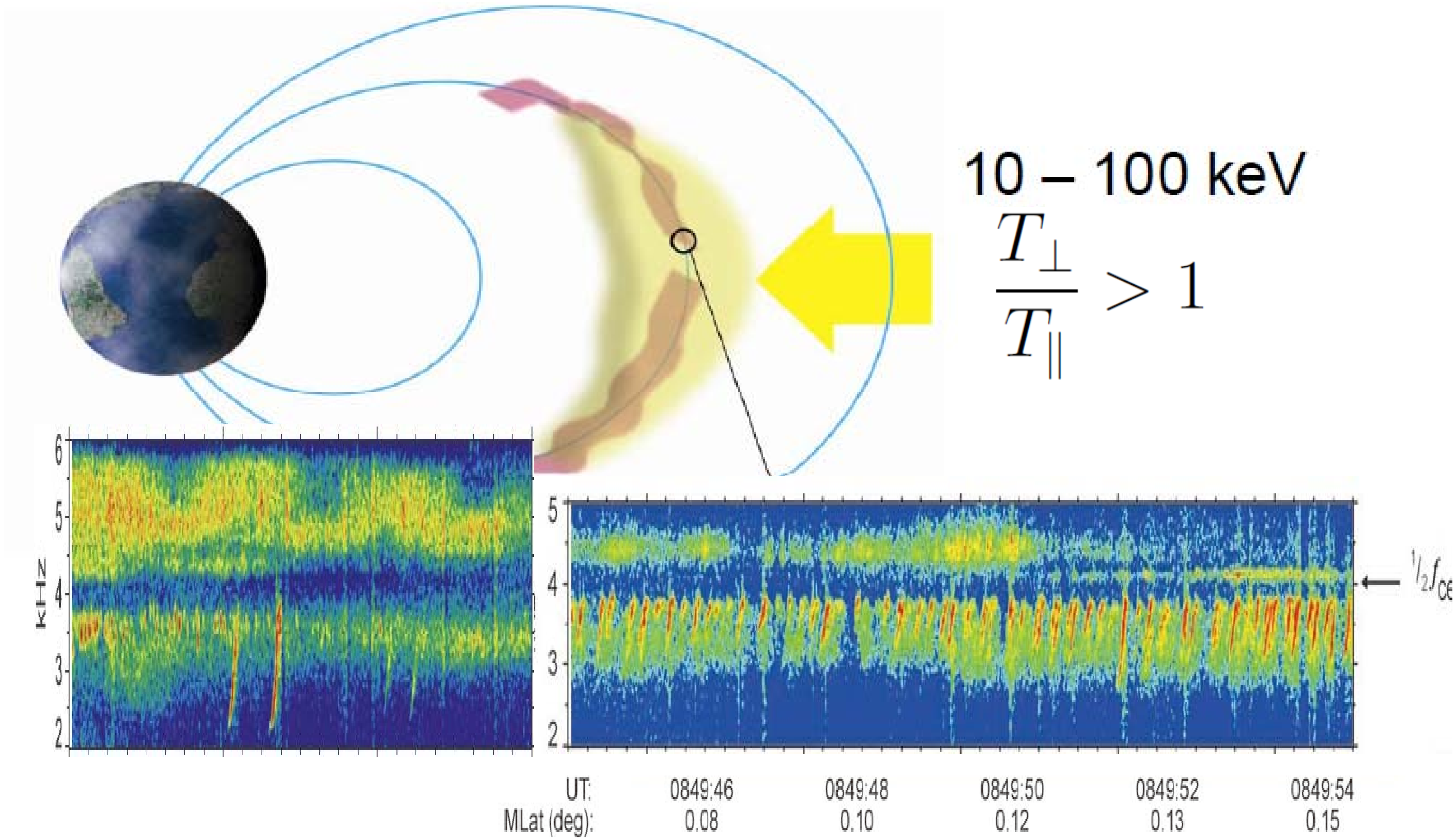
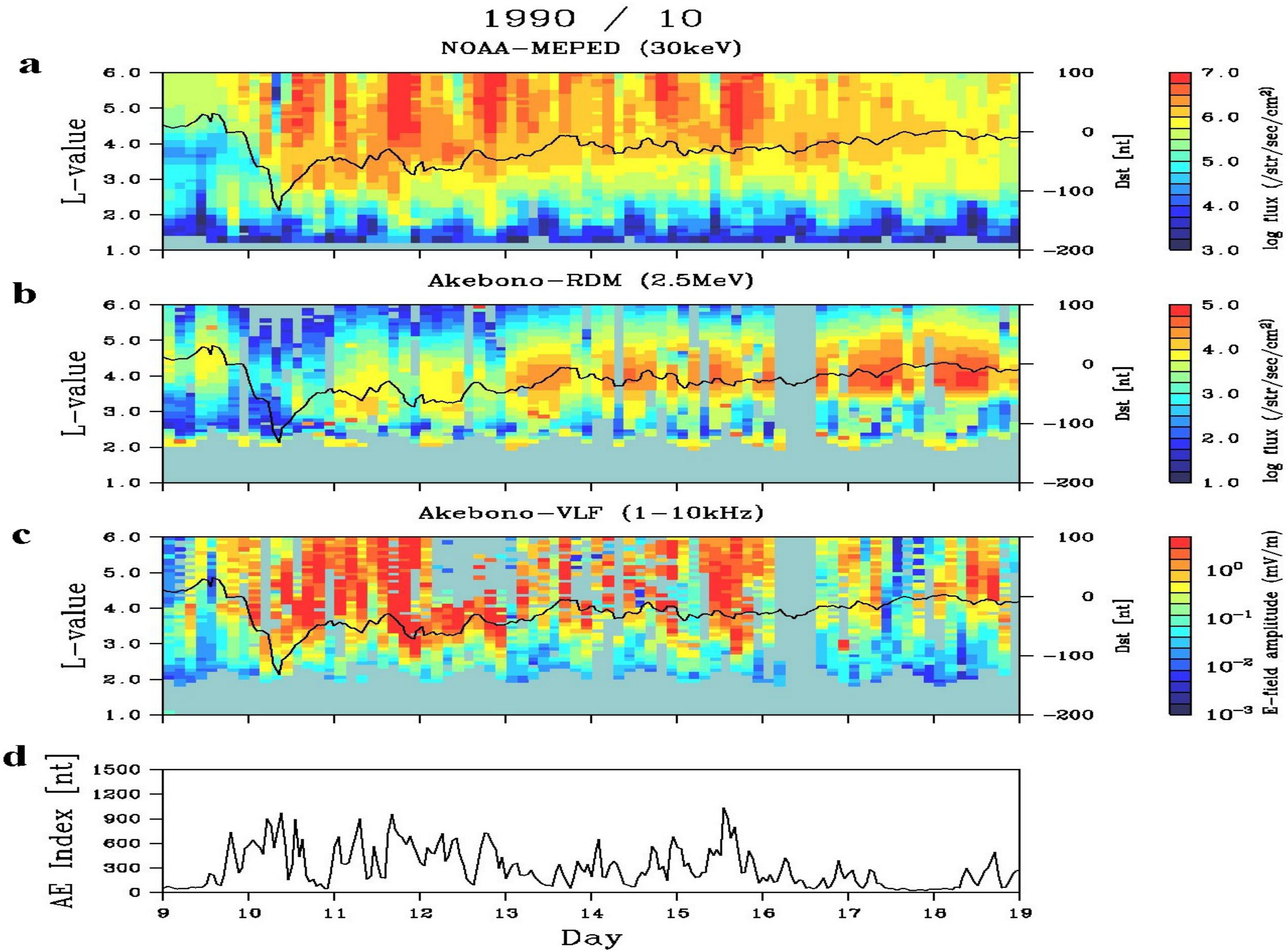
Modern Challenges in Nonlinear Plasma Physics, A Conference honoring the Career of Dennis Papadopoulos, Sani Resort, Halkidiki, Macedonia, Greece, June 15- 19, 2009

Chorus Emission due to Injection of Energetic Electrons



[Cluster, *Santolik and Gurnett,*]



[*Kasahara et al., GRL, 2008*]

Overview

Constant Frequency Wave

$$\frac{\partial \omega}{\partial t} = 0$$

- Linear growth rate
- Quasi-linear diffusion
- Nonlinear trapping and acceleration in an inhomogeneous medium: RTA and URA

Rising-tone Chorus Element

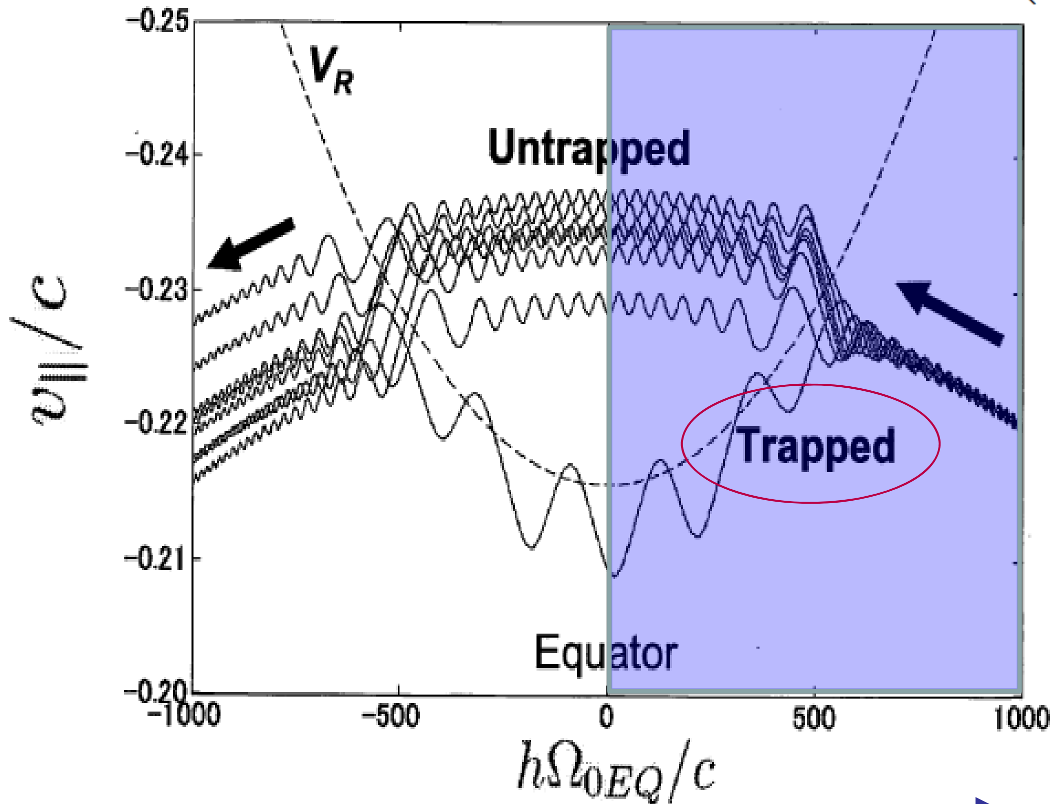
$$\frac{\partial \omega}{\partial t} > 0$$

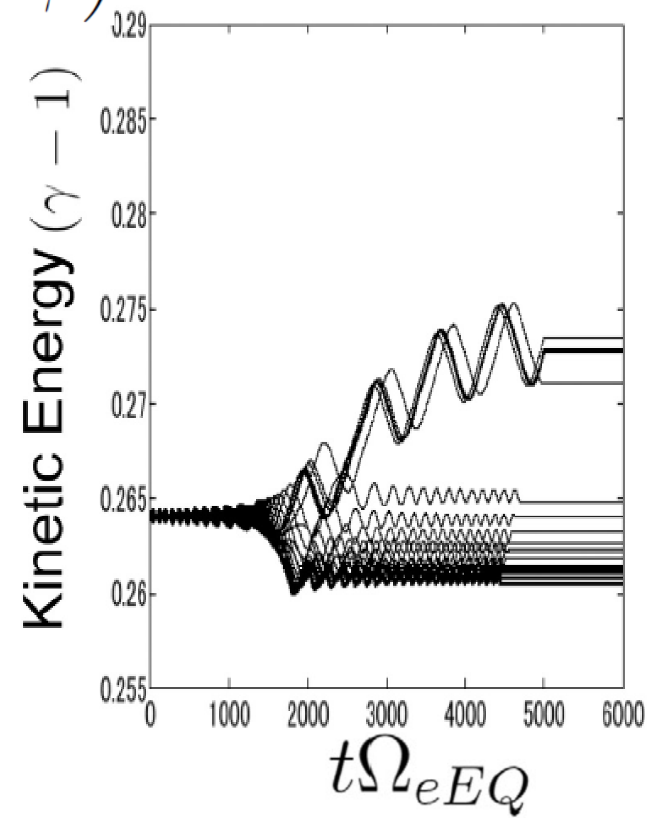
- Nonlinear-wave growth
- Generation of upper-band chorus emissions
- Efficient acceleration by nonlinear trapping and formation of the pancake distribution

Test Particle Simulation of Nonlinear Wave Trapping and Acceleration of Trapped Particles (10 – 100 keV)

$$\frac{\partial \omega}{\partial t} = 0$$

$$\omega - k v_{\|} = \Omega_e(h) / \gamma \quad \Rightarrow \quad V_R = \frac{1}{k}\left(\omega - \frac{\Omega_e}{\gamma}\right)$$



