# STATISTICAL PROPERTIES OF THE EVOLUTION OF SOLAR MAGNETIC FIELDS

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ABSTRACT

We are making an attempt to associate the formation and evolution of active regions with the heating and flaring of the plasmas in the solar atmosphere above the active region. The working hypothesis in our review is that "*active regions are open dynamical systems away from equilibrium, driven by the turbulent convection zone*". New magnetic loops **emerge** from the convection zone and are subject to surface diffusion **(random motion)**, which leads to cancellation of magnetic energy when they collide with magnetic structures of opposite polarity. In the course of their evolution from birth to disappearance, active regions reach quickly the stage of Self-Organized Criticality (SOC), heat the corona by driving nano-flares, micro-flares and flares. We believe that the statistical properties of the active regions at the photosphere (size distribution, fractal dimension etc) are correlated with the statistical properties of all aspects of solar activity (Ellerman Bombs, Bright Points, flares, particle acceleration, etc). Surprisingly, all these phenomena share one common characteristic, they are self-similar and their statistical behaviour follows well defined power laws. This last point reinforces our belief that the solar atmosphere is coupled, through the magnetic field, with the convection zone, which drives all the observed activity at the photosphere, chromosphere, corona and interplanetary space.

Key words: Convection Zone; Active Regions; Flares; Coronal Heating.

## 1. INTRODUCTION

The most energetic phenomena above the solar surface are associated with "active regions". It has become apparent recently that the sun is never "quiet." As the quality of the observations improved the last few years the term "quiet-sun" has gradually disappeared. It is for these reasons that the study of active regions (of all sizes) has attracted the attention of many observers and theorists the recent years.

In this review we will attempt to address the question: How the subphotospheric activity is mapped into the formation and subsequent evolution of active regions? The formation and evolution of magnetic flux tubes inside the convection zone is an important theoretical problem, which still remains open. We believe today that flux tubes are formed at the base of the convection zone and rise to the surface through buoyant forces. During their buoyant rise, flux tubes are influenced by several physical effects, such as the Coriolis force, magnetic tension, drag and large scale convection motion. The lack of understanding of the relative importance of these forces in the formation and evolution of active regions has forced many observers to develop large scale statistical studies for the characteristics of active regions (see review by Howard (1996)). Several statistical regularities are observed and a number of them will be discussed in the following section.

We will argue in this review that the magnetic field becomes the main avenue were the turbulent convection zone influences the evolution of the entire solar atmosphere. The coupling of a low-$\beta$ turbulent convection zone with the high-$\beta$ atmosphere is a common astrophysical system. Energy is transmitted from the high-$\beta$ convection zone to the low-$\beta$ solar atmosphere and dissipated (in the form of waves and current sheet formation) in order to heat the corona and to drive explosive phenomena and flows. The complexity of the magnetic field in active regions suggests that all solar phenomena are interdependent and the well known say for the evolution of non-linear systems (attributed to Lorentz) "the sensitivity to the initial conditions in non-linear systems is such that the flopping of the wings of a butterfly in Brazil will influence the weather in Santorini" is applicable.

We propose that the active regions are strongly coupled "*systems*" which are gradually reaching a Self-Organized Critical state (see Bak et al (1987)). Their evolution follows simple physical rules, which emerge as the new regularities which control the evolution. This phenomenon is very common in nature and appears in many complex systems (see Jensen (1998)). Active regions are some times over driven, leaving the SOC state, producing very large explosions and resetting the system. These departures from the SOC state have not been studied carefully.

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## 2. KEY OBSERVATIONS

Numerous observational studies have investigated the statistical properties of active regions, using full-disc magnetograms and Ca II plage regions observations from the Mount Wilson Observatory and from the National Solar Observatory.

