Evolution of Whistler Turbulence in the Magnetosphere

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Introduction

• Whistler waves are ubiquitous in the space plasma environment
  – Transports momentum and energy
  – Signatures of important space plasma processes; e.g., reconnection, lightning, plasmaspheric hiss, etc.

• Whistlers are also injected into the near-Earth space by man-made VLF transmitters

• Both natural and man-made whistlers affect the space plasma environment
  – Hence, important to “space weather”

• Need to understand the evolution of whistler turbulence in the space plasma environment accurately
  – Numerical simulations are necessary
  – Prerequisite for accurate simulation is knowledge of whistler wave properties
Linear Whistler Wave Properties: Homogeneous Plasma

- Whistler dispersion relation in cold plasma for $\Omega_e > \omega > \Omega_i$ obtained by:

$$\vec{V} \cdot \vec{j} / e = \vec{V} \cdot n_0 \left( \frac{c\vec{E} \times \vec{B}}{B_0} + i\omega \left( \frac{\omega^2_{lH}}{\Omega_e^2} - 1 \right) \frac{c\vec{E}_\perp}{B_0} + i\frac{cE_zB_0}{m\omega} \right) = 0$$

Relative drift ($v_j - v_e$)

- From Maxwell equations with Coulomb gauge it can be shown that:

$$E_x = -ik_x\varphi \frac{(k_z^2 + k_\perp^2)}{k_\perp^2}, \quad E_y = E_x \frac{i\omega}{\Omega_e k^2}, \quad E_z = -ik_z\varphi \frac{k^2}{(1+k^2)}$$

Electrostatic for $k_\perp > k_z$

Electromagnetic

Electrostatic for $k > 1$

- For $\omega_{pe} > \Omega_e$ the dispersion relation is,

$$\omega^2 = \left( \frac{k_\parallel^2}{(1+k_\perp^2)} + \frac{m_e}{m_i} \right) \frac{k^2\Omega_e^2}{1+k^2}$$

$$\vec{k}^2 = \vec{k}_\perp^2 + \vec{k}_\parallel^2$$

$$\vec{k} = kc / \omega_{pe}$$

- Frequency in limiting cases:

- LH limit: $k_\perp >> 1, \quad k_\perp >> k_\parallel, \quad k_\parallel / k_\perp \ll \sqrt{m_e / m_i} \rightarrow \omega^2 = \Omega_e \Omega_i$

- Whistler limit: $k << 1, \quad \vec{k}_\parallel >> m_e / m_i \rightarrow \omega^2 = \vec{k}_\parallel^2 \vec{k}_\perp^2 \Omega_e^2$

- Magnetoacoustic limit: $k << 1, \quad \vec{k}_\parallel << m_e / m_i \rightarrow \omega^2 = k^2 V_A^2$

Whistlers and Lower Hybrid are the same wave at different propagation angle
Linear Whistler Wave Properties: Inhomogeneous Plasma

• With density inhomogeneity the (E X B) drift gives a large term
  - Inhomogeneity could be external or self-consistent

\[
\vec{\nabla} \cdot \vec{j} / e = \vec{\nabla} \cdot \left( n_0 + \delta n(x, y) \right) \left( \frac{c \vec{E} \times \vec{b}}{B_0} + i \frac{\omega}{2} \left( 1 - \frac{\omega_{LH}^2}{\omega^2} \right) \frac{c \vec{E}_i}{B_0} + i \frac{eE_z \vec{b}_0}{m \omega} \right) = 0
\]

\[-n_0 \frac{\omega}{\Omega_e} \left( \frac{1+k^2}{k^2} - \frac{\omega_{LH}^2}{\omega^2} - \frac{\Omega_e^2}{\omega^2} \frac{k_z^2}{1+k^2} \right) \frac{c k_y E_x}{B_0} - \frac{c}{B_0} (\vec{E} \times \vec{\nabla} \delta n) \cdot \vec{b}_0 = 0\]