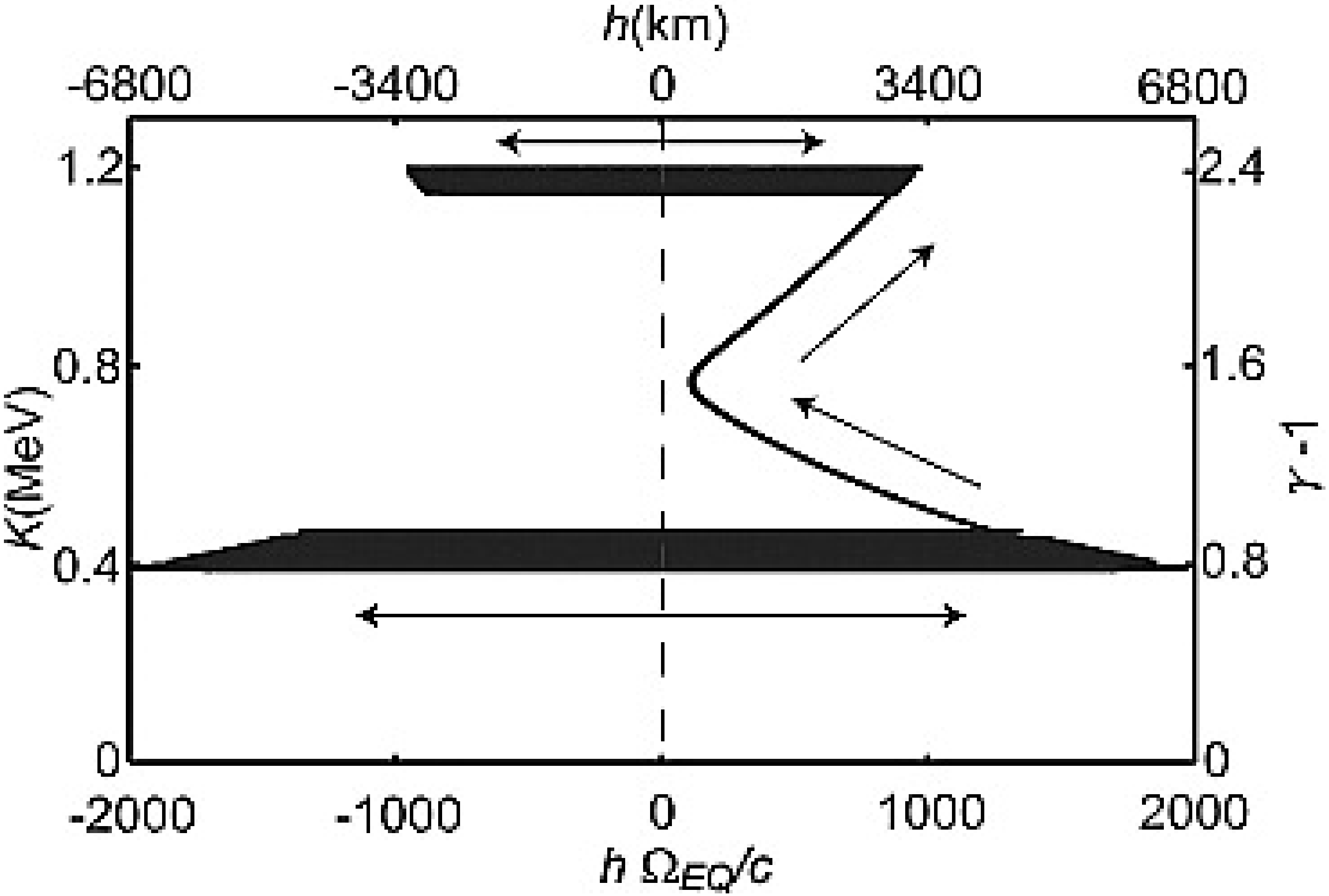


[*Omura and Summers, JGR, 2006*]

Trajectories of Resonant Electrons (400 keV)

$$\frac{\partial \omega}{\partial t} = 0$$

Relativistic Turning Acceleration （RTA)



Bw = 125 pT

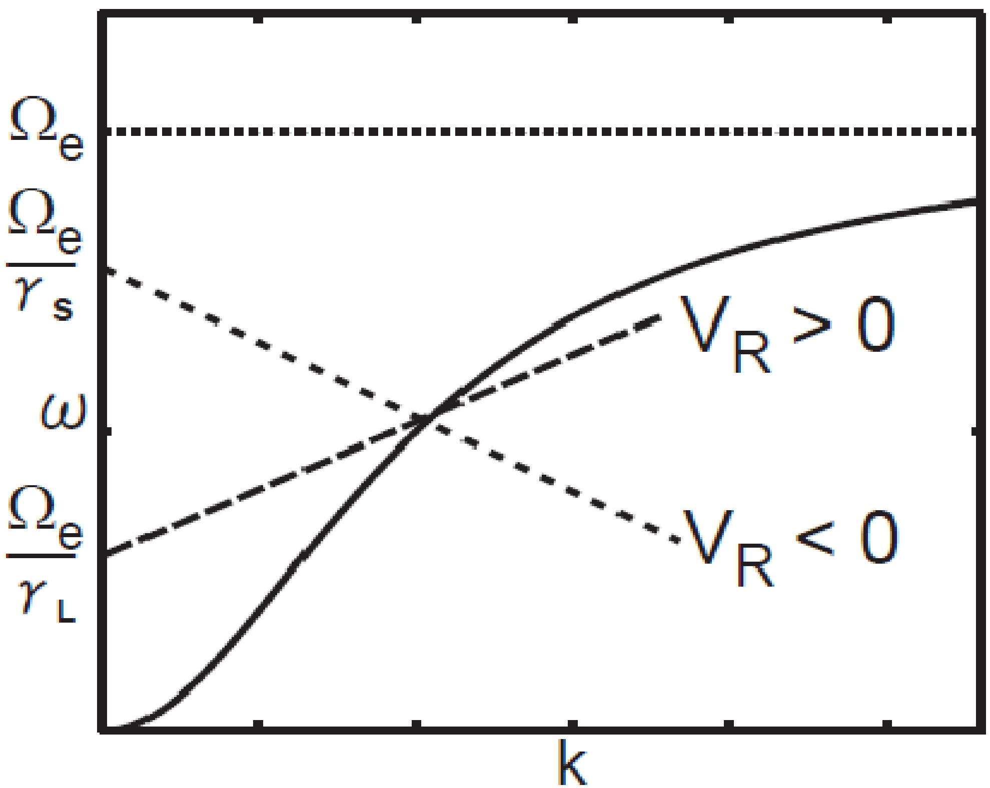
$$\omega_p = 2.0\Omega_{e0} \qquad \omega = 0.4\Omega_{e0}$$

[*Omura, Furuya, and Summers, JGR, 2007*]

Resonance Velocity

$$V_R = \frac{\omega}{k}\left(1 - \frac{\Omega_e(h)}{\omega\gamma}\right) \qquad \gamma = [1 - (v_{\|}^2 + v_{\perp}^2)/c^2]^{-1/2}$$

$$\gamma \geqq \Omega_e(h)/\omega \quad \Rightarrow V_R \geqq 0$$



$$K = m_0 c^2(\gamma - 1)$$

$$\frac{dK}{dt} = -eE_w v_\perp S$$

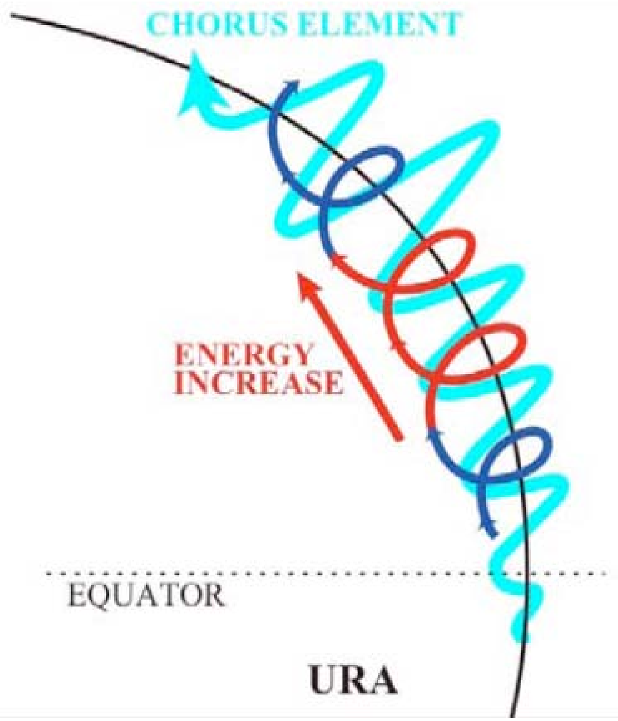
Trajectories of Resonant Electrons (> 1MeV)

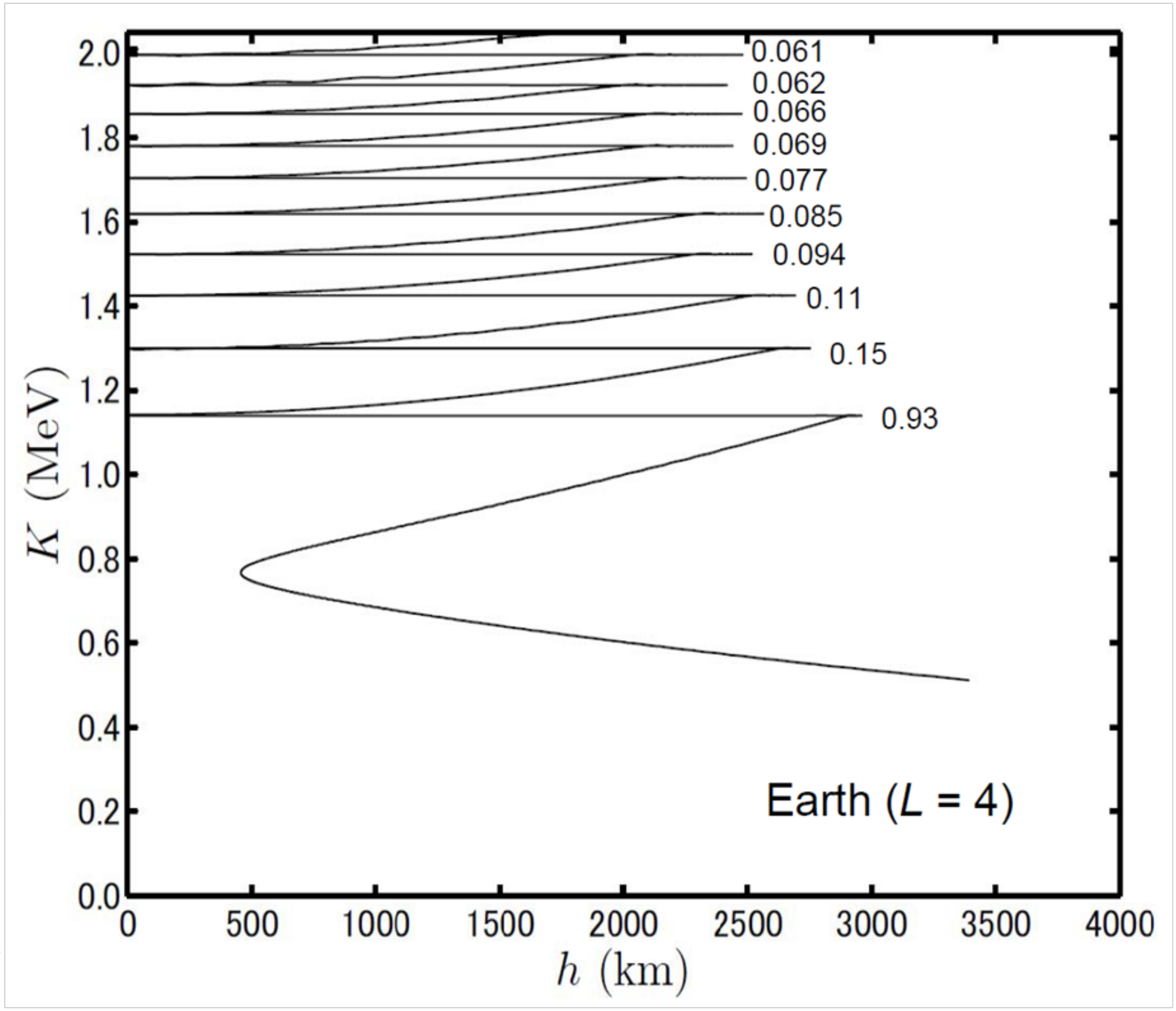
$$\frac{\partial \omega}{\partial t} = 0$$