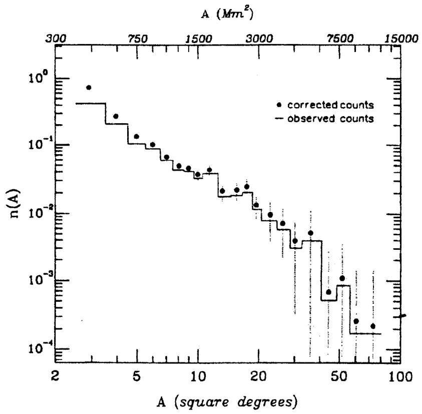


*Figure 1. The size distribution of young active regions(see Harvey & Zwaan (1993))*

These studies have examined among other parameters the size distribution of active regions, and their fractal dimension: *The size distribution function* of the newly formed active regions exhibits a well defined power law with index $\approx -1.94$ (see Figure 1), and active regions cover only a small fraction of the solar surface (around $\sim 8\%$) (Harvey & Zwaan, 1993). The *fractal dimension* of the active regions has been studied using high-resolution magnetograms by Balke, et al. (1993), and more recently by Meunier (1999). These authors found, using not always the same method, a fractal dimension $D_F$ in the range $1.3 < D_F < 1.8$.

The primary energy release through magnetic reconnection may well be the basic physical cause for the various types of observed emission events, such as coronal bright points (Parnell & Jupp, 2000; Krucker & Benz, 1998), X-ray networks flares (Krucker et al., 1997), transition region impulsive EUV emission (Benz & Krucker, 2002), and H$\alpha$ bright points (Ellerman bombs (Georgoulis et al., 2002)). All these emissions may play a role to coronal heating (this point is still under strong debate), and in particular it was recently claimed that impulsive, chromospheric EUV emissions are important signatures of not just the coronal heating process but also of mass supply to the corona (Brown et al., 2000).

It is interesting to note that all the mentioned observed explosive phenomena exhibit power-law distributions in energy, with slopes roughly in the range between $1.5$ and $2.5$. We believe that the observed discrepancies on the indices are directly related to the dynamical evolution of the active regions.

## 3. SUB-PHOTOSPHERIC ACTIVITY

The evolution of a single flux tube from the overshoot layer, where it is generated till it reaches the photosphere, is a very difficult and challenging problem. Studies of the dynamics of a 1-D (slender) flux tube are useful since they permit calculation of the evolution as the flux-tube propagates through the convection zone. The 2-D and 3-D characteristics of the rise of magnetic flux tubes allow us to understand the role of the twist in the properties of a rising flux-tube. A recent review of the main results on the evolution of 1-D, 2-D and 3-D flux tubes was given by Moreno-Inseris (1997). It is well documented from numerical simulations that untwisted flux tubes will never reach the photosphere and twisted flux tubes can rise almost without change through the convection zone, if the azimuthal magnetic field is strong. Two very important questions should be addressed: (1) How are flux-tubes formed from a large scale magnetic field, and (2) which is the origin of the twist in the flux tubes. Hughes et al. (1997) address this problem, their main conclusion is that the non-linear evolution of the magnetic Rayleigh-Taylor instability is responsible for the formation of magnetic flux tubes. The origin of the twist is a much more complex problem. There are several possibilities: (1) the flux-tubes are formed with a large twist in the overshoot layer, (2) the twist is built-up during the propagation inside the convection zone. It is still an open question if the required twist can be there from “birth” or is added later. We are far from understanding the details both of the formation and the origin of the twist. Large scale 3-D simulations of the formation and evolution of magnetic flux tubes will solve many of the open problems that still exist.

We move from the single flux-tube scenario to the idea that magnetic flux-tubes of all sizes and twists are generated in the overshoot layer and propagate towards the photosphere. The idea to study the formation of active regions as the outcome of the statistical evolution of $N$ randomly moving flux-tubes was proposed by Bogdan (1984) and developed subsequently in many articles (Bogdan, 1985; Bogdan & Lerche, 1985). Similar studies on the statistical mechanics of a gas of vortices, embedded in a two-dimensional inviscid fluid, were performed by Fröhlich & Ruelle (1982). The evolution of a collection of $N$ flux-tubes can address the statistical properties of the observed data.

