WHISTLER TRIGGERING ASSOCIATED WITH SINGULAR ELECTRON VELOCITY DISTRIBUTIONS

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OVERVIEW OF THE PROBLEM

- Whistler scattering causes precipitation of electrons out of the radiation belts. Interested in unstable growth and amplification of whistlers in the magnetosphere.

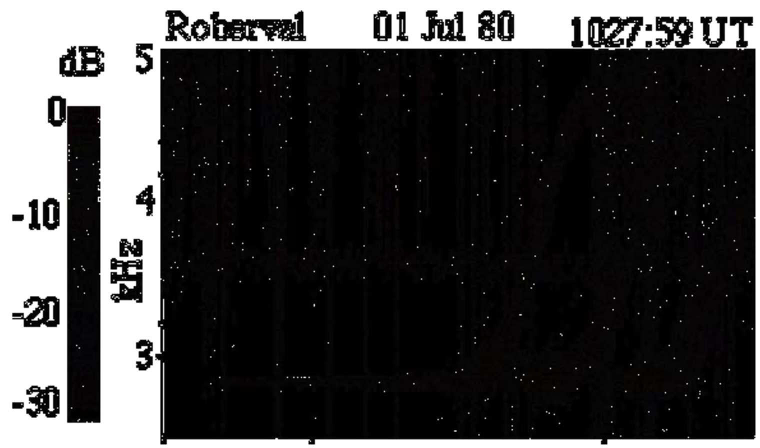
- **Important features:**
  - Instability driver: Strongly nonlinear cyclotron resonant electrons, $\omega - \mathbf{k} \cdot \mathbf{v} - \Omega_e = 0$
  - Powerful coherent whistlers "trigger" *nonlinear* amplification, distinct from the linear instability of broad-band whistler noise
  - The nonlinear evolution depends strongly on slow spatio-temporal variation, e.g.
    - Non-uniformity of the geomagnetic field
    - Finite duration of the trigger pulse
    - Spatial variation of the ambient plasma density

    Infinite homogeneous models are not appropriate.
  - Field-aligned plasma density ducts can lead to essentially 1-D propagation. In the absence of ducts, obliquely propagating whistlers can be important.

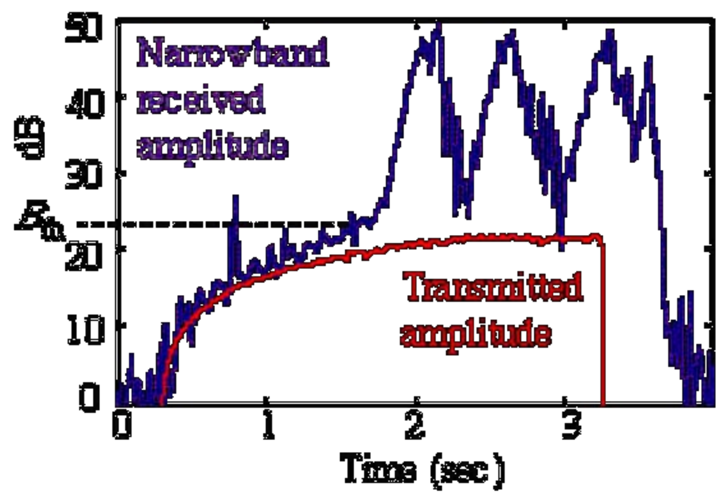
- This talk presents 1-D simulations of coherent parallel-propagating whistlers. We further simplify the problem by considering a ring distribution of fast electrons.

WHISTLER AMPLIFICATION: WHAT WE NEED TO EXPLAIN

- Decades worth of data (Helliwell et al, Siple expts; Inan et al, HAARP) show that powerful coherent whistler pulses launched into the radiation belts can "trigger secondary emissions" up to 30 dB more powerful.



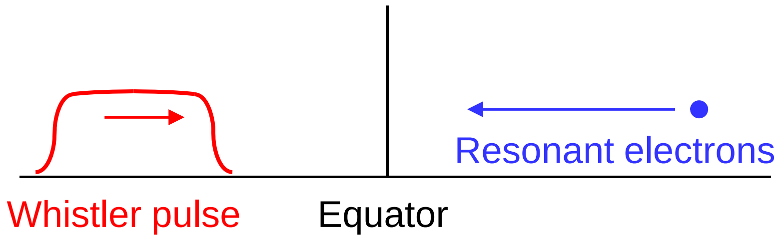
- **Observed extremely nonlinear features:**
  - Frequency lock followed by rise or fall
  - Exponential growth @ 25-250 dB/sec
  - Saturation @ 20 – 35 dB
  - Bursty behavior



