Nonlinear Whistler Waves in Earth’s Radiation Belts: THEMIS Observations

Chris Cully
Swedish Institute of Space Physics, Uppsala, Sweden
chris@irfu.se
Acknowledgements

Themis is the result of the work of many. In particular I’d like to acknowledge:

- Vassilis Angelopoulos
- Electric Field Instrument:
  - John Bonnell, Forrest Mozer, Robert Ergun
- Particle Instruments:
  - Jim McFadden, Davin Larson
- Search Coil Magnetometer:
  - Alain Roux, Olivier LeContel
- Fluxgate Magnetometer:
  - Karl-Heinz Glassmeier
Outline

- Early observation:
  - Larger E amplitudes than expected

- Event study
  - Where quasilinear theory works
  - Where nonlinear theory is required
    - Chorus sweep
      - Omura’s chorus generation mechanism
      - Experimental test
  - Cyclotron harmonic emissions
Background

- Wave-particle interaction is a major controlling process in the Earth’s radiation belts
  - Acceleration
  - Loss (scattering into loss cone)
- Whistler waves
  - Largely at dawn
  - Equatorial generation
  - Bursts of rising tones (chorus)

Li et al., GRL, 2009
Themis

- 5 satellites in equatorial orbits
- Plasma instruments
  - Low energy (<30 keV) + high energy (<6 MeV) ions and electrons
  - Electric and magnetic fields
Unexpected amplitudes

- Early Themis result: Whistler electric fields larger than expected
- Heavy-tailed distribution
- >100 mV/m
  - Deterministic dynamics rather than stochastic
- Occur in large (hours of MLT) and persistent (days) region
- Agrees with other recent data (eg. STEREO)

Cully et al, GRL, 2008
Many of the largest amplitude events are obliquely propagating
- Even near the equator
- Lack the characteristic chorus chirp

Propagation effect?

CASE STUDY
Outline

- Early result:
  - Larger E amplitudes than expected

- Event study
  - Where quasilinear theory works
  - Where nonlinear theory is required
    - Chorus sweep
      - Omura’s chorus generation mechanism
        - Experimental test
  - Cyclotron harmonic emissions
Thermis-D, L~5 R_E, MLAT~2°

- Bursts of whistler activity up to 0.5 f_ce
  - Gap at 0.5 f_ce (common)
  - Parallel propagation
- Strong cyclotron harmonics
Resonant ellipses

- Electrons moving counter to the wave motion at

\[ V_R = \frac{\omega}{k} \left(1 - \frac{\omega_{ce}}{\gamma \omega}\right) \]

are cyclotron-resonant with the wave

- Ellipses at \( f = 0.2 \, f_{ce} \) and \( f = 0.5 \, f_{ce} \) demarcate interaction boundaries
**Diffusion curves**

- Given broad spectrum of waves:
  - In frame moving with wave phase velocity, (relativistic) energy of resonant particle is conserved
  - As energy changes, particle resonates with different frequencies
- Result: diffusion curves shown
  - Hot, relativistic curves shown; cold and non-relativistic very similar

Summers, Thorne, Xiao, JGR, 1998
Horne and Thorne, GRL, 2003
Gendrin and Roux, JGR, 1980
Experimental test

- Diffusion surface = phase space density iso-surface
  - Valid between the resonant ellipses for the observed wave spectrum
  - Marginal stability criterion
Diffusion curves: result

- Very good agreement with observed distribution function
- Diffusion curve = f iso-surface in interaction region
- Isotropic core
  - Resonant frequency above $f_{ce}/2$ (no wave power available)
- Sometimes see shoulder at $f_{ce}/2$
Nonlinear questions

Linear theory predicts wave growth in a broad band up to $f_{ce}$. So:

1. Why does the chorus element chirp?
2. Why is the wave absorbed at $f_{ce}/2$?
Growth by nonlinear trapping

- Omura, Katoh and Summers, JGR, 2008
- Wave trapping results in electromagnetic phase space hole
- For specific conditions, resonant currents cause wave growth
  - Rising frequency
  - Quantified by inhomogeneity ratio $S$
Omura et al.’s prediction

- Fastest-growing wave is the one that maximizes the resonant current
- Inhomogeneity ratio $S \sim -0.4$

\[
S = -\frac{m_0}{kv_\perp eB_w} \delta^2 \left\{ \gamma \left( 1 - \frac{V_R}{V_g} \right)^2 \frac{\partial \omega}{\partial t} + \left[ \frac{k\lambda v^2}{2\Omega_e} - \left( 1 + \frac{\delta^2 \Omega_e - \gamma \omega}{2\Omega_e - \omega} \right)V_R \right] \frac{\partial \Omega_e}{\partial h} \right\}
\]

\[
\delta^2 = 1 - \frac{\omega^2}{c^2 k^2}
\]

Linear relation (if everything else constant and 1st term dominant)
Making it testable