Assuming a gas of flux-tubes moving inside the convection zone (see Fig. 2), Bogdan & Lerche (1985) developed a two-dimensional ($\partial/\partial z = 0$) theory of their evolution. The properties of the magnetic flux-tubes are described by the distribution function $f(t, r, \theta, v_r, v_\theta; \Gamma)$, where $\Gamma$ characterises the internal structure of the flux tubes. The evolution of $f$ is described by a collisional Boltzmann equation,

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} + \dot{\vec{v}} \cdot \frac{\partial f}{\partial \vec{v}} = S \qquad (1)$$

where $S$ is the collision integral. The collision integral contains two terms: (a) the evolution of the internal structure, and (b) changes in the velocities without alertion of the internal structure. It is more interesting to work with

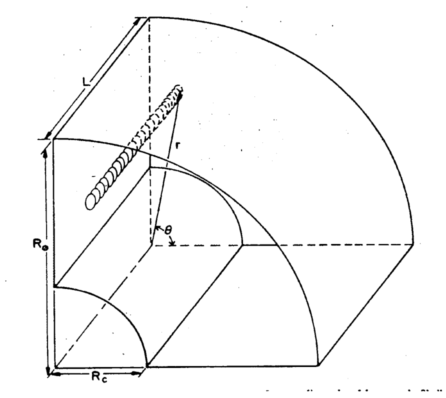


*Figure 2. Idealization of the solar convection zone. A twisted magnetic fibril is located at $\vec{r}$.(see Bogdan & Lerche (1985))*

the density of flux tubes $N(t, r, \theta; \Gamma) = \int d^3\vec{v} f$. A new evolution equation can be written

$$\frac{\partial N}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}[ru(r,\theta,t)N] - \vec{\nabla}\cdot[\vec{\vec{k}}(r,\theta,t)\cdot\vec{\nabla}N] = S \quad (2)$$

where $u(r, \theta, t)$ is the average velocity with which flux tubes move towards the surface. The complicated chaotic motion of a single flux tube is superposed on this mean velocity, and $\vec{\vec{k}}$ is the spatial diffusion tensor. The diffusion coefficient is difficult to estimate, since it includes all the information of the turbulent velocity-field. The velocity $\vec{u}$ and the parameter $\vec{\vec{k}}$ remain free parameters, so the transport equation is never going to be an "accurate" procedure for the estimation of the evolution of $N$. In order to work further with Eq. (2), we should discuss the collision integral $S$. We can describe a flux tube with its flux $\Phi$. The magnetic flux $\Phi$ remains constant throughout the motion and serves to distinguish one flux-tube from another. Assuming that the collision integral does not produce large changes in velocity space and expresses mostly the annihilation and coalescence of flux tubes, the diffusion Eq. (2) will now evolve to the equation

$$\frac{\partial N}{\partial t} + \frac{N}{T} + \frac{1}{r}\frac{\partial}{\partial r}[ruN] - \vec{\nabla}\cdot[\vec{\vec{k}}\cdot(\vec{\nabla}N)] = S \quad (3)$$

where $T$ is the rate at which flux tubes will spontaneously decouple and precipitate rapidly to the surface forming the photospheric patterns. Unfortunately, the diffusion Eq. (3) has so many unknown parameters $(u, \vec{\vec{k}})$, and it can hardly predict any of the measured quantities discussed in Section 2. This field of research has remained for almost twenty years undeveloped. New tools have since then appeared (e.g. Lattice Boltzman method) which can give a new light in this problem. In the mean time, our only hope are the 3-D MHD simulations which have been proven an invaluable tool for the analysis of subphotospheric flux tube dynamics.

## 4. THE PHOTOSPHERIC ACTIVITY

Several models have been developed using the anomalous diffusion of magnetic flux in the solar photosphere in order to explain the fractal geometry of the active regions (Schrijver et al, 1992; Lawrence, 1991; Lawrence and Schrijver, 1993; Milovanov & Zelenyi, 1993). Last, a percolation model was used to simulate the formation and evolution of active regions (Wentzel & Seiden, 1992; Seiden & Wentzel, 1996). In this model, the evolution of active regions is followed by reducing all the complicated solar MHD and turbulence to three dimensionless parameters. This percolation model explains the observed size distribution of active regions and their fractal characteristics (Meunier, 1999).