• Density fluctuations introduces new solution (sort of drift waves)

\[
\frac{\omega}{\Omega_e} = -\frac{\nabla_y \delta n}{2 n_0 k_x} \frac{k^2}{1+k^2} \left[ \left( \frac{\nabla_y \delta n}{2 n_0 k_x} \frac{k^2}{1+k^2} \right)^2 + \frac{m}{M(1+k^2)} + \frac{\bar{k}_y^2 \bar{k}_z^2}{(1+k^2)(1+k^2)} \right]^{1/2}
\]

\[
\omega \rightarrow -\frac{\Omega_e \nabla_y \delta n}{n_0 k_x} \frac{k^2}{1+k^2}
\]

• Nonlinear pondermotive force along B_0 can lead to second order density fluctuations

\[
\delta n(x, y) \propto k_\parallel \equiv \partial / \partial z
\]

Small $\delta n/n (> \text{max}[(m/M)^{1/2},k_z/k])$ leads to big change in whistler mode character
Nonlinear Whistler Properties

• Nonlinear quasi-electrostatic Lower Hybrid (LH) waves extensively studied
  – Porkolab, 1974
  – Hasegawa and Chen, 1975
  – Shapiro, Shevchenko, Papadopoulos and Sagdeev, 1977-1993

• Simulations based on EMHD equation (no density perturbation)

• 2D PIC simulations
  \[
  (\vec{k} \times \vec{\nabla} \delta n) \cdot \vec{B}_0 = 0
  \]
Plasma Weak and Structural Turbulence

- Induced whistler wave scattering while radiating low frequency wave
  - Waves energy and momentum are conserved
    \[ \omega_1 = \omega_2 + \omega_3, \quad \vec{k}_1 = \vec{k}_2 + \vec{k}_3 \]
    \[ \omega_1 \gg \omega_3 \quad (= LH / MS, \vec{k}c_s, l\Omega_i) \]

- Induced wave scattering by plasma particles
  - Wave momentum need not be conserved if particles are magnetized (principal momentum conserved)

\[ \Delta \vec{P}_\perp = (ne/c) \Delta \vec{A}_\perp + \Delta n\vec{v}_\perp = \left( m \nu_e (\Delta \vec{r}_\perp / \rho_e) + \sum_k (\Delta \vec{k}_\perp) N_k \right) = 0 \]

Resonance Condition \[ \nu || e = \frac{\omega_1 - \omega_2}{k_1|| - k_2||} \]

W-P interactions are less restrictive than W-W interactions
Waves Spectra Nonlinear Evolution

• Calculate nonlinear conversion of W/LH waves ($E_1$) into LH/W waves ($E_2$)
  - Maxwell Equation,
  - Fluid equations for ion
  - Vlasov equation for electrons in drift approximation

• Whistler waves in a medium with slowly varying density perturbation induced by beat waves ($\omega_1 - \omega_2$):

$$\nabla \cdot \vec{j} = n_0 \frac{\omega}{\Omega_e} \left( 1 + \frac{k^2}{k_z^2} - \frac{\omega_{iH}^2}{\omega^2} - \frac{\Omega_e^2}{\omega^2} \frac{k_z^2}{1 + k_z^2} \right) c \nabla \cdot \vec{E}_{\perp}^{(1)} + \frac{c}{B_0} (\vec{E}_{\perp}^{(1)} \times \nabla \delta n_e^{(2)}) \cdot \vec{b}_0 = 0$$

• 2nd order density perturbation due to pondermotive force along $B_0$.
  - Maxwell electrons and unmagnetized ions

$$\frac{\delta n_e^{(2)}}{n_0} = \left( - \frac{c}{B_0} (\vec{E}_{k_2} \times \vec{k}_1) \frac{k_{1\perp} e \varphi_{k1}}{\omega_{k1}(1 + k_{1\perp}^2) \nu_e} (1 + \zeta(Z(\zeta))) \right), \quad \zeta = \frac{\omega_{k1} - \omega_{k2}}{(k_{1z} - k_{2z}) \nu_{ie}}$$

- Subsonic ion condition: $(\omega_{k1} - \omega_{k2})^2 < (k_{1\perp} - k_{2\perp})^2 c_s^2$
Nonlinear Scattering: Short Wavelength