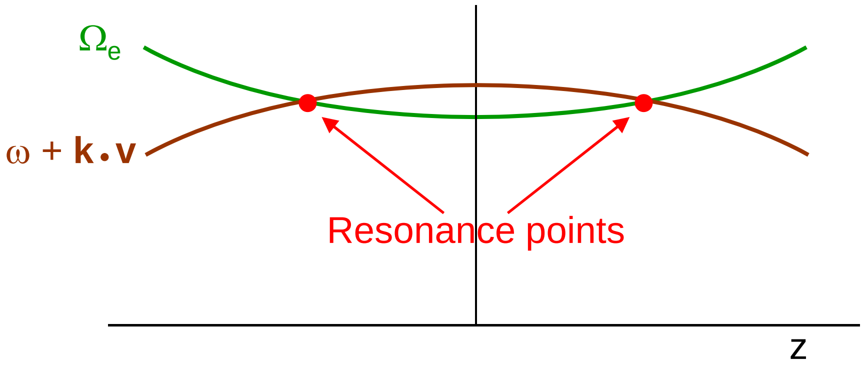
- **Threshold behavior**
  - Power
  - Pulse length
  - Narrow bandwidth

SIMULATIONS: NONLINEAR EVOLUTION OF COHERENT WHISTLER PULSE

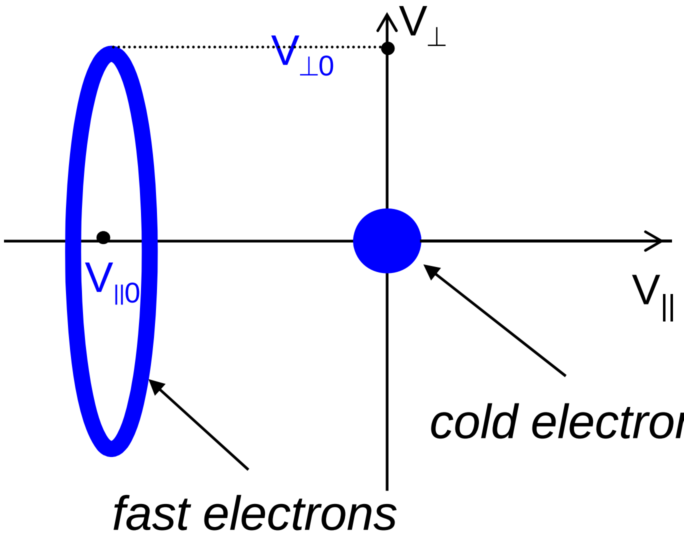
- Hybrid code HEMPIC, particle fast electrons, quasineutral, no displacement current, no fast waves
- Non-uniform geomagnetic field
- Inflow of fresh electrons
- Finite wave packet
- Ring distribution of constants of motion v^2 , $v_\perp^2 / B_0(z)$



Cyclotron resonance condition: $\omega + \mathbf{k}(z) \cdot \mathbf{v}(z) = \Omega_e(z)$