Vlahos et al. (2002a) developed further the percolation model for the emergence and evolution of magnetic flux on the solar surface using a 2-D cellular automaton (CA), following techniques developed initially by Seiden & Wentzel (1996). The dynamics of this automaton is probabilistic and is based on the competition between two "fighting" tendencies: **stimulated** or **spontaneous** emergence of new magnetic flux, and the disappearance of flux due to **diffusion** (i.e. dilution below observable limits), together with random **motion** of the flux tubes on the solar surface. The basic new element they add to the Seiden & Wentzel (1996) model is that they keep track of the **energy release** through flux cancellation (reconnection) if flux tubes of opposite polarities collide. They concentrate their analysis only on the newly formed active regions, since the old active regions undergo more complicated behavior.

The main physical properties of active regions, as derived from the observations of the evolving active regions, can be summarized in simple CA rules: A 2-D quadratic grid with $200 \times 1000$ cells (grid sites) is constructed, in which each cell has four nearest neighbors. The grid is assumed to represent a large fraction of the solar surface. Initially, a small, randomly chosen percentage ($1\%$) of the cells is magnetized (loaded with flux) in the form of positively (+1) and negatively (-1) magnetized pairs (dipoles), the rest of the grid points are set to zero. Positive and negative cells evolve independently after their formation, but their percentage remains statistically equal. The dynamical evolution of the model is controlled by the following probabilities:

**P**: The probability that a magnetized cell is stimulating the appearance of new flux at one of its nearest neighbors. Each magnetized cell can stimulate its neighbors only the first time step of its life. This procedure simulates the stimulated emergence of flux which occurs due to the observed tendency of magnetic flux to emerge in regions of the solar surface in which magnetic flux had previously emerged.

$\mathbf{D_m}$: The flux of each magnetized cell has a probability $D_m$ to move to a random neighboring cell, simulating motions forced by the turbulent dynamics of the underlying convection zone. If the moving flux meets oppositely polarized flux in a neighboring cell, the fluxes cancel (through reconnection), giving rise to a "flare". If equal polarities meet in a motion event, the fluxes simply add up.

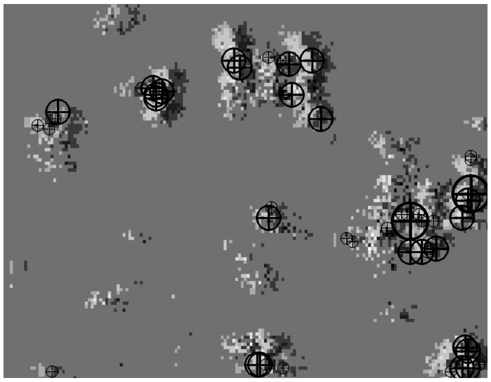

*Figure 3. A small portion of the modeled grid is presented. The dark areas represent negative and the white ones positive magnetic flux. The explosions ("flares") appear randomly at the interface of regions of oppositely polarized magnetic flux. The circles represent the positions of the "flares", with their radius being proportional to the logarithm of the released energy.*

$\mathbf{D_d}$: The probability that a magnetized cell is turned into non-magnetized in one time-step if it is next to a non-magnetized cell. This rule simulates two effects, the direct submersion of magnetic flux and the disappearing of flux below observational limits due to dilution caused by diffusion into the empty neighborhood.

$\mathcal{E}$: The probability that a non-magnetized cell is turned into magnetized spontaneously, independently of its neighbors, simulating the observed spontaneous emergence of new flux.

Every newly appearing flux tube is accompanied by an oppositely polarized mate, taking into account the fact that flux appears always in the form of dipoles.

A detail discussion of the connection between the parameters $P, D_m, \mathcal{E}$ and the physical mechanisms acting in the evolution of active regions was established in the articles of Wentzel & Seiden (1992) and Seiden & Wentzel (1996).

Vlahos et al. (2002a) performed a series of numerical experiments using the above model. The parameters used for the results reported here are $P = 0.185, Dd = 0.005$, $D_m = 0.05$ and $\mathcal{E} = 10^{-6}$. They are chosen such that, when following the evolution of our model and recording the percentage of magnetized cells, we find that it takes around 1000 time steps before the percentage of active cells is stabilized to a value which is close to the observed one (around $8\%$). In Fig. 3, we present a small portion of the grid. Dark areas correspond to negative and bright ones to positive polarity. The spatial location of the "flares" is marked with circles, with the size of the circles proportional to the logarithm of the locally released energy.