- For narrow frequency band ($\delta\omega < \gamma_{NL}$) Re $Z$ leads to modulation instability, NLS equation and collapse of localized LH 3D wave packets (if $T_e >> T_i$)

$$i \frac{\partial E_{k_2}}{\partial t} \sim -E_{k_2} \omega_{NL} \frac{M}{m} \frac{W_{k_2}}{n_0 T_e} + E_{k_2} (\omega - \omega_{NL})$$

$$W_k \equiv \frac{\omega_{pe}^2 |E_k|^2}{\Omega_e^2 8\pi}$$

- For broad frequency band turbulence Im $Z$ leads to nonlinear scattering by plasma electrons
  - Short wavelength electrostatic case discussed by Hasegawa and Chen, 1975

$$\gamma_{NL\rightarrow LH} \equiv \frac{\partial \ln W_{k_2}}{\partial t} = \omega_{NL} \frac{M}{m} \sum_{k_1} \frac{(\vec{k}_1 \times \vec{k}_2)^2}{k_{1\perp}^2 k_{2\perp}^2} \zeta \text{Im} Z(\zeta) \frac{W_{k_1}}{n_0 T_e}$$

- Frequency decreases while wave scatters

$$\Delta\omega \sim \min \left\{ k_{1z} - k_{2z} | v_e, | k_{1\perp} - k_{2\perp} | c_s \right\}$$

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Nonlinear Scattering: Long Wavelength Generalization

- Generalization of Hasegawa and Chen
  
  - Long wavelength electromagnetic regime

  \[ \gamma_{NL} = \frac{dN_{k_2}}{N_{k_2} dt} \sim \Omega_z^2 \frac{k_{2\perp}^2}{1 + k_{2\perp}^2} \sum_{k_1} \frac{(k_1 \times k_2)_z^2}{k_{1\perp}^2 k_{2\perp}^2} \frac{k_{1\perp}^2}{1 + k_{1\perp}^2} \zeta \text{Im} Z(\zeta) \frac{N_{k_1}}{n_0 T_e} \]

  \[ N_k = W_k / \omega_k \]

  - Scattering rate decreases frequency slightly and conserves “plasmons” \( N \)

  \[ \Delta \omega / \omega_{LH} < |k_{1\perp} - k_{2\perp}| \beta_e^{1/2} \]

  - Wave-particle resonance can be easily met for any combinations of \((k_{||}, k_{\perp})\) in a thin slot in which \( \omega \sim \text{const.} \)

  \[ \omega^2 = \left( \frac{k_{||}^2}{1 + k_{||}^2} + \frac{m_e}{m_i} \right) \frac{k^2 \Omega_e^2}{1 + k^2} \]

  \( \gamma_{NL} \) largest for \( k_1 \perp k_2 \)

Short wavelength can scatter into long wavelength and vice-versa: \( \gamma_{NL} \) largest for \( k_1 \perp k_2 \)
Electromagnetic 2D-3V PIC Simulation

- Simulation box (X-Y) 512 x 256, equals 51.2 and 25.6 electron inertial lengths

- Magnetic field in (X-Z) plane with inclination $b_x = B_x/B_0$

\[ k_{||} = k_x \sin \theta \]

- Simulation parameters

\[ m_i = 100m_e, \ \omega_{pe}^2 / \Omega_e^2 = 5, \ v_{te} = 0.14c, \ \beta_e = 0.1, \ T_e = T_i \]

- Whistlers self-consistently generated by “heavy ring electrons”

\[ n_r / n_e = 0.25; \]

\[ V_r / c = 0.2 \]

\[ m_r / m_e = 3 \& 10 \]
Instability Generation In Simulation

- **Hydro:** Whistlers generated by ring beam for $\Omega_e > \omega > \omega_{LH}$
  
  [Ganguli et al., JGR, 2007]
  
  - Large $k_\perp V_r / \Omega_r > 1$ necessary

  \[
  \omega = l \Omega_r \quad \frac{\gamma}{l \Omega_r} = \frac{1}{2} \left[ n_e m_e dJ^2_i(\sigma_r) \right] \frac{b_e}{\Gamma_i(b_e)} \left( \frac{\Omega_e^2 - l^2 \Omega_r^2}{\Omega_e^2} \right)^2 \frac{k^2}{1 + k^2}
  \]