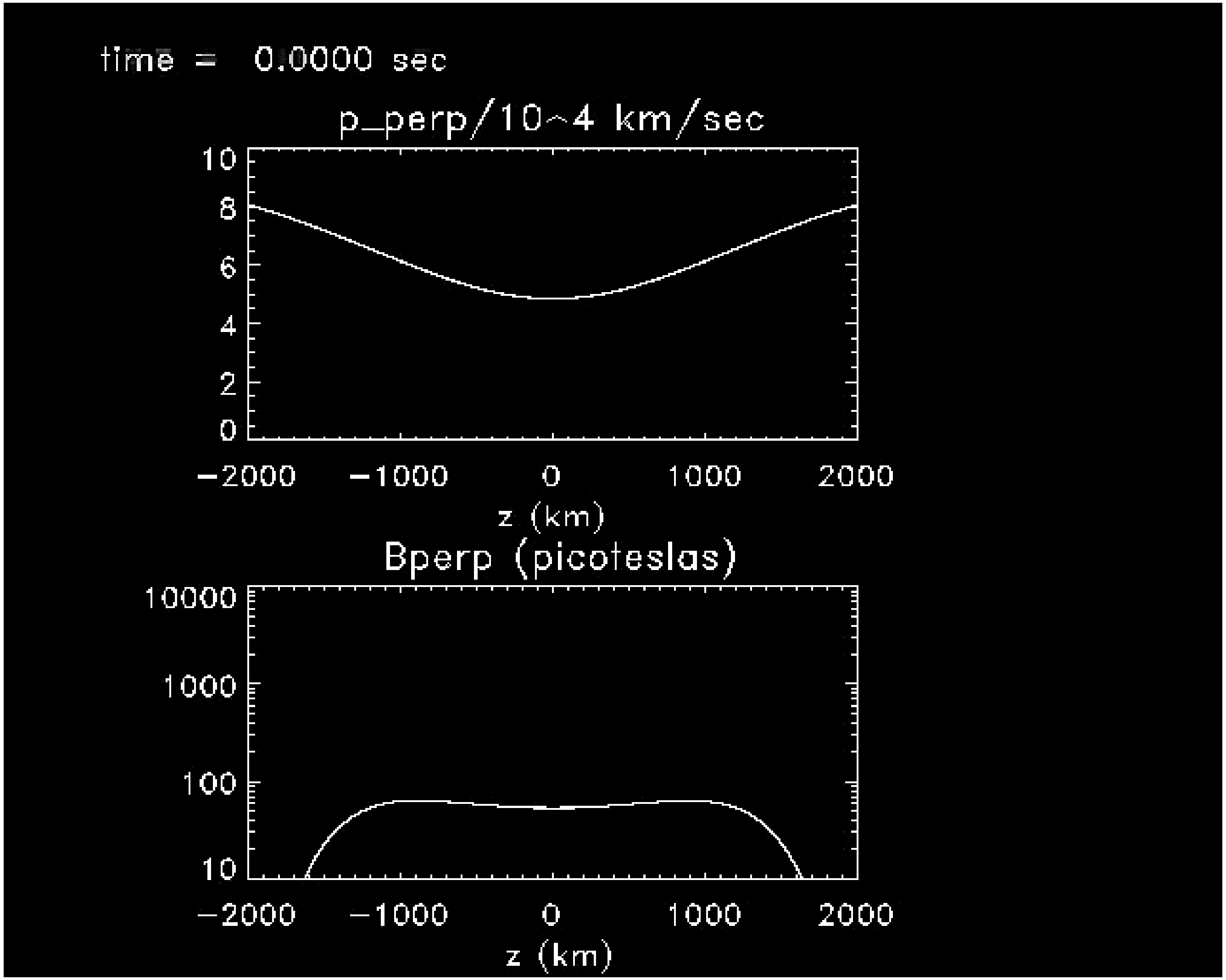


Electron distribution

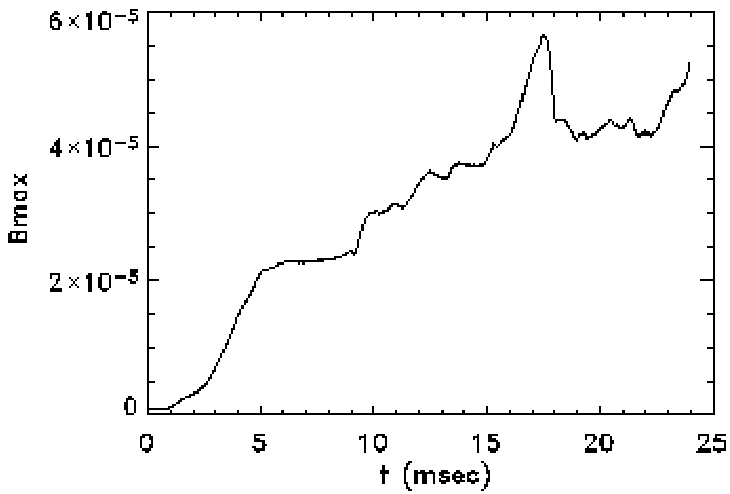


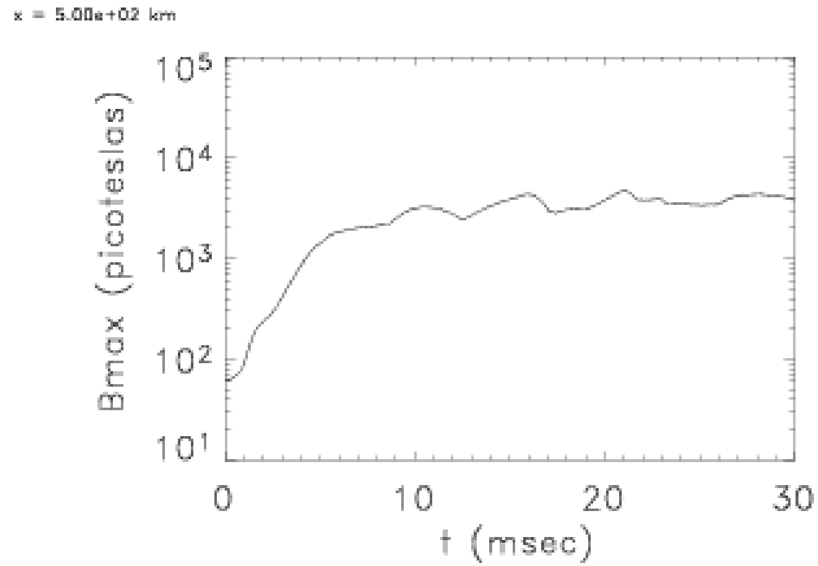
Ring distributions are probably unphysical, but can be regarded as Green's function sources for building step discontinuities in the distribution, as suggested by Trakhtengerts and emphasized by Dennis.