The size distribution of the simulated active regions is estimated and is approximated by a power law fit of the form $N(s) \sim s^{-k}$, with $k = 1.93 \pm 0.08$. Finally, we estimate the fractal dimension $D_F$ of the set of magnetized cells with the box counting algorithm (Falconer, 1990), finding $D_F = 1.42 \pm 0.12$.

The cancellation of magnetic flux due to collisions of oppositely polarized magnetic flux tubes leads to the release of energy, whose amount we assume to be proportional to the difference in the square of the magnetic flux before and after the event. In Fig. 4a, we plot the released energy $E(t)$ as a function of time. Fig. 4b shows the energy distribution of the recorded "flares": It follows a power law, $f(E) \sim E^{-a}$, with $a = 2.5 \pm 0.13$, for energies $E > 20$. For energies $E < 20$ finite resolution effects must be expected to bias the distribution, so we do not draw conclusions for the small energies. It is important to note also that the power law in the distribution of energy is extended over three decades.

A variation of the parameters $P, D_d, D_m$ does not alter the power law behaviours and the fractality, they seem to be **generic** properties of the model. The exact values of the parameters, $k, a, D_F$ depend though on the free parameters but remain inside the observed limits even for a large variation of $P, D_d, D_m$. The results are also independent of $\mathcal{E}$ as long as it remains small enough.

In several independent simulations, we used different values for the stimulation probability $P$ and found that $k$ and $a$ are closely correlated (see Figure 5).

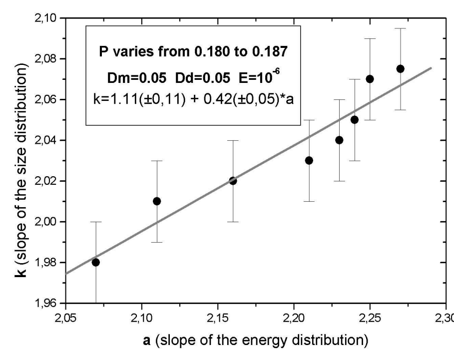


*Figure 5. There is a linear dependence of the size distribution active region index k with the energy release index a.*

## 5. THE CORONAL ACTIVITY

Observations of solar X-ray corona are reported in the literature since the early seventies. In the early eighties several authors showed that the peak-luminosity distribution of flares displays a well-defined, extended power law with an index $-1.8 \pm 0, .05$ (see Lin et al. (1984); Crosby et al. (1992)). Deviations from the power law behaviour appear in the lowest and highest energies. It has been pointed out that the lower energy deviations from the power law are due to instrumental limitations.

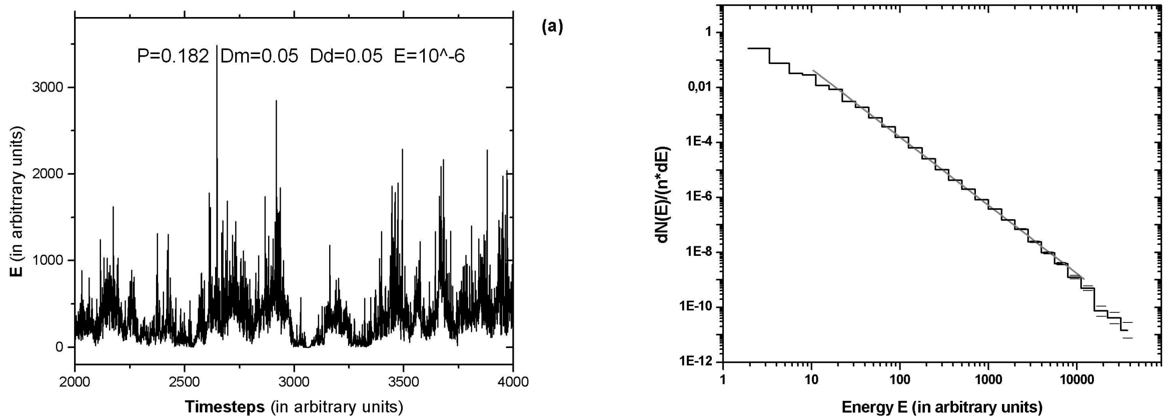


*Figure 4. (a) The energy released in the cancellation of magnetic flux as a function of time, using the parameters* $P = 1.185, D_d = 0.005, D_m = 0.05, \mathcal{E} = 10^{-6}$. *(b) The energy distribution of the recorded "flares". The power-law index is* $2.5 \pm 0.13$.