Ultra-Relativistic Acceleration (URA)

$$\gamma_0 > \Omega_{EQ}/\omega$$

$$V_{R0} > 0$$





[*Summers and Omura, GRL, 2007*]

$$\frac{d\zeta}{dt} = \theta$$

$$\frac{d\theta}{dt} = \omega_t^2(\sin\zeta + S)$$

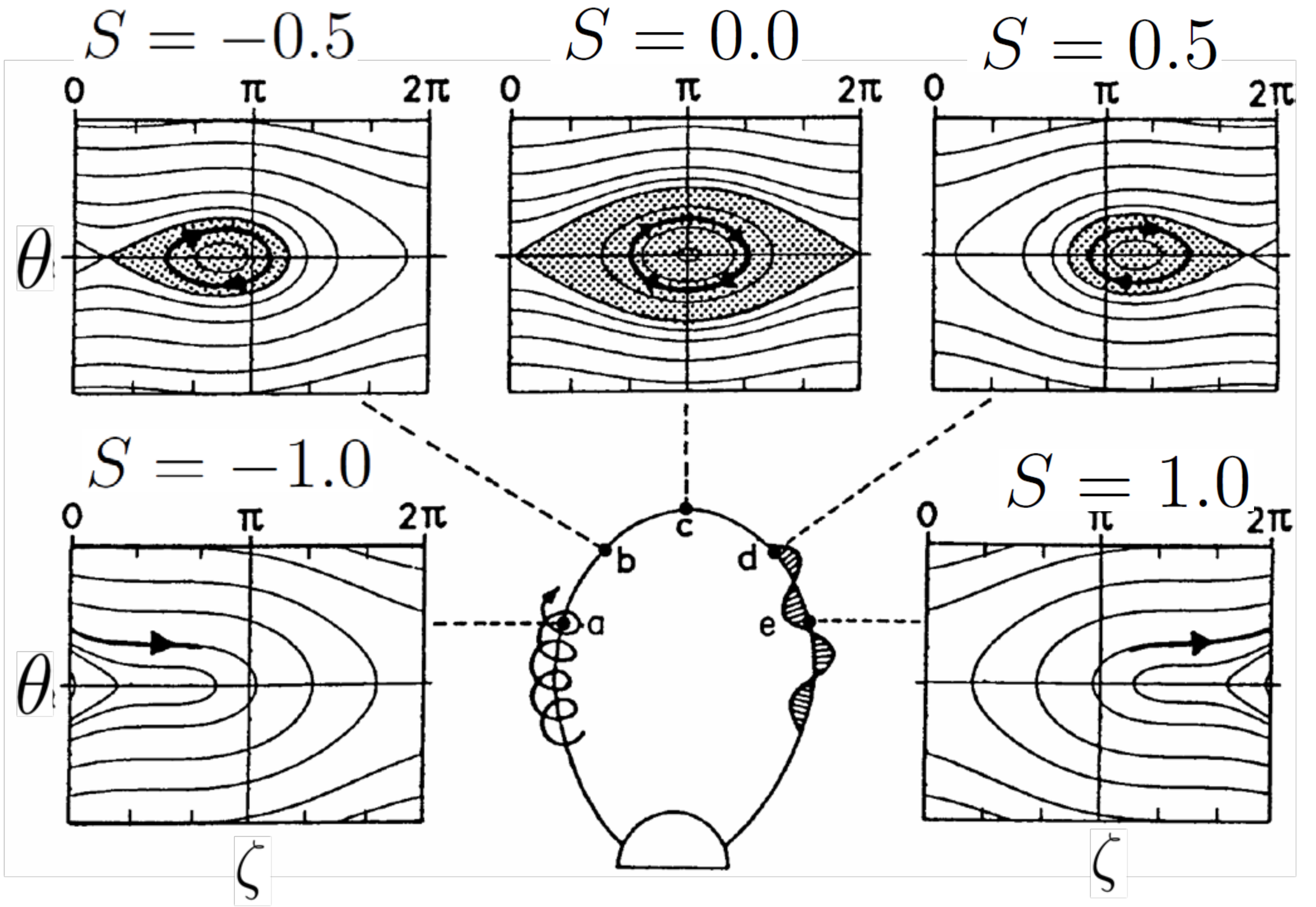
$$\theta = k(v_\parallel - V_R)$$

for Whistler-mode Wave

$$\theta = -k(v_\parallel - V_p)$$

for Longitudinal Wave

$$\frac{\partial\omega}{\partial t} = 0$$



$$\frac{\partial \omega}{\partial t} > 0$$

Inhomogeneity Ratio

$$S = -\frac{1}{s_0 \omega \Omega_w}\left(s_1 \frac{\partial \omega}{\partial t} + c s_2 \frac{\partial \Omega_e}{\partial h}\right)$$

$$s_0 = \frac{\delta}{\xi}\frac{V_{\perp 0}}{c} \qquad s_1 = \gamma\left(1 - \frac{V_R}{V_g}\right)^2$$

$$s_2 = \frac{1}{2\xi\delta}\left\{\frac{\gamma\omega}{\Omega_e}\left(\frac{V_{\perp 0}}{c}\right)^2 - \left[2 + \Lambda\frac{\delta^2(\Omega_e - \gamma\omega)}{\Omega_e - \omega}\right]\frac{V_R V_p}{c^2}\right\}$$

$\Lambda = \omega/\Omega_e$ for inhomogeneous density model

$\Lambda = 1$ for constant density model

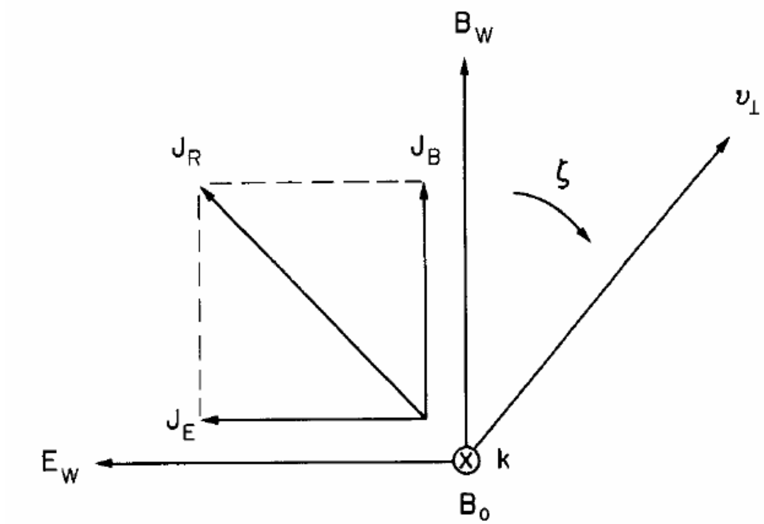
[Omura et al., 2008; *Omura et al., JGR, in press*]

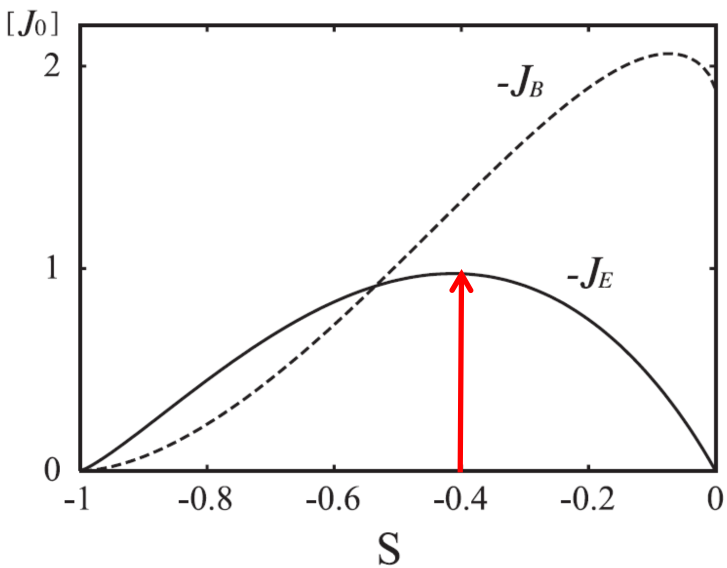
Nonlinear Wave Growth due to Formation of Electromagnetic Electron Hole

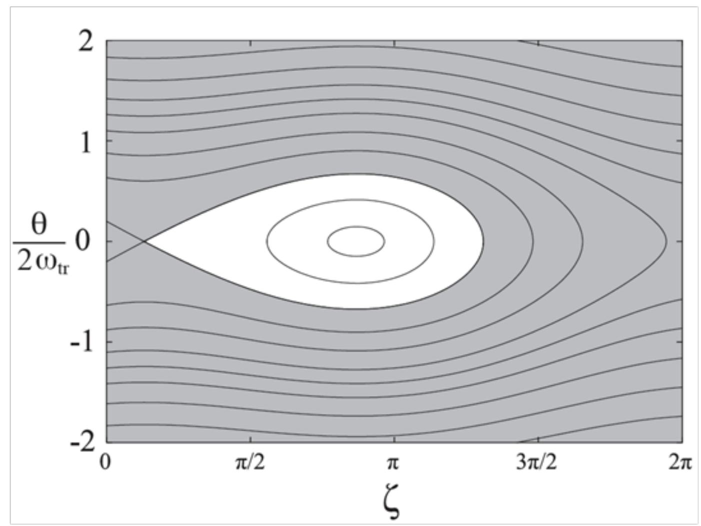
$$\frac{\partial \omega}{\partial t} > 0$$

$$\frac{\partial B_w}{\partial t} + V_g \frac{\partial B_w}{\partial h} = -\frac{\mu_0 V_g}{2} J_E$$

$$\frac{\partial \omega}{\partial t} + V_g \frac{\partial \omega}{\partial h} = 0$$





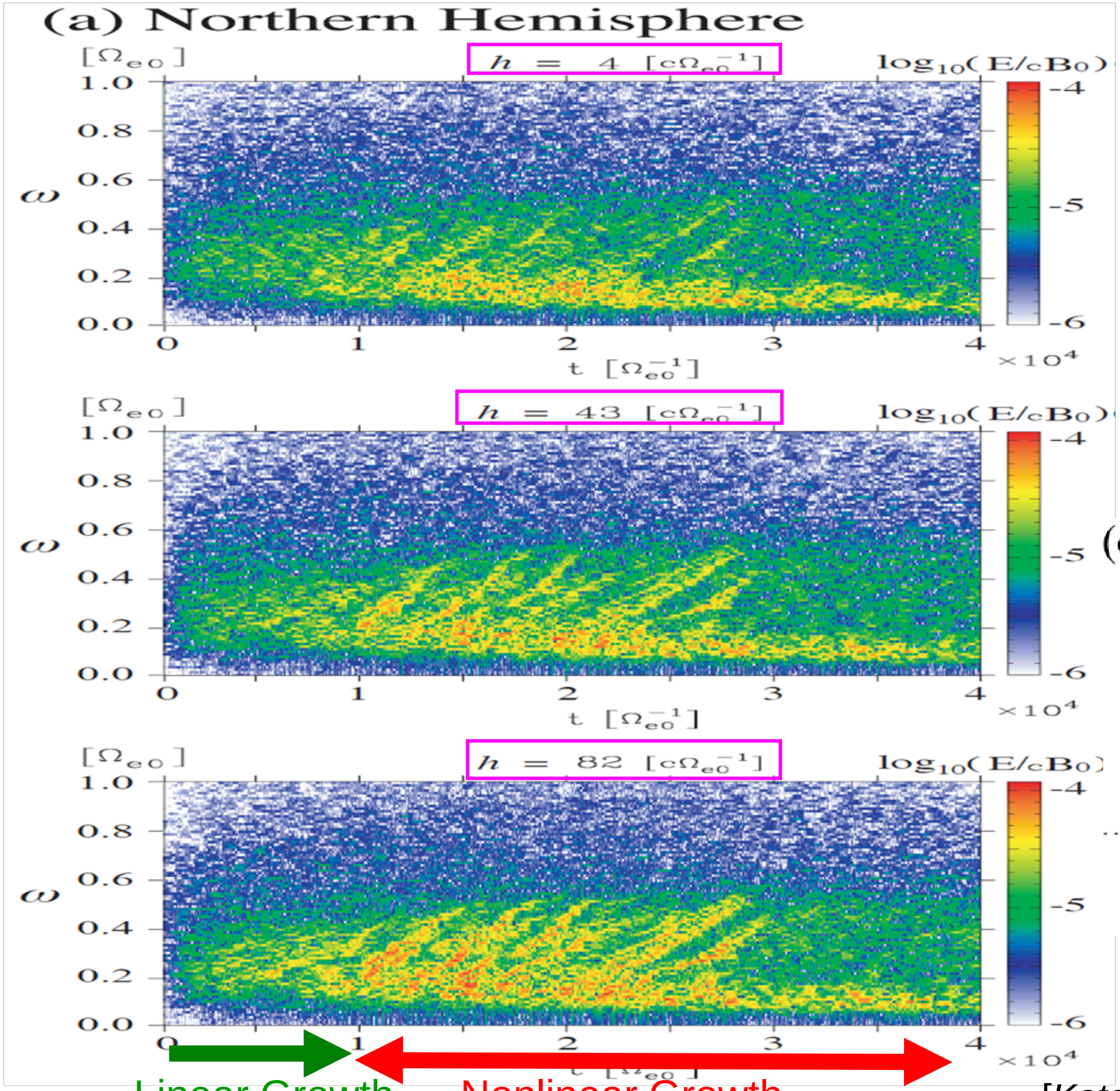


Maximum ***J$_E$***

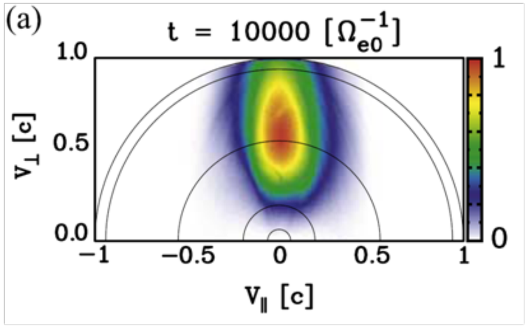
$$S_{EQ} = -0.4$$

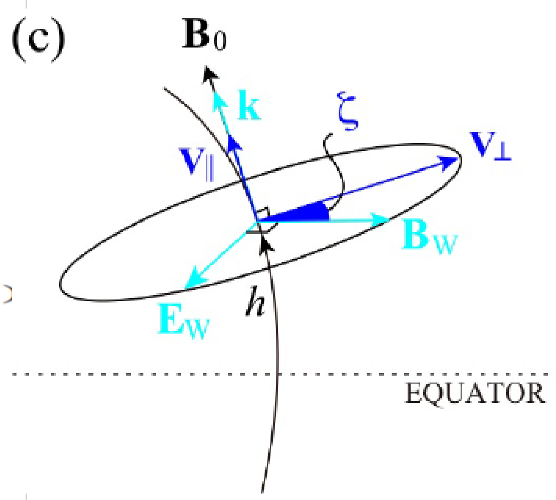
[*Omura , Katoh, Summers, JGR, 2008*]

Electron Hybrid Simulation (cold electrons : fluid)