GROWTH OF THE INSTABILITY



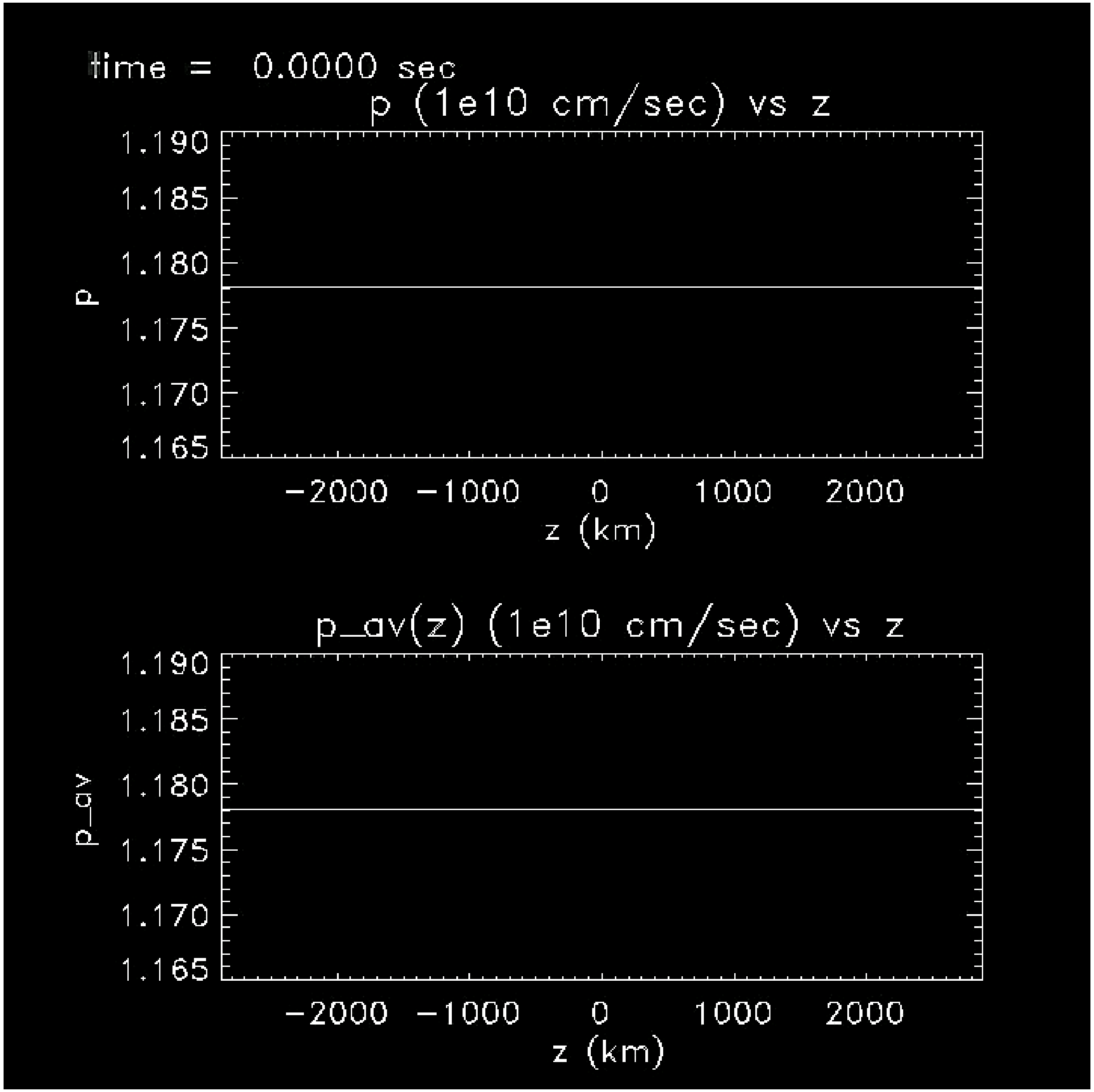
INSTABILITY GROWTH: $B_{max}(t)$



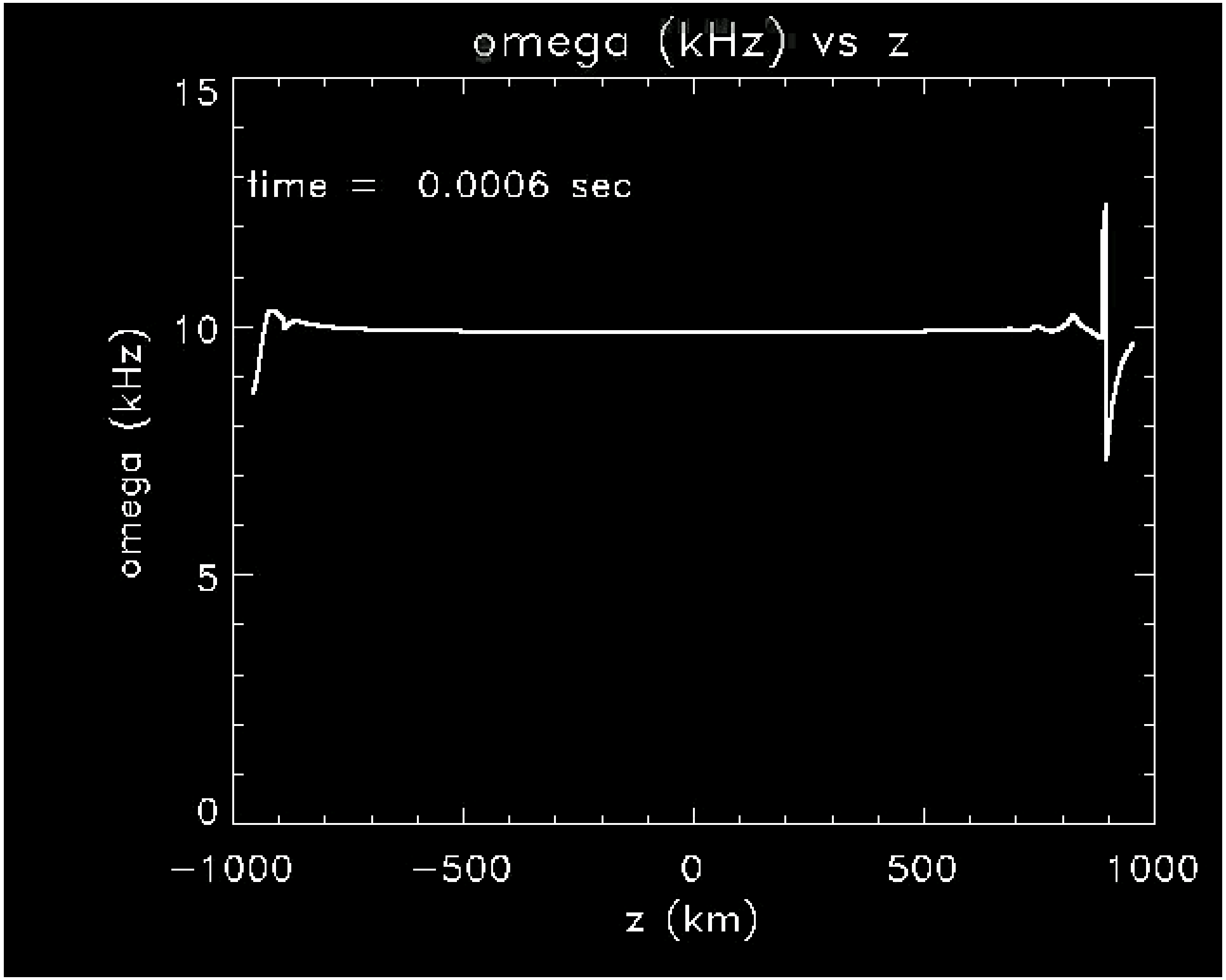


- Linear stage: Exponential growth with growth rate $\Gamma \propto n_{res}^{2/3}$