The existence of power laws in the frequency distribution of the explosive activity may suggest that flares are a self-oganization phenomenon in the active region. Lu & Hamilton (1991) were the first to realize that active regions may be in the Self-Organized Critical state and propose that flares are caused by small magnetic perturbations $(\delta B)$ which cause a local reconstruction of the unstable magnetic topology causing flares (in the form of avalanches in the initial concept introduced by Bak et al (1987)) of all sizes (nano-flares, micro-flares, flares). This model opened the way for a series of similar models developed the last ten years (see review by Charbonneau et al. (2001)).

There are two basically different ways of developing CA models for flares further: (i) Either one considers CA models *per se*, tries to change the existing models further or invent new ones, with the only aim of adjusting them to reproduce still better the observations, i.e. one makes them a tool the results of which explain and maybe predict observed properties of flares. In this approach, one has not to care about possible inconsistencies with MHD or even Maxwell's equations, the various components of the model are purely instrumentalistic. (ii) On the other hand, one may care about the physical identification and interpretation of the various components of the model, not just of its results, and one may want the CA model to become consistent with the other approach to solar flares, namely MHD. In the approach (ii), some of the freedom one has in constructing CA models will possibly be reduced, since there are more 'boundary conditions' to be fulfilled in the construction of the model: the observations must be reproduced and consistency with MHD has to be reached. (Trials to construct new CA models which are based on MHD and not on the sand-pile analogy were recently made by Einaudi and Velli (1999); Macpherson & MacKinnon (1999); Longcope & Nooman (2000))

Isliker et al. (2000, 2001) propose a set-up which can be superimposed onto each classical solar flare CA model, and which makes the latter interpretable in a MHD-consistent way (by *classical* CA models we mean the models of Lu & Hamilton (1991) (LH91), Vlahos et al. (1995), Georgoulis & Vlahos (1996), Georgoulis & Vlahos (1998), Galsgaard (1996), and their modifications, which are based on the sand-pile analogy). The set-up thus specifies the physical interpretation of the grid-variables and allows the derivation of quantities such as currents etc. It does not interfere with the dynamics of the CA (unless wished): loading, redistributing (bursting), and the appearance of avalanches and self-organized criticality (SOC), if the latter are implied by the evolution rules, remain unchanged. The result is therefore still a CA model, with all the advantages of CA, namely that they are fast, that they model large spatial regions (and large events), and there with that they yield good statistics. Since the set-up introduces all the relevant physical variables into the context of the CA models, it automatically leads to a better physical understanding of the CA models. It reveals which relevant plasma processes and in what form are actually implemented, and what the global flare scenario is the CA models imply. All this was more or less hidden so far in the abstract evolution rules. It leads also to the possibility to change the CA models (the rules) at the guide-line of MHD, if this should become desirable. Not least, the set-up opens a way for further comparison of the CA models to observations.

The specifications the set-up meets are : The vector $\vec{A}_{ijk}$ at the grid sites $\vec{x}_{ijk}$ denotes the local vector-field, $\vec{A}(\vec{x}_{ijk})$. Note that this was not specified in the classical CA models. Lu et al. (1993) for instance discuss this point: it might also have been thought of as a mean local field, i.e. the average over an elementary cell in the grid.

Guided by the idea that we want to assure $\nabla\vec{B} = 0$ for the magnetic field $\vec{B}$, which is most easily achieved by having the vector-potential $\vec{A}$ as the primary variable and letting $\vec{B}$ be the corresponding derivative of $\vec{A}$ ($\vec{B} = \nabla \wedge \vec{A}$), we furthermore assume that the grid variable $\vec{A}$ of the CA model is identical with the vector-potential.