Need to eliminate some variables:

\[ B_w = F\left( S, \frac{\partial \omega}{\partial t}, k, \omega, V_g, \Omega_e, \frac{\partial \Omega_e}{\partial h}, v_\perp, v_\parallel, V_R \right) \]
Making it testable

- Need to eliminate some variables:
  \[ B_w = F \left( S, \frac{\partial \omega}{\partial t}, k, \omega, V_e, \Omega, \frac{\partial \Omega_e}{\partial h}, v_\perp, v_\parallel, V_R \right) \]

- Growth in range \(-0.6 < S < -0.2\)
- Dispersion relation (introduces \(\omega_{pe}\))
- Growth in range \(0.2 < f/f_{ce} < 0.5\)
- Resonance condition

\[ B_w = G \left( \frac{\partial \omega}{\partial t}, \Omega_e, \omega_{pe}, \frac{\partial \Omega_e}{\partial h}, v_\perp, v_\parallel \right) \]

Plasma parameters
Velocities (???)
Maximization in velocity space

Fastest-growing wave is the one that maximizes the resonant current

Maximize \( n' = \int_C f d^3v \) over domain bounded by:
- \(-0.6 < S < -0.2\)
- \(0.2 < f/f_{ce} < 0.5\)
Maximization in velocity space

\[ B_w = 1 \text{nT}, \quad \frac{\partial \omega}{\partial t} = 2 \text{ kHz/s} \]

\[ B_w = 1 \text{nT}, \quad \frac{\partial \omega}{\partial t} = 0.5 \text{ kHz/s} \]

Integration domain depends on \( B_w \) and \( \frac{\partial \omega}{\partial t} \) through \( S \)
Testable result

- By integrating over the observed particle distributions, arrive at

\[ n' = F\left(\frac{\partial \omega}{\partial t}, B_w\right) \]

- Maximizing n’ yields a very specific testable prediction:
  - Relationship between \( B_w \) and \( \partial f/\partial t \)
  - NO adjustable parameters!!
Testing it...

- Need $B_w$ and $\partial f/\partial t$.
  Either:
  - Select multiple chorus elements, use average $B_w$ and $\partial f/\partial t$ for each element
    - Amplitude and sweep rates vary throughout sweep
  - Select one element
    - Use zero crossings to get “instantaneous” $B_w$ and frequency
The result

- Observed chorus elements agree in detail with the theoretical prediction
  - Lines should follow maximum in n’ (red)
**Cyclotron harmonics**

- **Electrostatic wave between** \( f_{ce} \) and \( 3/2 f_{ce} \)
  - Linear polarization
  - Perpendicular to \( B_0 \)
- **Amplitude** \( \sim 100 \text{ mV/m} \)
  - Greater than the whistlers
  - Greater than expected
ECH: Scattering rates

- Pitch angle diffusion rates for Electron Cyclotron Harmonic waves (ECH) calculated by Horne and Thorne.

- For 100 mV/m:
  - Strong scattering limit over several tens of degrees for \(~\text{keV}\) particles
    - Loss cone filled on each bounce

Horne and Thorne, JGR, 2000
**ECH: effect**

- ECH waves remove “extended” loss cone
- Whistler waves fill it back in
Conclusions

- **Observation:** maximum electric field amplitudes larger than previously seen (>100 mV/m)
- **Quasilinear theory:**
  - Good fit to diffusion curves
  - Featureless spectrum up to $f_{ce}$ not observed
- **Nonlinear theory:**
  - Amplitude vs. sweep rate (Omura)
    - Specific testable prediction verified
- **Open (?):**
  - What absorbs the waves at $f_{ce}/2$?
  - What causes the high-amplitude unstructured oblique whistlers?
  - Interaction between Electron Cyclotron Harmonic waves and chorus whistlers?
EXTRA SLIDES
Dispersion curves

- 2-component plasma: cold isotropic core + hot anisotropic
- Good match to dispersion curve
  - Test of instrument performance

- Thermal effects not important below $0.5 \, f_{ce}$
  - Wave power confined to range $0.2-0.5 \, f_{ce}$
STATISTICAL RESULTS
Electric field amplitude distribution

- Probability distribution function is "heavy"-tailed, with substantial probability out to tens of mV/m.
  - 4-s averaged data: values up to tens of mV/m
    - Instantaneous values much larger, but limited statistics
Themis: nonlinear whistlers

Spatial/temporal distribution

- Multiple satellites yield information on spatial and temporal distribution
- Enhanced activity can last for days
- Confined to a few hours of local time
Statistics: conclusions

- Whistler electric field amplitudes have a heavier tail than previously reported
  - > 100 mV/m bursts
  - High-amplitude regions are:
    - Spatially extended (several $R_E$
    - Temporally persistend (several days)
  - Relevant for acceleration/loss processes

- Many of the highest-amplitude events are obliquely propagating
  - Even at magnetic latitudes < few degrees
  - Lack the characteristic chorus chirp