- Saturation of linear growth: $B_{wave} \propto n_{res}^{0.85\pm0.15}$
- Subsequent nonlinear growth is roughly linear in time

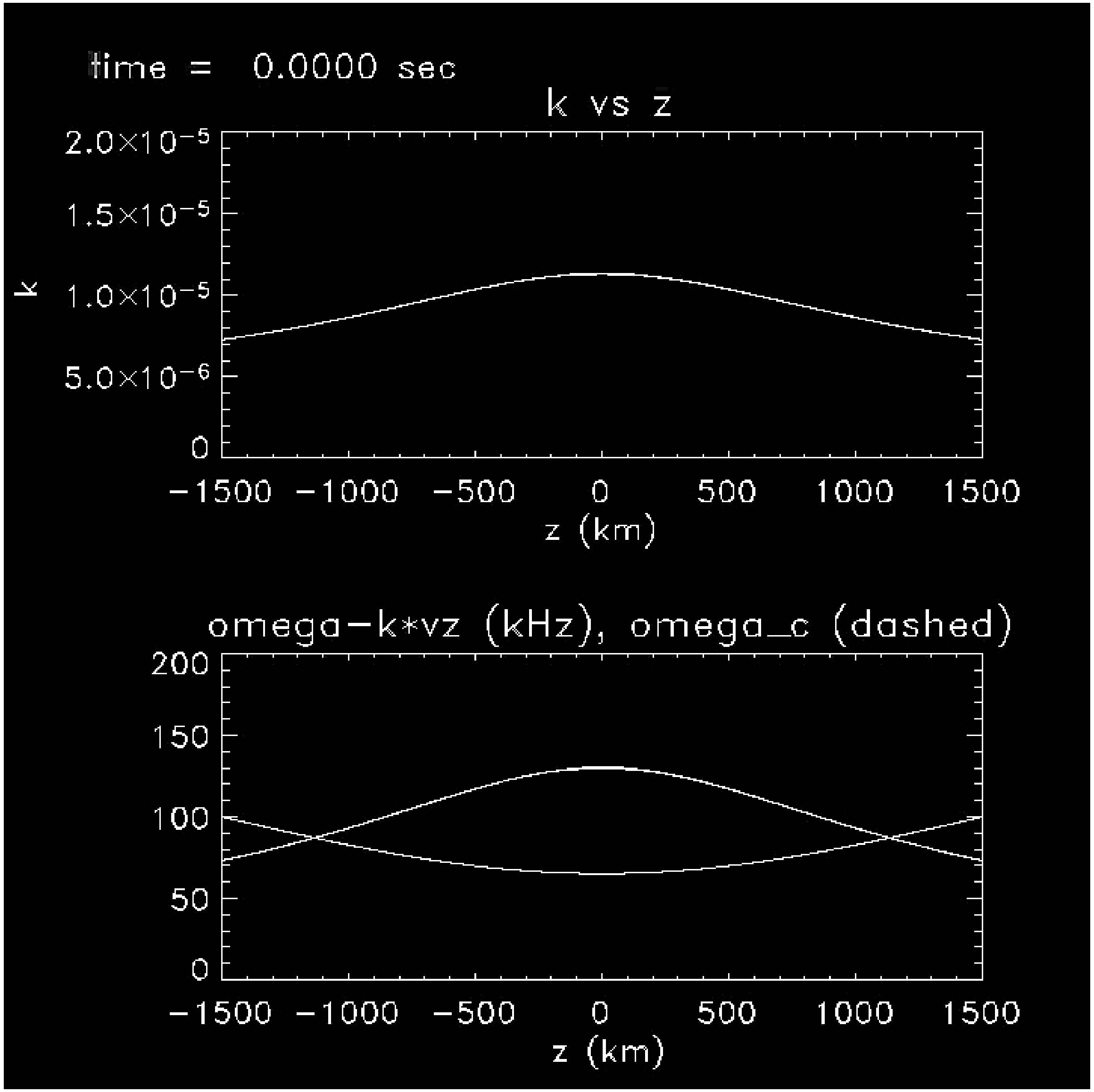
ELECTRON ENERGY LOSS



FREQUENCY ω(z,t)