The remaining and actually most basic problem then is to find an adequate way to calculate derivatives in the grid. In general, CA models assume that the grid-spacing is finite, which also holds for the CA model of Lu & Hamilton (1991) (as shown in detail by Isliker et al. (1998), so that the most straightforward way of replacing differential expressions with difference expressions is not adequate. Consequently, one has to find a way of continuing the vector-field into the space in-between the grid-sites, which will allow to calculate derivatives. The 3–D interpolation is performed as three subsequent 1–D interpolations in the three spatial directions (Press et al. (1992)). For the 1–D splines, we assume natural boundaries (the second derivatives are zero at the boundaries).

With the help of this interpolation, the magnetic field $\vec{B}$ and the current $\vec{J}$ are calculated as derivatives of $\vec{A}$, according to the MHD prescription:

$$\vec{B} = \nabla \wedge \vec{A}, \tag{4}$$

$$\vec{J} = \frac{c}{4\pi} \nabla \wedge \vec{B}. \tag{5}$$

According to MHD, the electric field is given by Ohm's law, $\vec{E} = \eta\vec{J} - \frac{1}{c}\vec{v} \wedge \vec{B}$, with $\eta$ the diffusivity and $\vec{v}$ the fluid velocity. Since the classical CA models use no velocity-field, our set-up can yield only the resistive part,

$$\vec{E} = \eta\vec{J}. \tag{6}$$

In applications such as to solar flares, where the interest is in current dissipation events, i.e. in events where $\eta$ and $\vec{J}$ are strongly increased, Eq. refres can be expected to be a good approximation to the electric field. Theoretically, the convective term in Ohm's law would in general yield just a low-intensity, background electric field.

Eq. refres needs to be supplemented with a specification of the diffusivity $\eta$: citetIsl98 have shown that in the classical CA models the diffusivity adopts the values $\eta = 1$ at the unstable (bursting) sites, and $\eta = 0$ everywhere else. This specifies Eq. refres completely. This set-up of Isliker et al. (2000, 2001) for classical solar flare CA models yields, among others, consistency with Maxwell's equations (e.g. divergence-free magnetic field), and availability of secondary variables such as currents and electric fields in accordance with MHD. Both are new for solar flare CA models. The set-up specifies the so far open physical interpretation of the CA models. This specification is to some extent unavoidably arbitrary, and it would definitely be interesting to see what alternative interpretations would yield — if they can be derived consistently. One can claim, however, that the interpretation we chose is reasonable, it is well-behaved in the sense that the derivatives of analytically prescribed vector-potentials are reproduced and that the abstract stress-measure of the CA models is related to the current, due to general properties of spline interpolation. The central problem which was to solve is how to calculate derivatives in a CA model, i.e. how to continue the primary grid-variable in-between the grid sites, since the notion of derivatives is alien in the context of CA models quite in general.

The main aim in Isliker et al. (2000, 2001) with the introduced set-up was to demonstrate that the set-up truly extends the classical CA models and makes them richer in the sense that they contain much more information, now. The main features we revealed about the CA models, extended with this set-up, are:

**1. Large-scale organization of the vector-potential and the magnetic field:** The field topology during SOC state is bound to characteristic large-scale structures which span the whole grid, very pronounced for the primary grid variable, the vector-potential, but also for the magnetic field . Bursts and flares are just slight disturbances propagating over the large-scale structures, which are always maintained, also in the largest events. The magnitude of the current, as a second order derivative of the primary field, does not show any obvious large-scale structure anymore, it reflects more or less only the random fluctuations of the large-scale organized magnetic field. It is worthwhile noting that the large-scale structure of the primary grid-variable is not an artificial result of our set-up, but a consequence of the SOC state in which the system finds itself. The appearance of large-scale structures for the primary grid variable was demonstrated here for the first time. It may have been known to different authors, but it never has explicitly been shown: SOC models for flares are derived in analogy to sand-pile dynamics, and the paradigm of a pile reappears in the field topologies of the solar flare CA models.