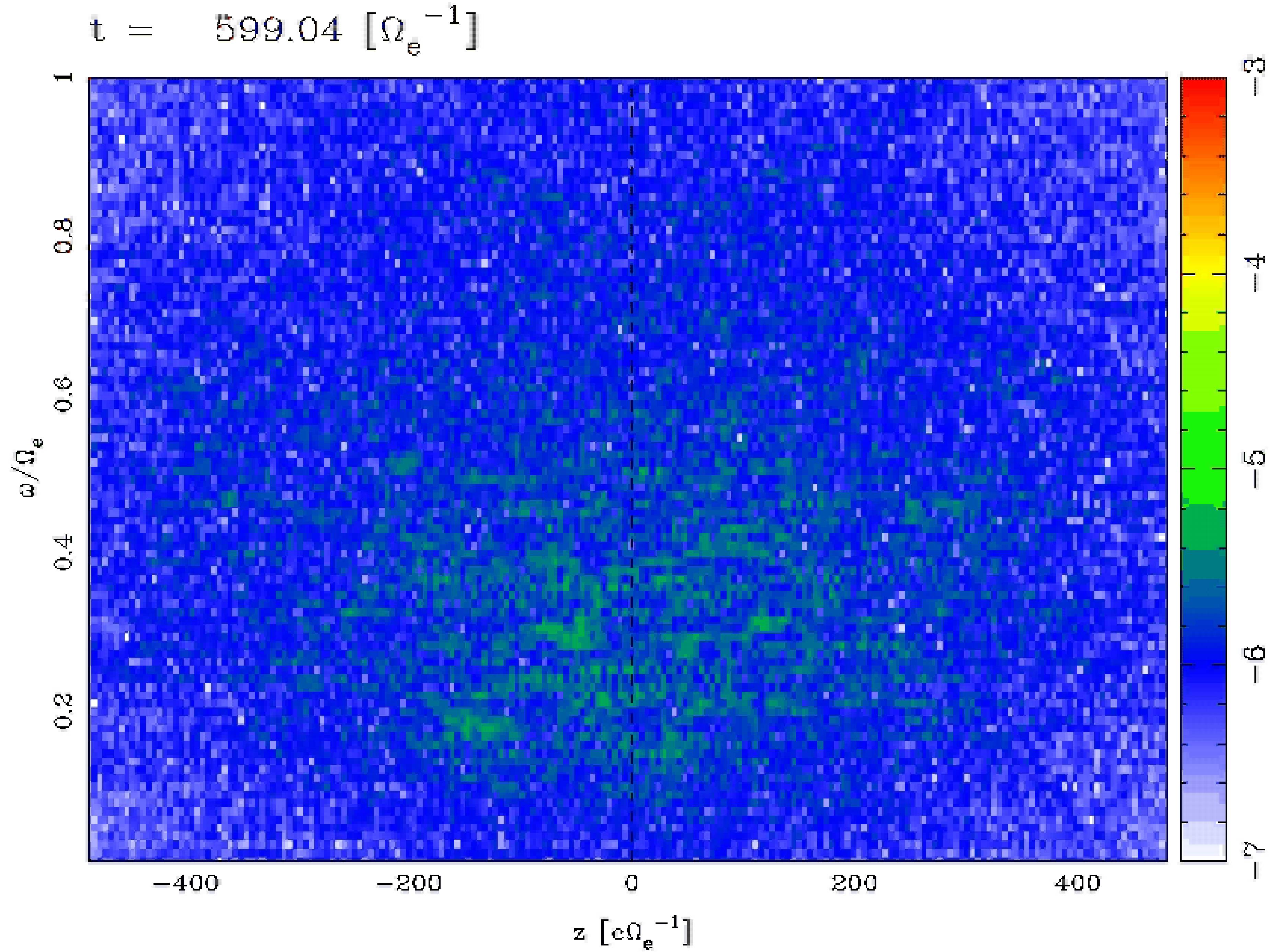


$T_\perp / T_\parallel = 9$





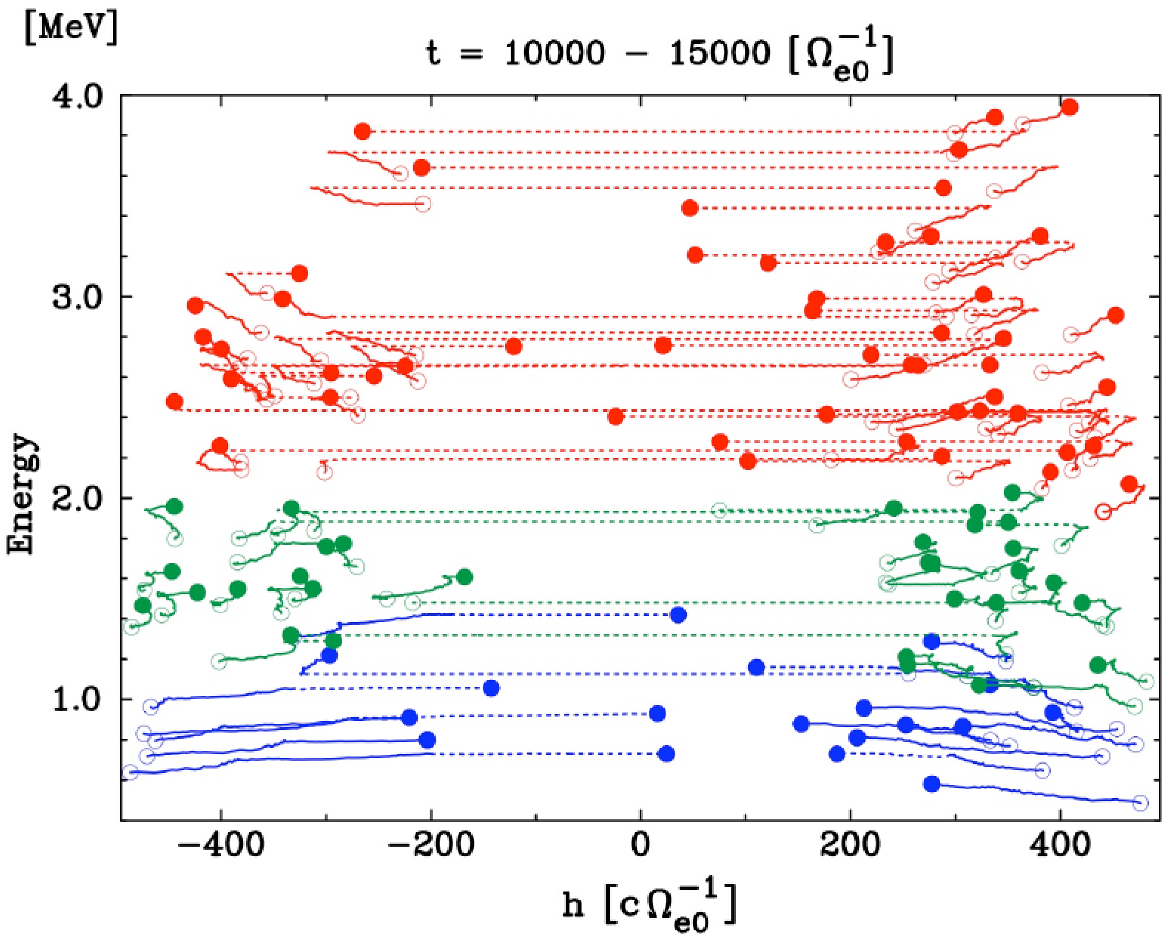
[*Katoh and Omura,GRL, 2007*]

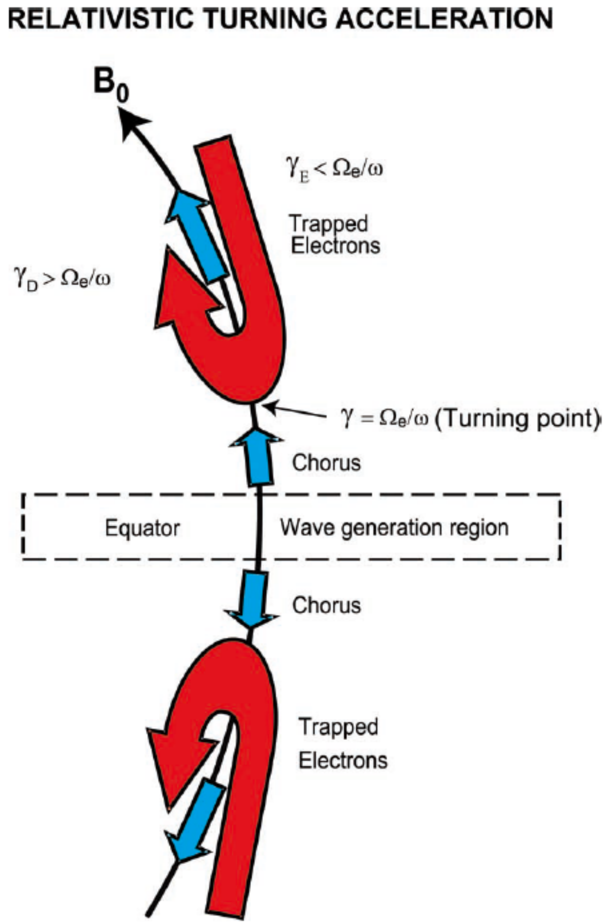


t = 599.04 [Ω_e^{-1}]
1
0.8
0.6
0.4
0.2
ω/Ω_e
−400
−200
0
200
400
z [$c\Omega_e^{-1}$]
−3
−4
−5
−6
−7

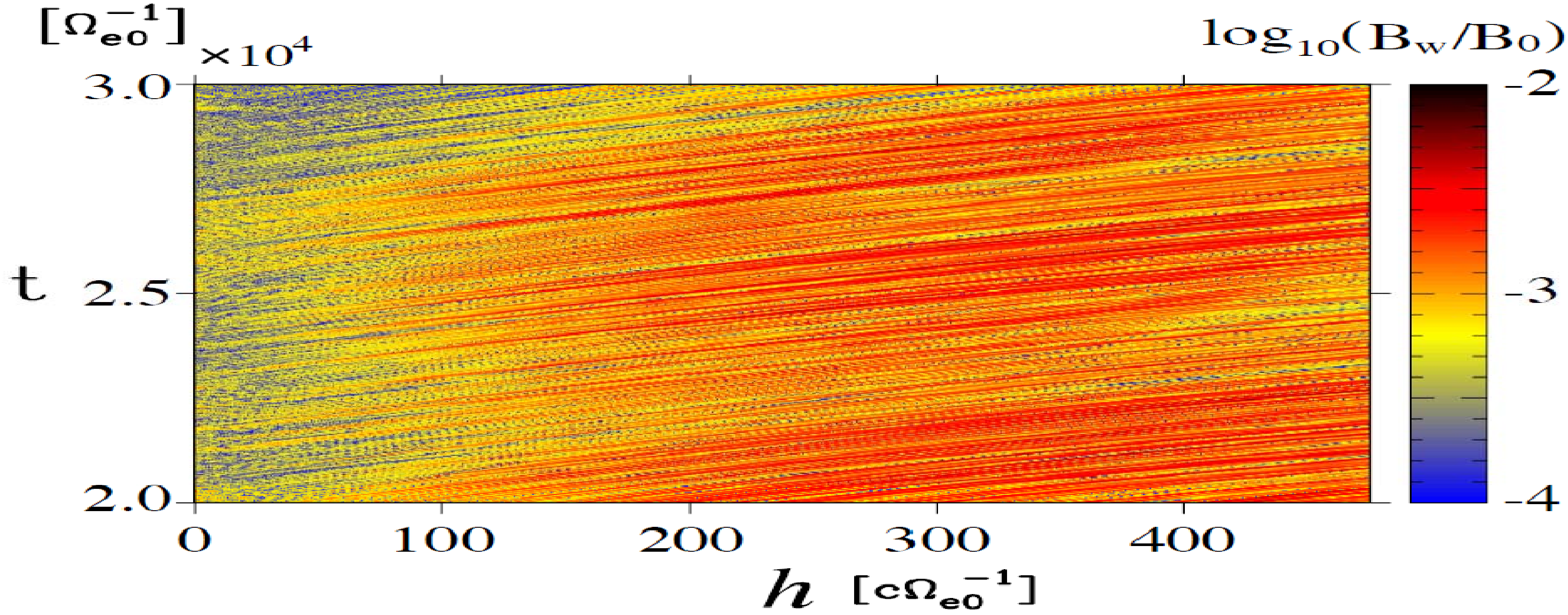
$$\frac{\partial \omega}{\partial t} > 0$$

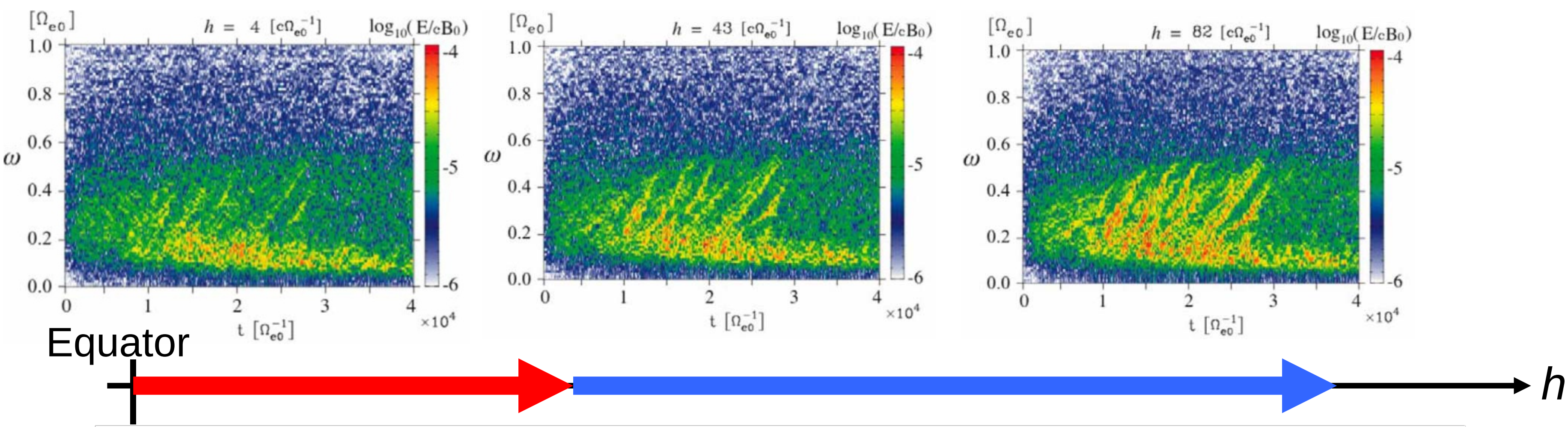
RTA and URA in the chorus generation process