DOPPLER SHIFTED FREQUENCY $\omega(z,t)-kv_z$



NOTABLE FEATURES OF THE INSTABILITY

- The nonlinear instability occurs only for waves propagating away from the equator.

- The instability propagates backward toward the equator.
  It is driven by the modulated wake of electrons that have passed through resonance.
  It thus resembles a feedback-driven absolute instability.

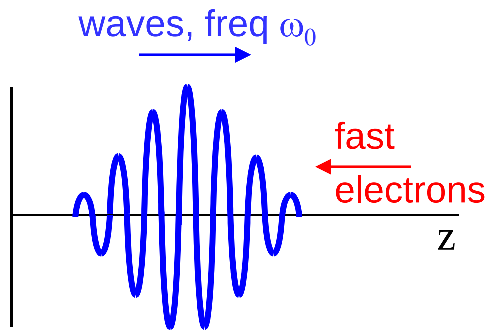
- Electrons interact strongly with the wave throughout $0 < z < z_+$.

- ω falls sharply as the instability grows and propagates.

- In the nonlinear stage, amplitude grows roughly linearly, and saturates at a large amplitude.

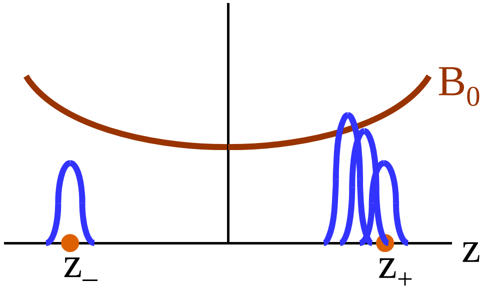
- Most of the resonant electrons are driven to sharply lower pitch angle.

PHENOMENOLOGY OF THE TRIGGERING PROCESS (1)



- Instability grows linearly at the two resonant points $z_\pm$ where $\omega_0 - k(z)\,\mathbf{v}(z) = \Omega_e(z)$.

- Electrons stream through $z_\pm$, are modulated at freq ω_0, and form a wake carrying current at freq ω_0.
  Free-streaming of these electrons, with gyration at frequency $\Omega(z)$, leads to a current pattern $\exp[ik(z)z - i\omega_0 t]$, where k(z) satisfies
  ω_0 - k(z) v(z) = $\Omega_e(z)$, i.e. $k(z) = [\Omega_e(z)-\omega_0]/|v(z)|$. Note $k(z) < k_0(z)$.

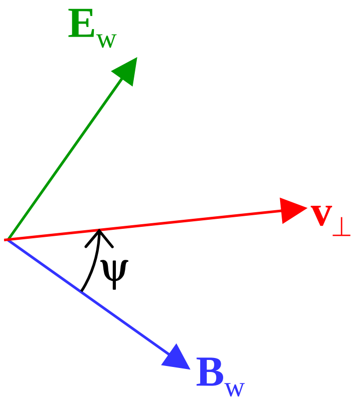


- A little downstream from the resonance point, wake current drives a new wave with wavenumber k(z), and freq $\omega(k(z))$ given by the linear dispersion relation.
  $\omega(k(z)) < \omega_0$, so the driven wave is a "faller."

- This wave drives a wake current at frequency $\omega(k(z))$, which then excites waves further downstream at still lower $\omega(k(z))$.
  Electron remains in resonance with the new waves, everywhere from the original resonance point to the equator.

- Furthermore, there is an unstable nonlinear interaction between the resonant electrons and the new waves, ***but only for electrons moving toward the equator***.

ENERGY TRANSFER BETWEEN ELECTRONS AND WAVES



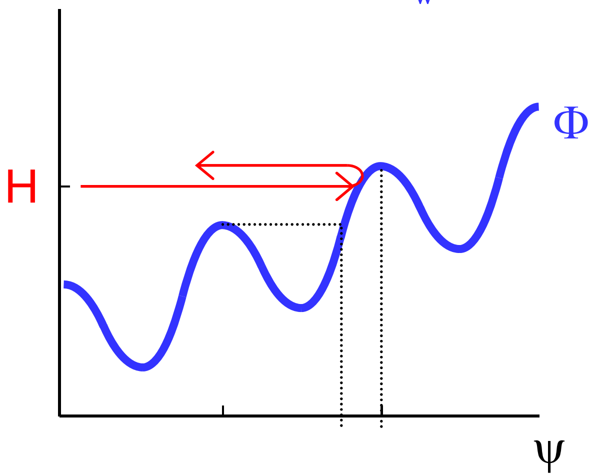
- Define ψ as the phase difference between $\mathbf{v}_\perp$ and $\mathbf{B}_w$.