  - For the simulation parameters and for $l = 1$

    \[
    \sigma_r = k_\perp V_r / \Omega_r = 0.45 \bar{k}_\perp (m_e / m_r)
    \]

    \[
    b_e = (k_\perp \rho_e)^2 / 2 \ll 1 \Rightarrow b_e / \Gamma_1(b_e) \sim 2
    \]

- **Kinetic:** Whistlers generated by temperature anisotropy
  
  [Kennel and Petschek, JGR, 1966]
  
  - Small $k_\perp V_r / \Omega_r < 1$ necessary

  \[
  \omega = \frac{1}{\Omega_r} \quad \frac{\gamma}{\Omega_r} = \frac{\sqrt{\pi}}{\theta^{-1/2} \beta_\perp^{1/2} k^2} \exp \left( - \frac{1}{\theta^{-1} \beta_\perp \kappa^6} \right)
  \]

  \[
  \kappa = k_\parallel c / \omega_{pr} \quad \theta = m_r V_r^2 / 2 \Theta_r \quad \beta_\perp = \frac{4 \pi m_r m_e V_r^2}{B_0^2} \quad \frac{\gamma_{\max}}{\Omega_r} \sim \sqrt{\beta_\perp \theta}
  \]
Simulation Results 1: Evidence Of Wave-Wave Interaction

B field: $\omega_{pet}(0-830)$

Daughter/Mother whistlers

LH/MS

$\Delta \omega$

$m_r = 3m_e \, b_x = 1/2 \, (\theta = 60^0)$

$\omega \sim 0.15\omega_{pet} \sim 3.3\omega_{LH}$

E field: time 470

Whistler mother

Whistler

Whistler $\omega_M = 3.3\omega_{LH}$ scatters radiating LH/MS wave $\omega_D \approx 0.5\omega_{LH}$
Simulation Results 2: Evidence Of Wave – Particle Interaction

\[ m_r = 10m_e, \ b_x = 1/5 \ (\theta = 78^0) \]

Whistler scatters radiating daughter waves \( \Delta \omega/\omega_{LH} < \Delta k_\perp c/\omega_{pe} \beta^{1/2} < 0.2 \).

No third low frequency wave to satisfy 3 wave decay condition.

Daughters/Mother whistlers

\[ 0 < \omega_{pet} < 1000 \]
\[ 0 < \omega_{pet} < 2000 \]
\[ 0 < \omega_{pet} < 3000 \]
\[ 0 < \omega_{pet} < 4000 \]
Simulation Results 3: Evidence Of Large Angle Scattering

\[ m_r = 10m_e, \quad b_x = 1/5 \quad (\theta = 78^0) \]

Whistler radiates daughter waves with large angle rotation for which \( \gamma_{NL} \) is large.
Simulation Results 4:
No Evidence Of Nonlinear Scattering

$m_r = 10m_e, b_x=1 (\theta = 0^\circ)$

No nonlinear scattering on this time scale contrary to the $b_x = 0.2$ case
Simulation Results 5:
Whistlers Born With & Maintain Large $k_\parallel/k_\perp$

$m_r = 10m_e$, $b_x = 1$ ($\theta = 0^0$)

Only whistler with small $k_\perp/k_\parallel$ arise. No nonlinear scattering.
Conclusion

- Electromagnetic PIC simulations show that evolution of whistler turbulence is dominated by nonlinear ponderomotive force

- The ponderomotive force leads to higher (second) order density perturbation

- The density perturbation significantly changes the whistler evolution
  - Extends the instability relaxation time by orders of magnitude
  - Introduces an essentially 3 dimensional character
  - Nonlinear scattering (wave-wave and wave-particle) dominate the nonlinear phase

- Wave-particle interactions convert short wavelength quasi-em waves into long wavelength em waves and vice-versa
  - Large changes in wavelength possible because wave momentum need not be conserved