[*Katoh, Omura and Summers, Ann. Geophys*. 2008]



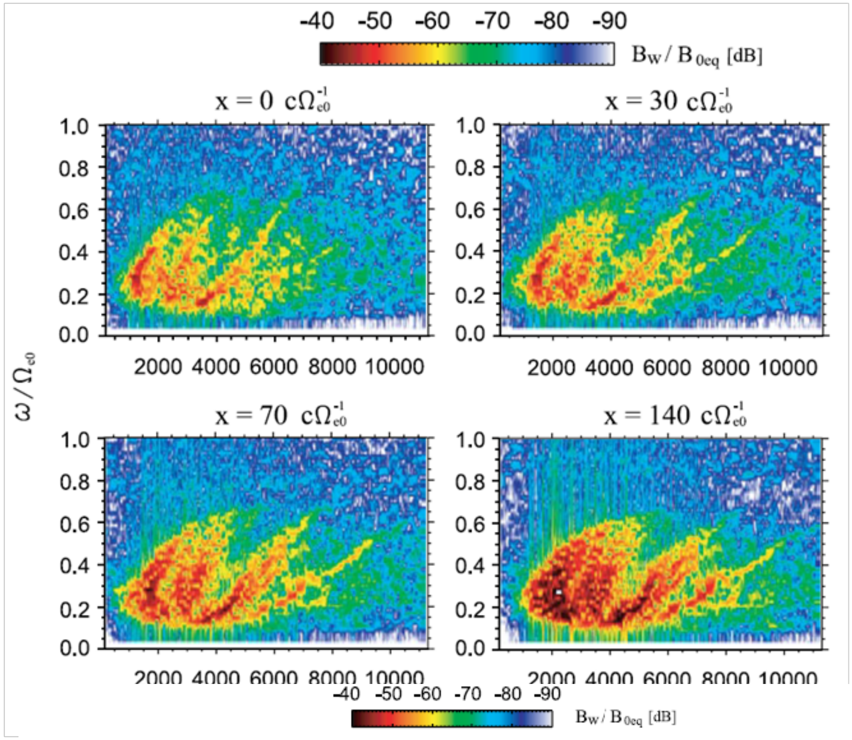


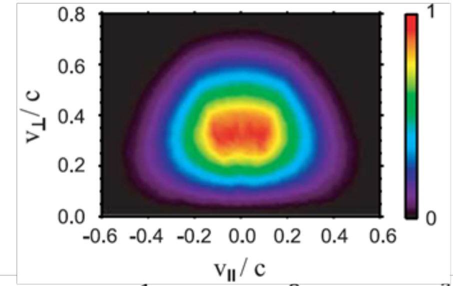
$$S = -\frac{1}{\omega_t^2 \delta^2}\left[\gamma\left(1-\frac{V_R}{V_g}\right)^2 \frac{\partial \omega}{\partial t} + \left\{\frac{k\gamma v_\perp^2}{2\Omega_e} - \left(1+\frac{\delta^2}{2}\frac{\Omega_e-\gamma\omega}{\Omega_e-\omega}\right)V_R\right\}\frac{\partial \Omega_e}{\partial h}\right]$$

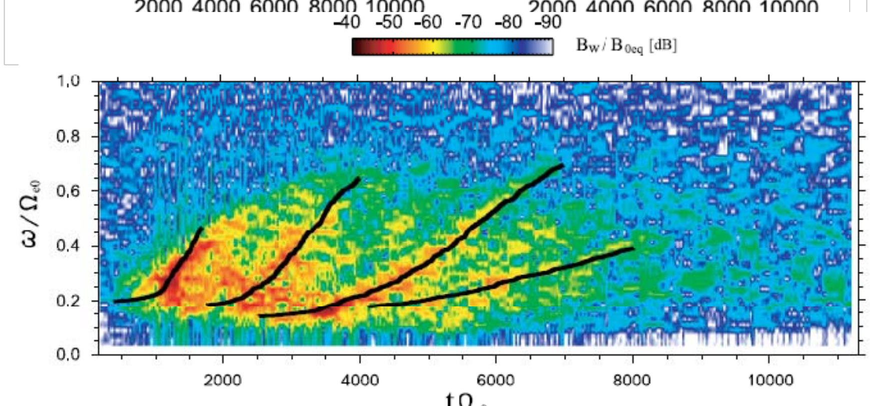
B_w

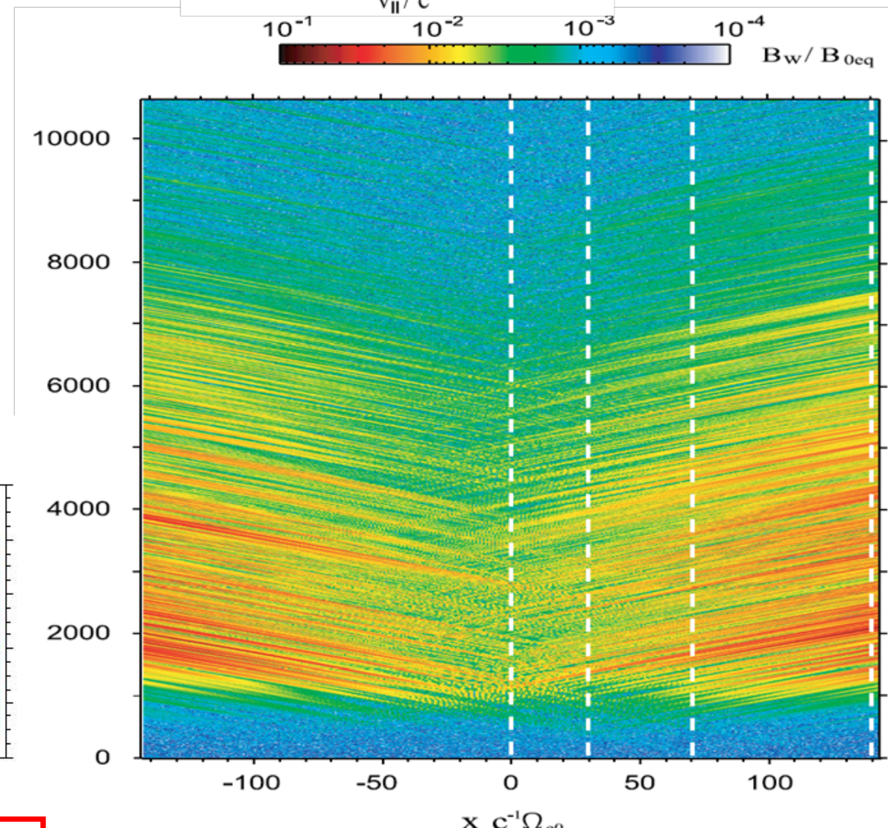
Full-Particle Simulation of Chorus Emission

$T_\perp / T_\parallel = 3$



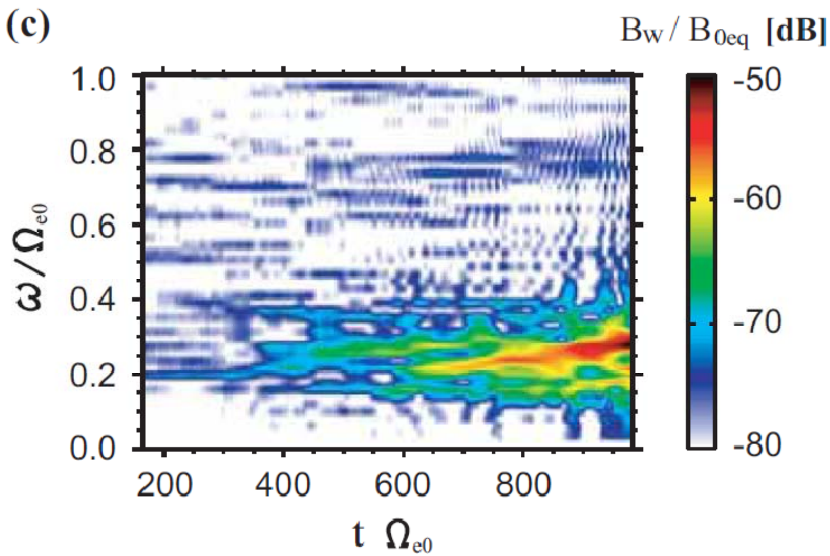


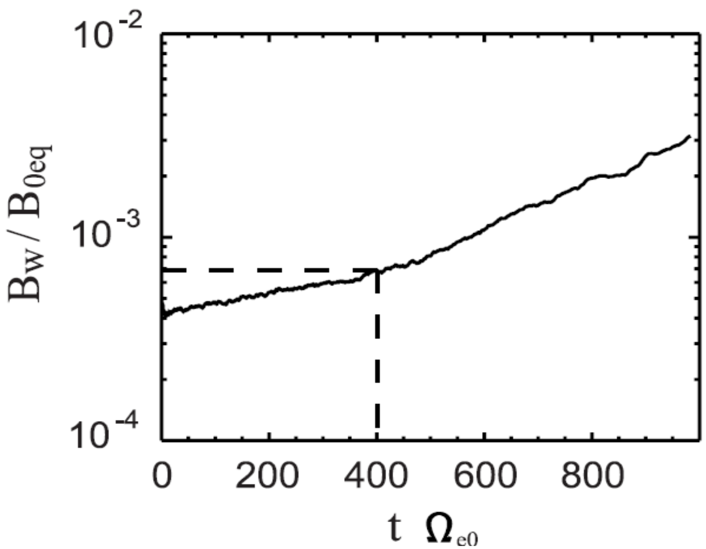




$$\frac{\partial \omega}{\partial t} = \frac{0.4\,\delta}{\gamma\,\xi}\frac{V_{\perp 0}}{c}\frac{\omega}{\Omega_{e0}}\left(1-\frac{V_R}{V_g}\right)^{-2}\frac{B_w}{B_{0eq}}\Omega_{e0}^2$$

[*Hikishima et al., JGR, 2009*]





The condition for absolute instability at the equator

$$\frac{\partial B_w}{\partial t} = -\frac{\mu_0 V_g}{2} J_E - V_g \frac{\partial B_w}{\partial h} > 0$$

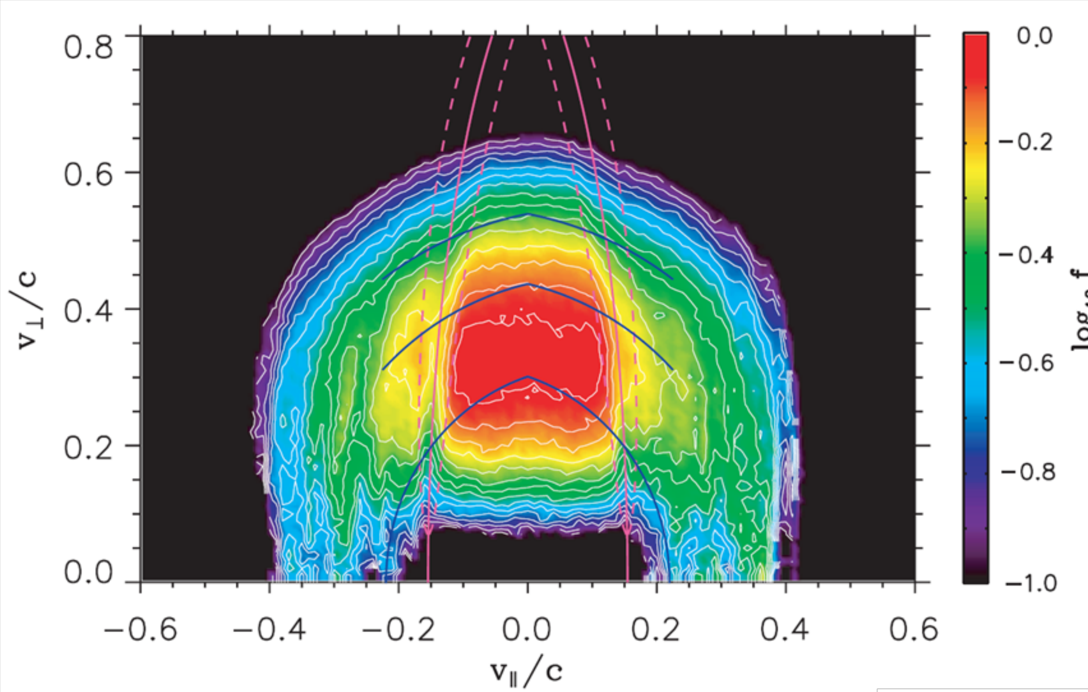
gives the amplitude threshold for nonlinear wave growth.