  Electron loses energy if $-e\mathbf{v}_\perp \cdot \mathbf{E}_w < 0 \;\Rightarrow\; -\pi < \psi < 0$.

- For an electron subject to the wave field as well as the mirror field, can reduce eqs of motion to

$$\frac{d^2\psi(t)}{dt^2} + \frac{d\Phi(\psi)}{d\psi} = 0, \quad \Phi \equiv \omega_{tr}^{\,2}\cos\psi + S\psi, \quad \omega_{tr} \equiv \sqrt{\frac{eBkv_\perp}{mc}}$$

- $H \equiv \frac{1}{2}\left(\frac{d\psi}{dt}\right)^2 + \Phi(\psi)$ is a constant of the motion.



- An electron beginning at $z > z_+$ starts with $d\psi/dt > 0$, and $S > 0$ when $dB_0/dz > 0$.

  The electron must "bounce off" the potential $\Phi(\psi)$.

  This corresponds to passing through resonance, then $d\psi/dt < 0$.

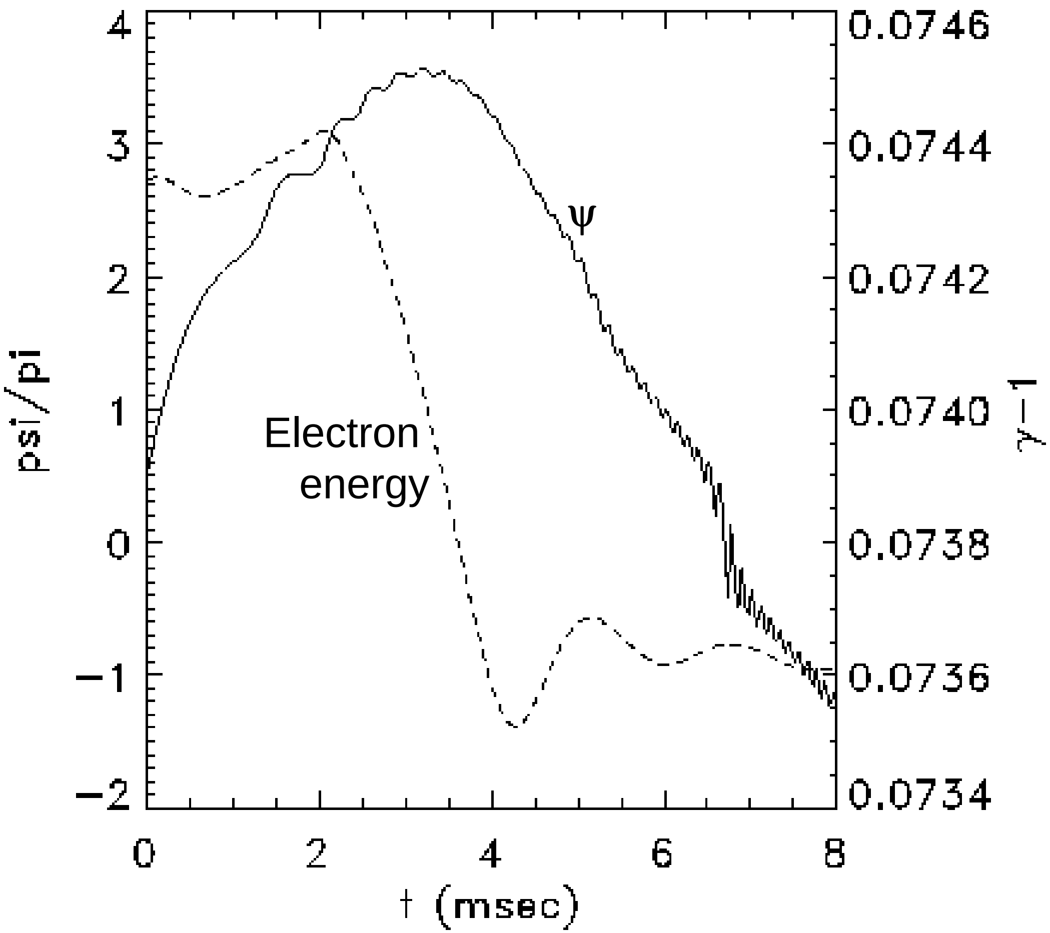
- An electron spends a lot of time at the "bounce," so this point dominates energy exchange.

  $\psi > 0$ at a peak of $\Phi(\psi)$, so electron gains energy if it bounces at a peak,