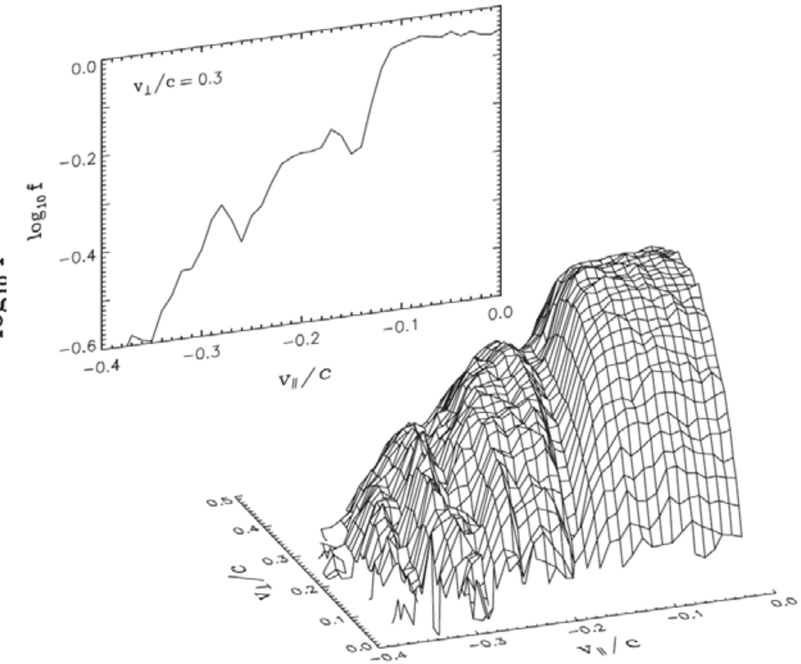
$$\tilde{\Omega}_w = \frac{B_w}{B_0} > \tilde{\Omega}_{th}$$

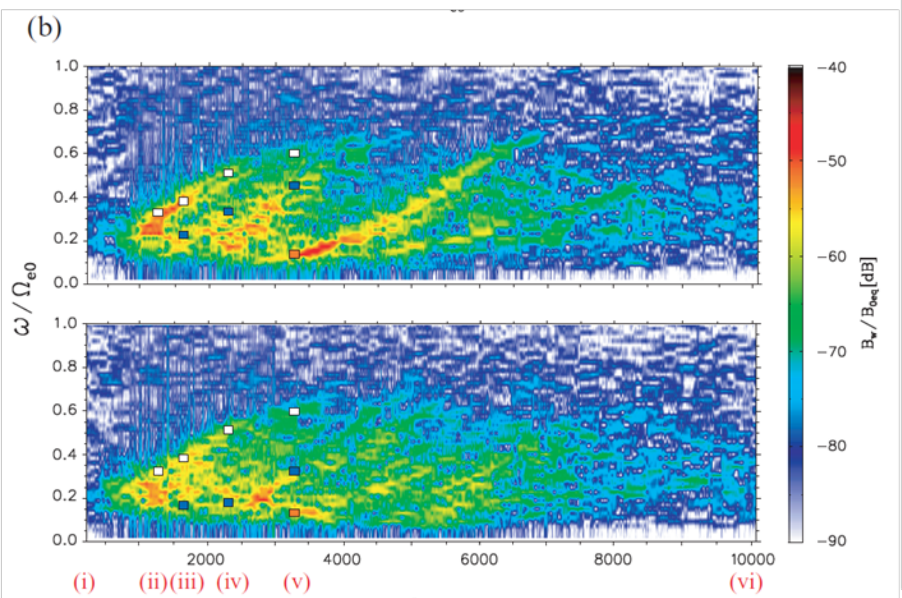
$$\tilde{\Omega}_{th} = \frac{100\pi^3\gamma^3\xi}{\tilde{\omega}\tilde{\omega}_{ph}^4 \tilde{V}_{\perp 0}^5 \delta^5} \left(\frac{\tilde{a} s_2 \tilde{U}_{t\|}}{Q} \right)^2 \exp\left(\frac{\gamma^2 \tilde{V}_R^2}{\tilde{U}_{t\|}^2} \right)$$

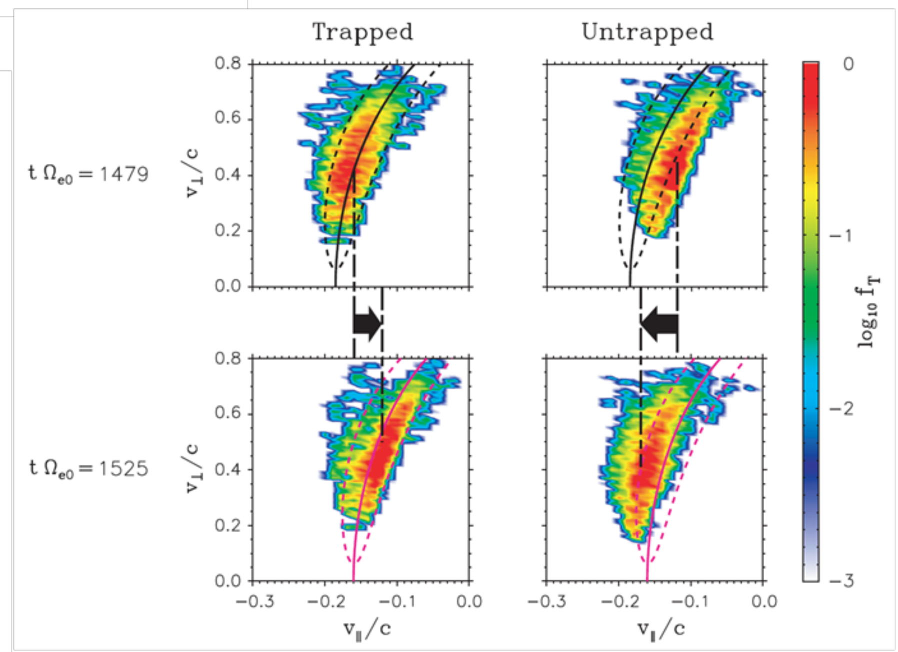
The nonlinear growth rate can be grater than the linear growth rate.

[*Omura et al., JGR, in press*]

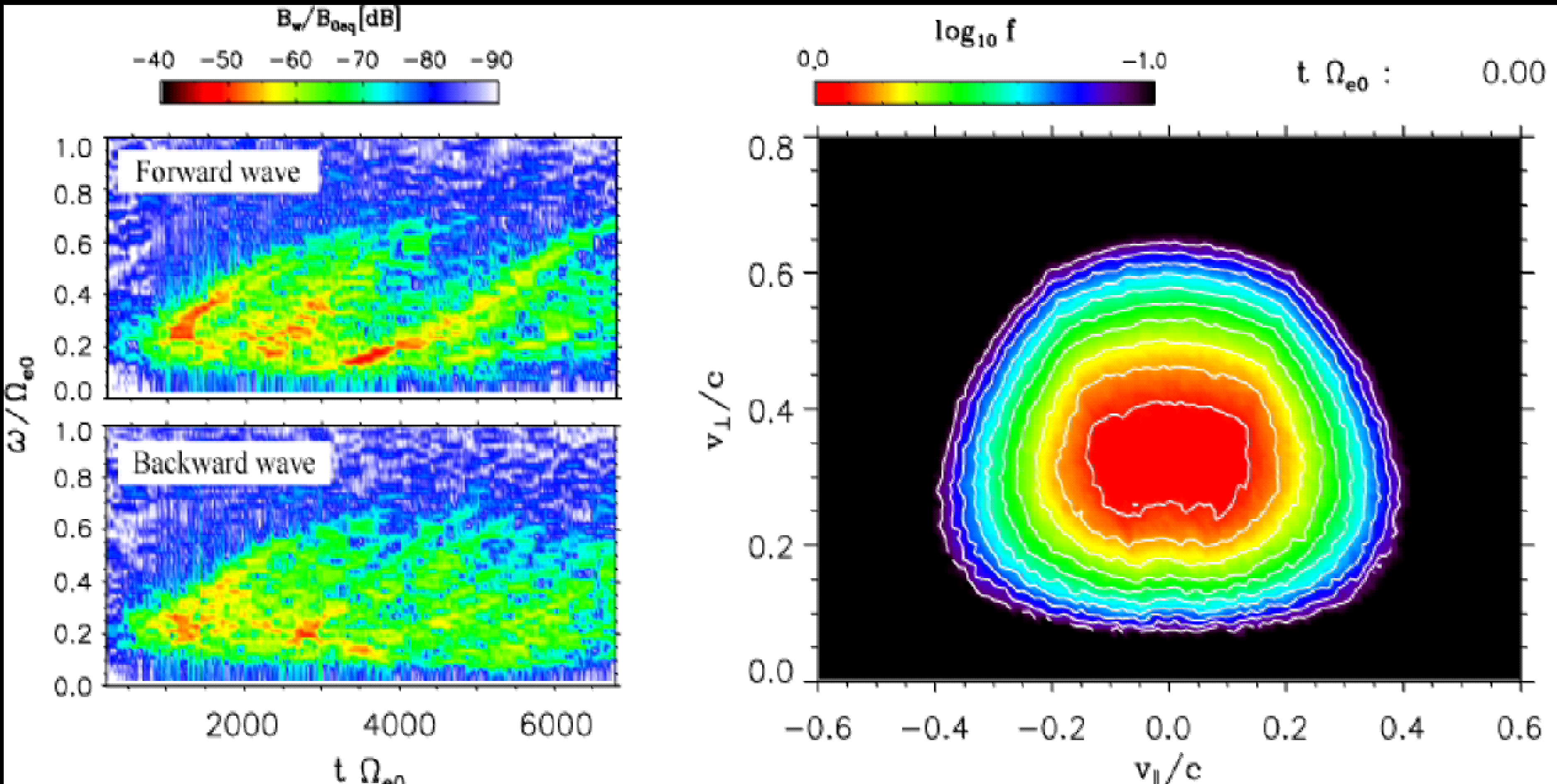








[*Hikishima et al., submitted to JGR*]

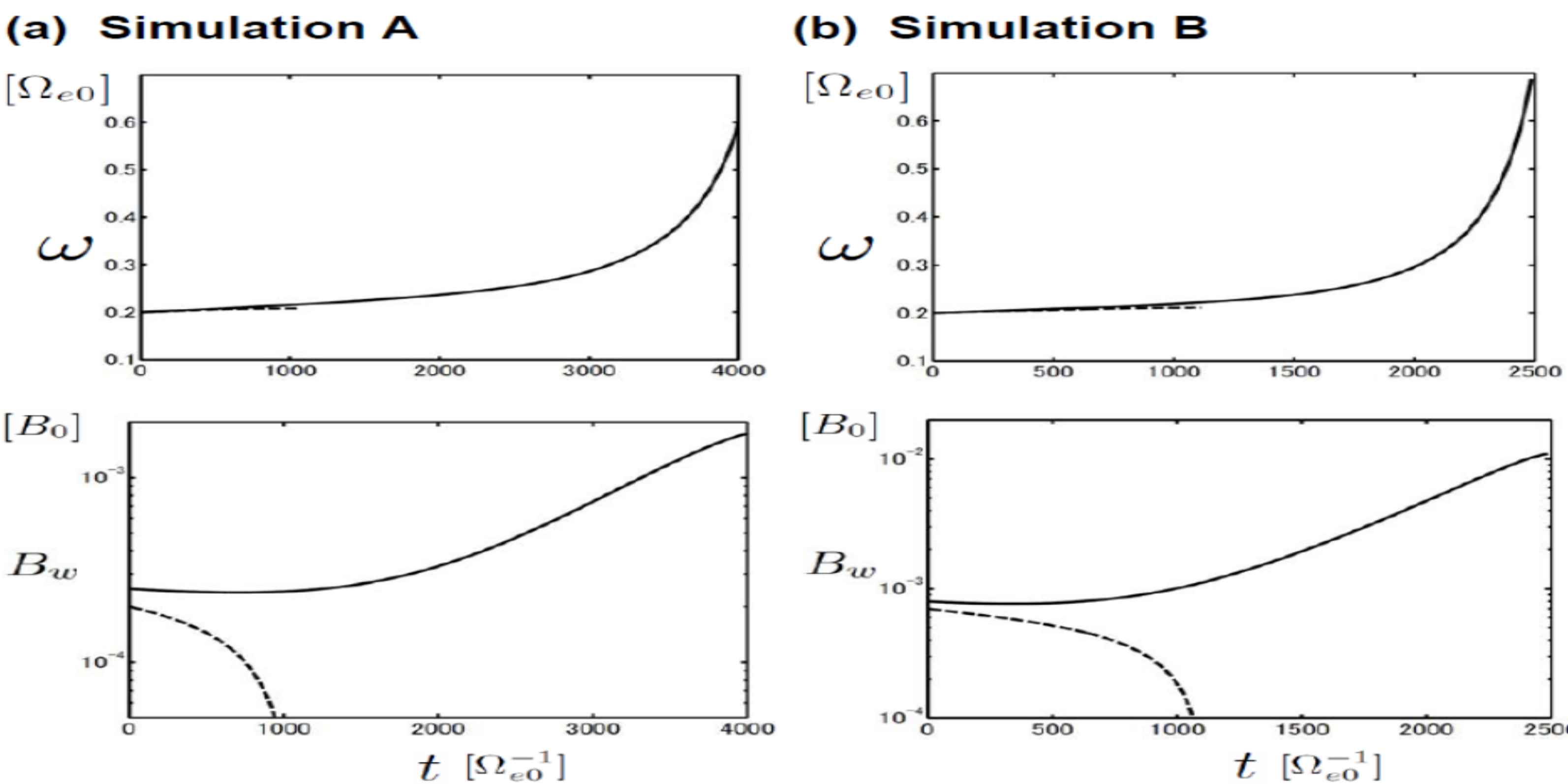


B_w/B_{0eq} [dB]
−40 −50 −60 −70 −80 −90
Forward wave
Backward wave
ω/Ω_{e0}
$t\ \Omega_{e0}$
$\log_{10} f$
0.0
−1.0
$t\ \Omega_{e0}$: 0.00
$v_\perp/c$
$v_\parallel/c$

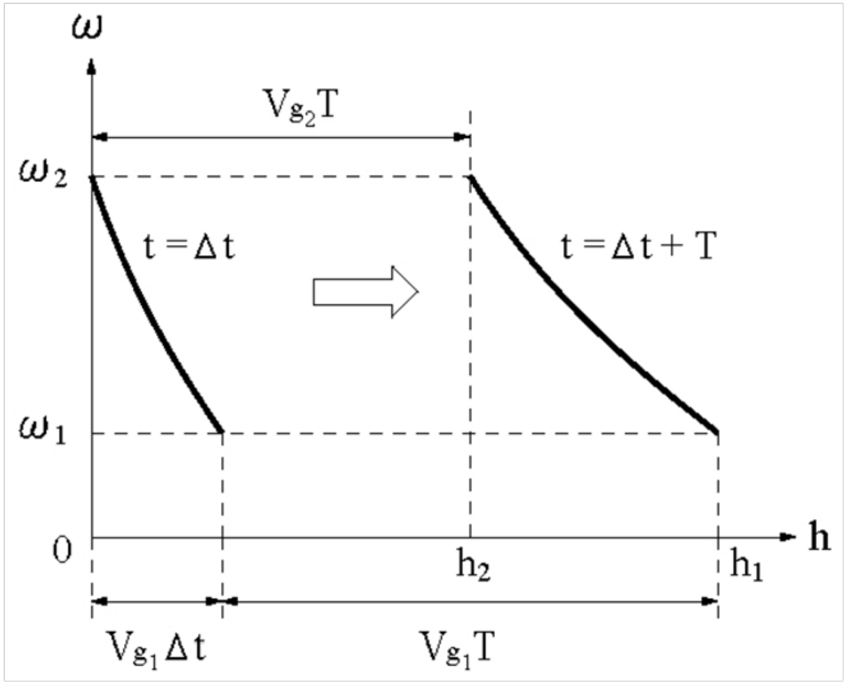
Chorus Equations

$$\frac{\partial \tilde{\Omega}_w}{\partial \tilde{t}} = \tilde{V}_g \left[\frac{Q\tilde{\omega}_{ph}^2}{2\tilde{U}_{t\parallel}} \left(\frac{\tilde{V}_{\perp 0}\delta}{\pi\gamma} \right)^{3/2} \left(\frac{\xi\tilde{\Omega}_w}{\tilde{\omega}} \right)^{1/2} \exp\left(-\frac{\gamma^2 \tilde{V}_R^2}{2\tilde{U}_{t\parallel}^2} \right) - \frac{5 s_2 \tilde{a}}{s_0 \tilde{\omega}} \right]$$

$$\frac{\partial \tilde{\omega}}{\partial \tilde{t}} = \frac{2 s_0}{5 s_1} \tilde{\omega} \tilde{\Omega}_w$$

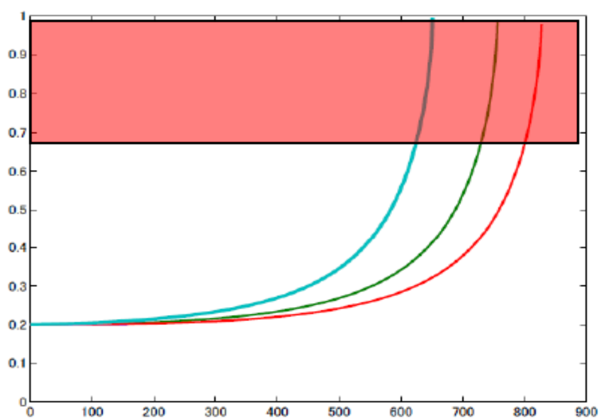


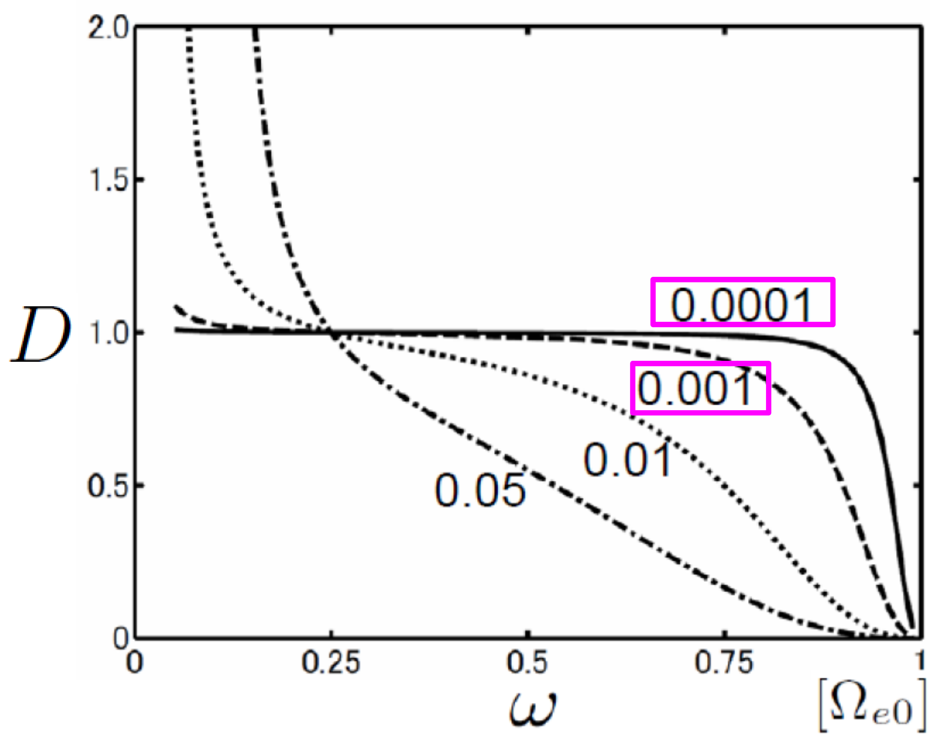
Spatial Variation of Frequency Sweep Rate (Dispersion Effect)



$$\left(\frac{\partial \omega}{\partial t}\right)_{h=h_T} = D\left(\frac{\partial \omega}{\partial t}\right)_{h=0}$$

(b)





$$D = \left[1 - \frac{\delta^3(\Omega_e^2 - 4\omega\Omega_e - 4\xi^2\omega^2)}{4c\xi\omega(\Omega_e - \omega)^2} h_T \left(\frac{\partial \omega}{\partial t}\right)_{h=0}\right]^{-1}$$

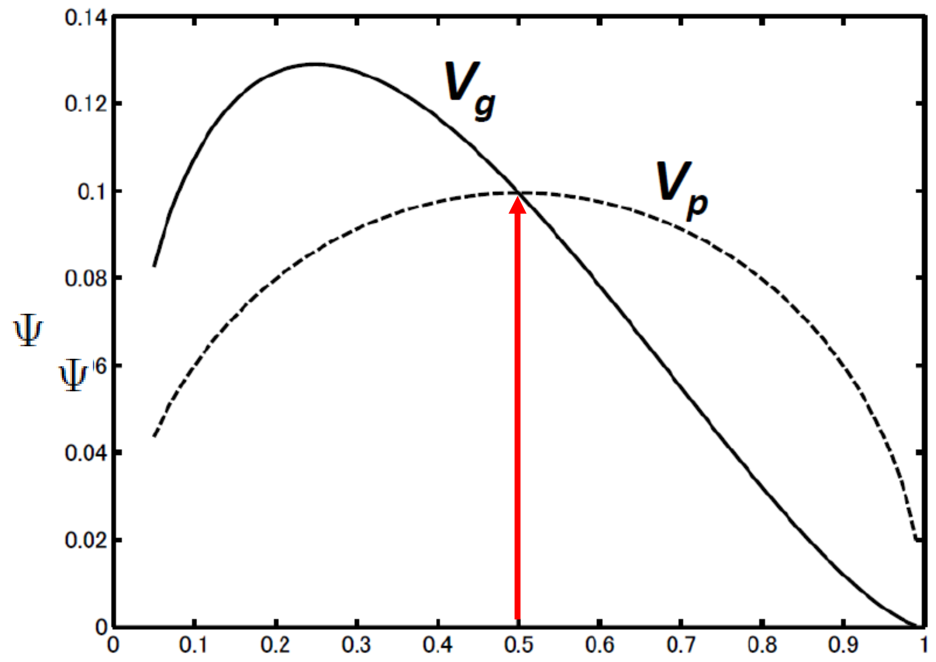
Quasi-parallel Propagation (Oblique) $\sin^2\Psi << 1$

Ψ: Wavenormal Angle

$$E_{w\|} = \frac{\omega \sin\Psi}{\delta^2\Omega_e - \omega} E_w$$

With $\omega = 0.5\Omega_e$

$$V_g = V_p$$



$$\tilde{v}_\| = v_\| - V_p$$

$$\frac{d\tilde{v}_\|}{dt} = -\frac{eE_{w\|}}{\gamma m_0}\sin\phi - \frac{v_\perp^2}{2\Omega_e}\frac{\partial\Omega_e}{\partial h}$$

Nonlinear Resonant Damping in the dipole magnetic field

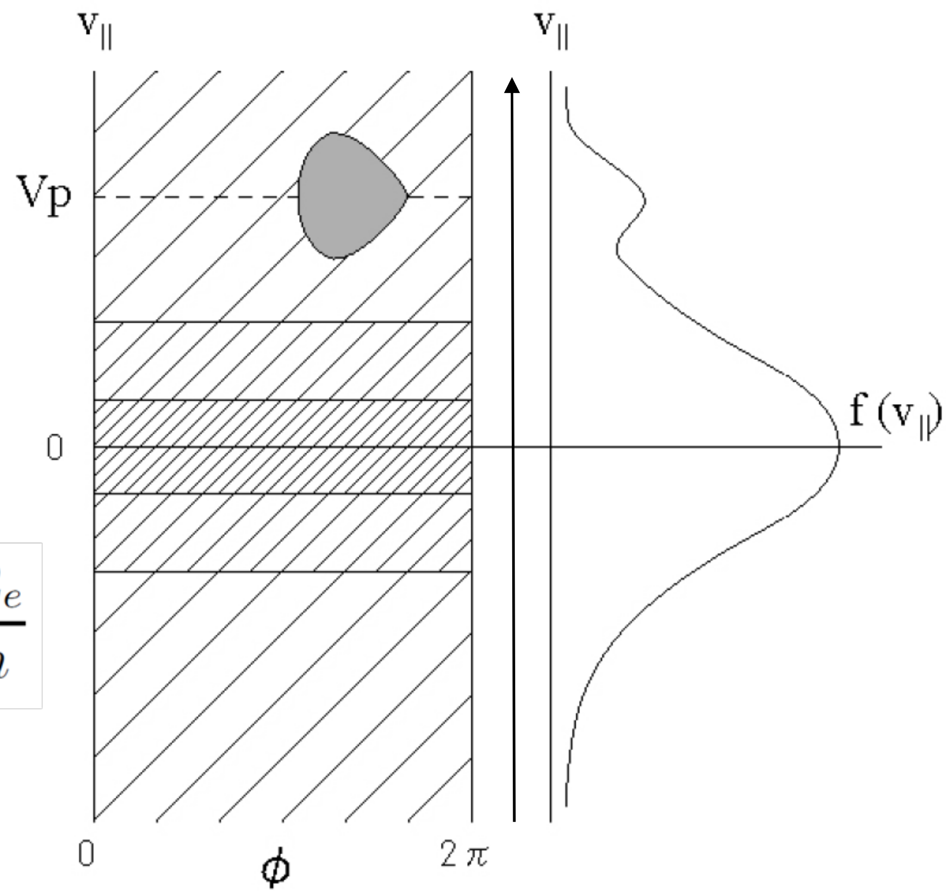
$$\theta = -k(v_\parallel - V_p)$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \theta$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega_{t\parallel}^2(\sin\phi + S_\parallel)$$

$$\omega_{t\parallel}^2 = \frac{ekE_{w\parallel}\delta^2}{\gamma m_0} \qquad S_\parallel = \frac{kv_\perp^2}{2\omega_{t\parallel}^2\Omega_e}\frac{\partial\Omega_e}{\partial h}$$

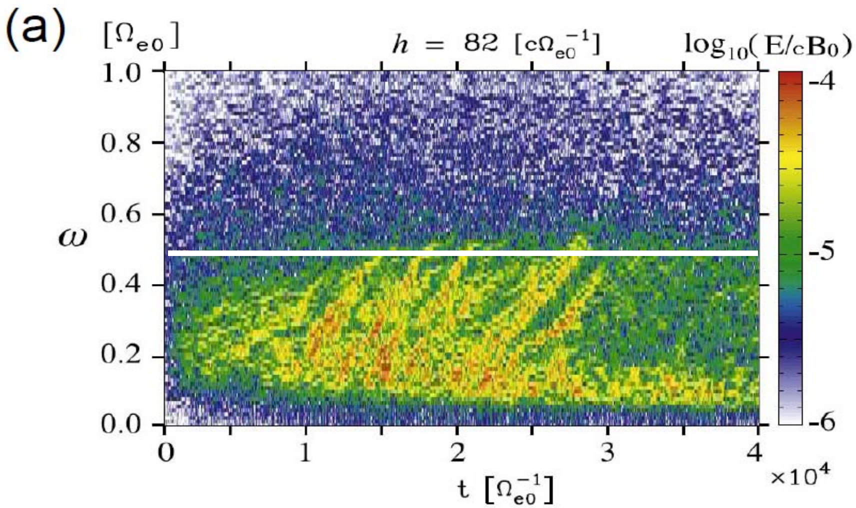
$$S_\parallel > 0 \qquad \sin\phi_c < 0$$