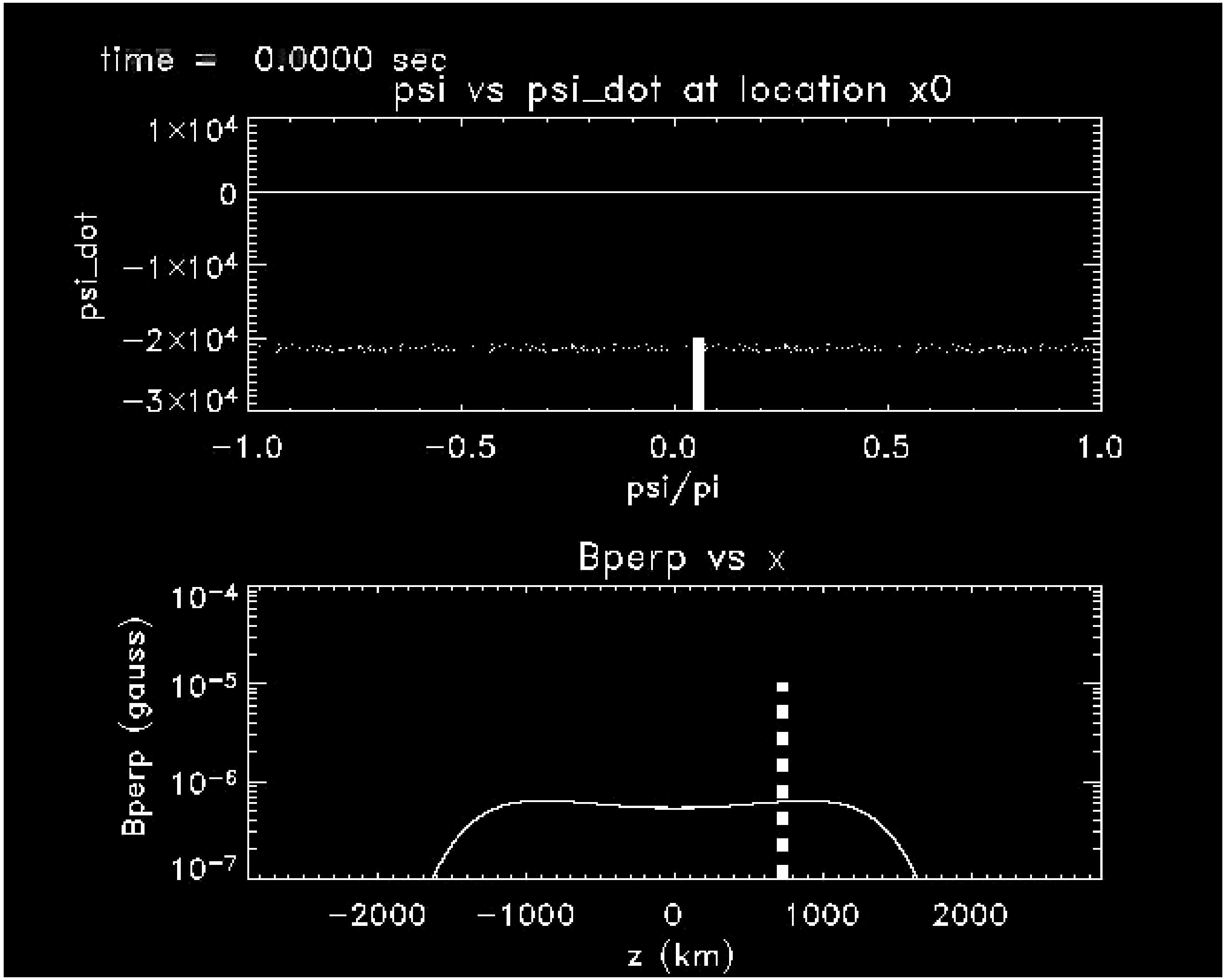
  but $\psi < 0$ for nearly all bounce positions. Thus electrons usually lose energy at the bounce.

  Can show that this energy goes into wave growth.

PHASE TRAJECTORY OF A TYPICAL ELECTRON



PSI/PSI_DOT PHASE SPACE



TEMPORAL GROWTH AND SATURATION

- Electron kinetic energy loss passing through resonance $\Delta K \sim B_w (\Delta t_{res}) (\sin \psi_{res})$.
  Dwell time at resonance $\Delta t \sim B_w^{-1/2}$, $\langle \sin \psi_{res} \rangle$ is difficult to predict,
  but the simulations indicate $\Delta K \sim B_w$.
  Wave energy $\sim B_w^2 \Rightarrow dB_w^2 \sim B_w \Rightarrow B_w \sim t$, as observed.

- Wave propagation plays a small role during the growth phase, since $v_{wave} << v_{electron}$.
  But the loss rate of wave energy at a given location, due to wave propagation,
  is proportional to $B_w^2 \sim t^2$. Wave growth $\sim t$.
  So eventually growth = convective loss $\Rightarrow$ saturation.
  Thereafter, electrons continue to feed energy to the waves,
  but the energy goes into broadening the wave packet rather than amplitude growth.

SUMMARY

- For a "beam-like" distribution of fast electrons interacting with a coherent whistler driver, an absolute instability is seen.
- The instability is initially excited at the resonant point where resonant electrons are moving ***toward*** the equator and the waves are propagating ***away*** from the equator. The instability is confined between the original resonant point and the equator.
- The instability is driven by the modulated wake electrons: hence it propagates backward (with the electrons) rather than forward (with the wave) from the original excitation point.
- The unstable waves are fallers, with frequency decreasing by about a factor of three from the driver frequency.
- We have seen instability of a somewhat similar nature (with weaker growth and less distinct features) in cases with a step discontinuity in $f(v_{||})$, This could lead to an instability sequence (previously suggested by Trakhtengerts et al) where a large-amplitude wave creates a step discontinuity, which then triggers nonlinear growth.