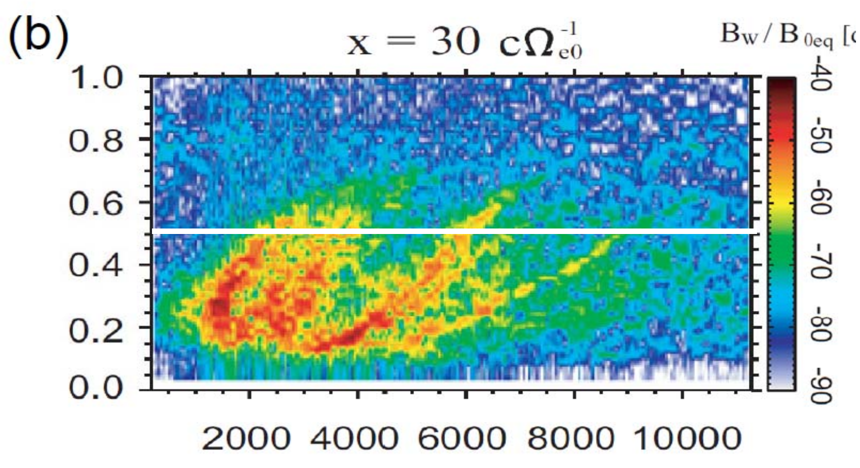


$$J_\parallel E_{w\parallel} = -eE_{w\parallel}\int\int v_\parallel g_t(v_\parallel,\phi)\sin\phi\ \mathrm{d}\phi\mathrm{d}v_\parallel > 0$$

[*Omura et al., JGR, in press*]

Simulations : parallel propagation

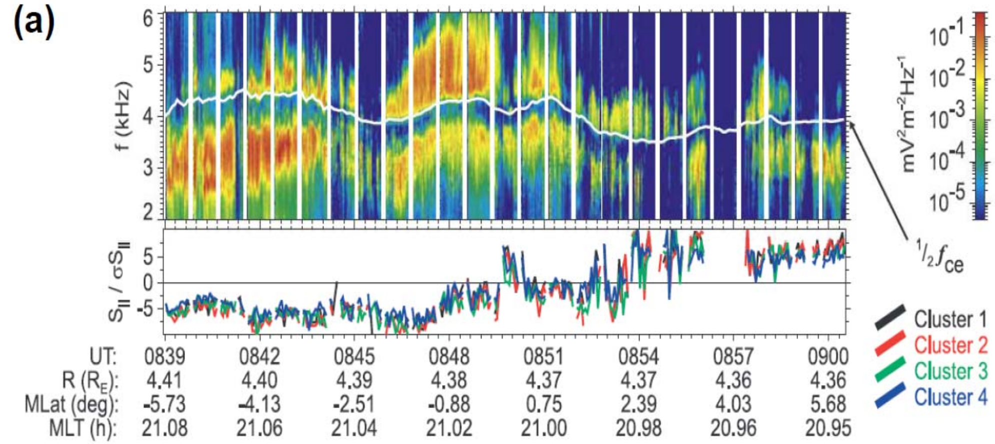


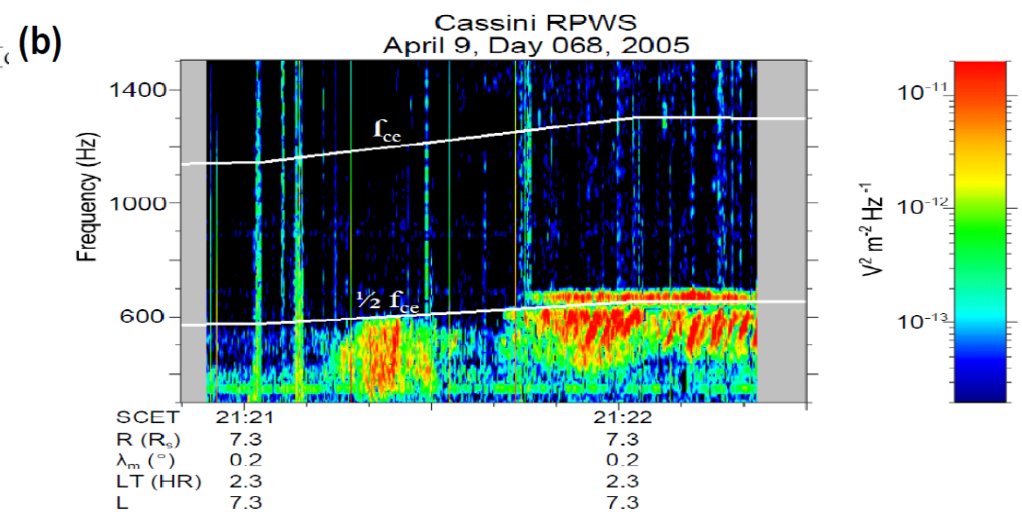


[*Omura et al., JGR, 2008*]

[*Hikishima et al., JGR, 2009*]

Observations: oblique propagation





[*Santolik, et al., JGR, 2003*]

[*Hosphodarsky et al., JGR, 2008*]

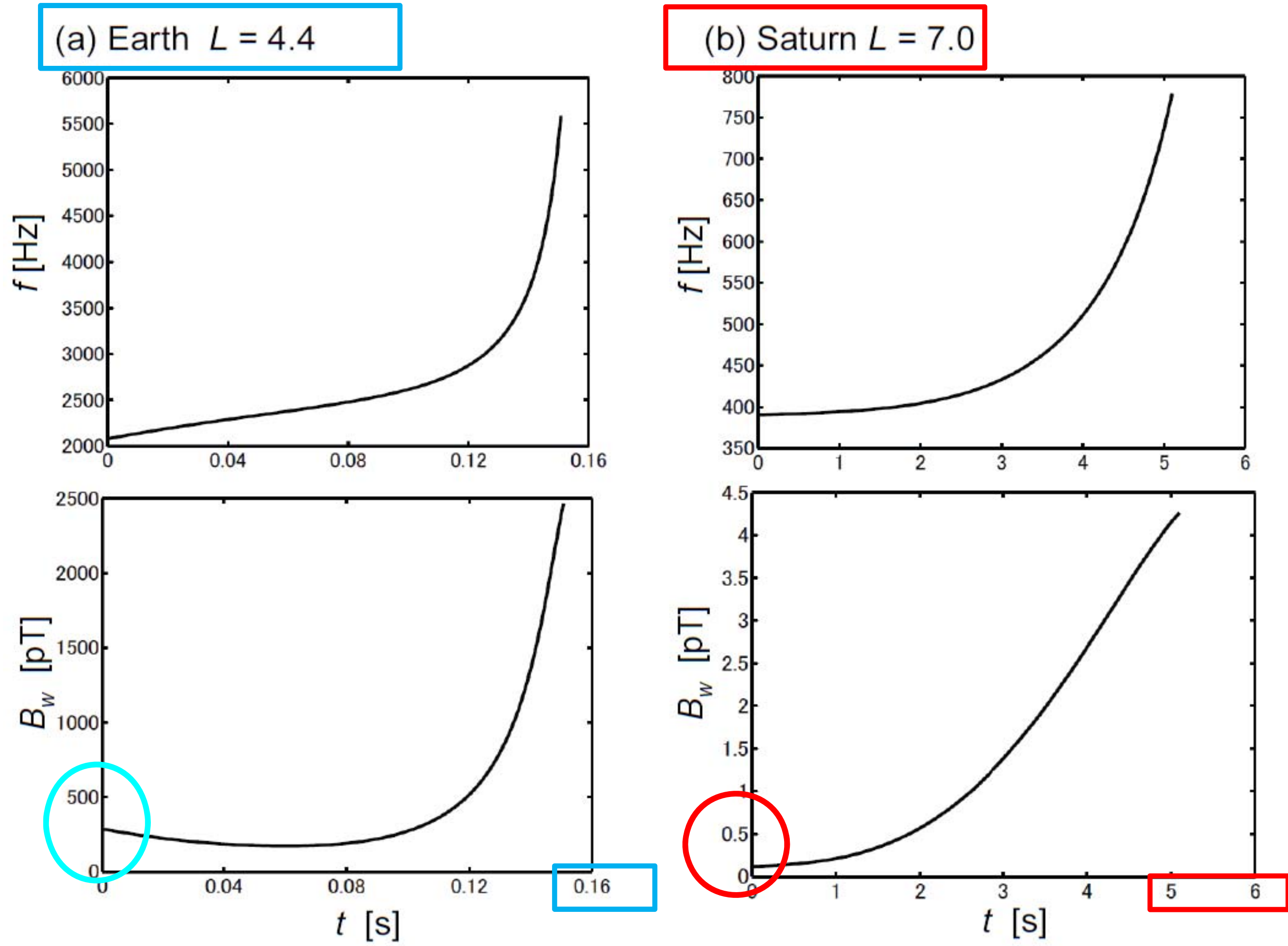
Chorus Equations

$$\frac{\partial \tilde{\Omega}_w}{\partial \tilde{t}} = \tilde{V}_g \left[\frac{Q\tilde{\omega}_{ph}^2}{2\tilde{U}_{t\parallel}} \left(\frac{\tilde{V}_{\perp 0}\delta}{\pi\gamma} \right)^{3/2} \left(\frac{\xi\tilde{\Omega}_w}{\tilde{\omega}} \right)^{1/2} \exp\left(-\frac{\gamma^2 \tilde{V}_R^2}{2\tilde{U}_{t\parallel}^2} \right) - \frac{5s_2\tilde{a}}{s_0\tilde{\omega}} \right]$$

$$\frac{\partial \tilde{\omega}}{\partial \tilde{t}} = \frac{2s_0}{5s_1}\tilde{\omega}\tilde{\Omega}_w$$

Threshold of Wave Amplitude B_w

Time Scale of Chorus T_c



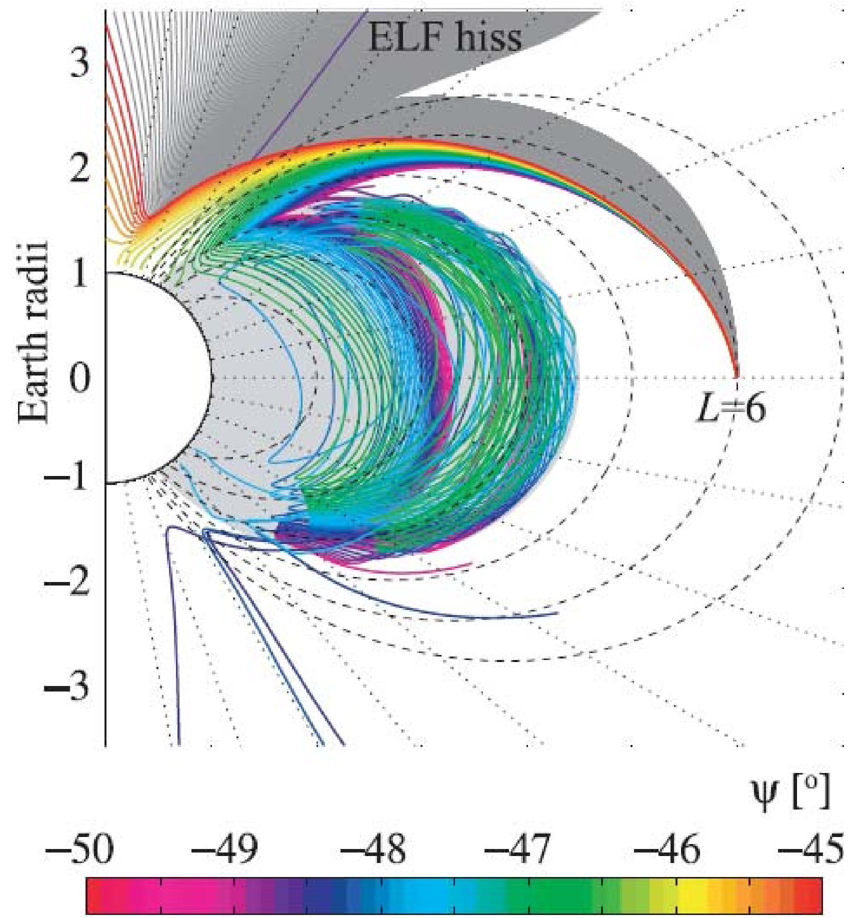
Summary

- The nonlinear wave growth takes place near the equator through formation of an electromagnetic electron hole for a seed rising tone with a wave amplitude above a threshold.
- The self-sustaining mechanism of nonlinear wave growth results in a rising tone limited only by the dispersion effect operating near the gyro-frequency.
- Oblique propagation in a dipole field induces strong nonlinear resonant damping at half the gyro-frequency due to trapping by the parallel wave electric field, resulting in the lower and upper band chorus emissions.
- Resonant electrons trapped by chorus emissions are accelerated to higher pitch angles, forming a pancake distribution. Some of them are effectively accelerated to relativistic energy by RTA and URA.

Origin of Plasmaspheric Hiss: Chorus?

Yes, but not by direct conversion from chorus emissions propagating to high latitudes. Chorus is generated in parallel propagation at the equator, not at wave normal angles 45 – 50 degrees.

Alternative interpretation: Chorus emissions effectively precipitate a wide range of energetic electrons into the auroral regions. The precipitated electrons can generate the lower-hybrid waves through Landau resonance. The lower-hybrid waves propagate into the plasmasphere as whistler-mode HISS emissions.



[*Bortnik et al., Science, 2009*]

A study suggested by Dennis 18 years ago.

Theory and Simulations on Chorus Emissions

1. Y. Omura and D. Summers, Dynamics of high-energy electrons interacting with whistler mode chorus emissions in the magnetosphere, *JGR*, Vol. 111, A09222, 2006.
2. Y. Katoh and Y. Omura, Computer simulation of chorus wave generation in the Earth's inner magnetosphere,*GRL*. Vol. 34, L03102, 2007.
3. Y. Omura, N. Furuya, D. Summers, Relativistic turning acceleration of resonant electrons by coherent whistler-mode waves in a dipole magnetic field, Journal Geophysical Research, Vol. 112, A06236, 2007.
4. Y. Katoh and Y. Omura, Relativistic particle acceleration in the process of whistler-mode chorus wave generation, *GRL*, 34, L13102, 2007.
5. D. Summers and Y. Omura, Ultra-relativistic acceleration of electrons in planetary magnetospheres, *GRL*., 34, L24205, 2007.
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