**2. Increased current at unstable grid-sites:** Unstable sites are characterized by an enhanced current, which is reduced after a burst has taken place, as a result of which the current at a grid-site in the neighbourhood may be increased.

**3. Availability of the electric field:** The electric field is calculated with the resistive part of Ohm's law, which can be expected to be a good approximation in applications where the interest is in current-dissipation events, e.g. in the case of solar flares.

**4. Energy release in terms of Ohmic dissipation:** We replaced the some-what *ad hoc formula* in the CA models to estimate the energy released in a burst with the expression for Ohmic dissipation in terms of the current. The distributions yielded in this way are very similar to the ones based on the ad hoc formula, so that the results of the CA models remain basically unchanged.

**5. CA as models for current dissipations:** As a consequence of point 2 and 4 in this list, and of the fact that there is an approximate linear relation between the current and the stress measure of the CA, we can conclude that the *extended* CA models can be considered as models for energy release through current dissipation.

It is important to mention here another attempt made by Nordlund & Galsgaard (1996) to simulate, using a 3-D MHD code, the sporadic development and evolution of current sheets of all sizes inside the active region. They used periodic "y-z" boundaries and perfectly conducting rigid x-boundaries with sinusoidal shear with randomly changing direction and phase, acting on an initial magnetic field with straight field lines. It is remarkable to note the appearance of non-steady current surfaces as it was the case with the CA model presented above (see

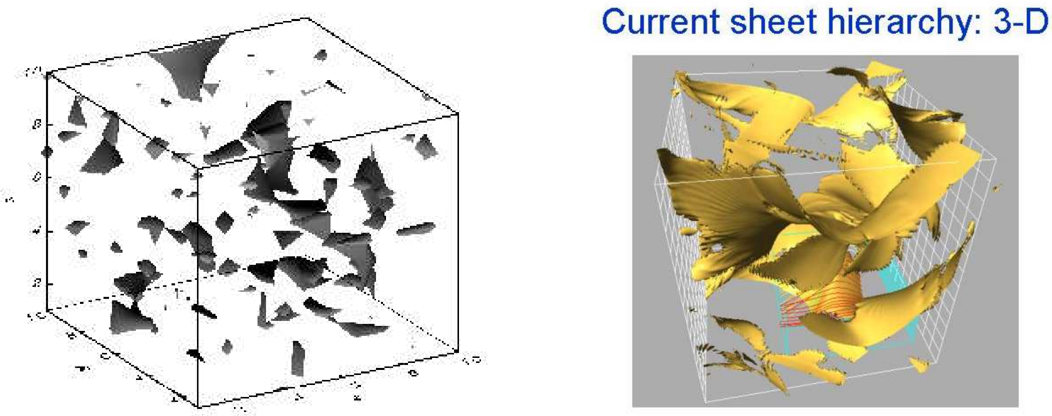


*Figure 6. (a) Three dimensional isosurfaces of electric current density derived from the CA model (Isliker et al. (2001)) (b) Isosurfaces of electric current density, from 3D MHD experiments with boundary driven magnetic dissipation (Nordlund & Galsgaard (1996))*

Fig. 6). The boundary motion used in these studies is still very simple and has no direct influence on the characteristics of the energy release. It is not clear if the statistical properties (distribution of energy released) agree with the observations.

## 6. DISCUSSION

We believe that a new model for the energy released of the complex active regions is suggested from the global analysis presented in this review. The new model seems to have several characteristics:

- Newly formed magnetic clusters on the solar surface follow a well defined statistical law which maps into 2-D internal characteristics of the turbulent convection zone. The magnetic patterns formed on the photosphere have also a well defined fractal dimension. Photospheric flows shuffle the emerged magnetic structures causing the release of non-potential magnetic energy due to collisional reconnection. Simple 2-D percolation models with three free parameters can represent remarkably well the formation of active regions in the solar surface, so these models can be used as drivers in 3-D active region complexes (see Georgoulis & Vlahos (1996)). These models are currently under development.

- The 3-D structures above the photosphere gradually evolve towards a global SOC state. The competition between formation of critical discontinuities and their sudden relaxation leads the system to a self organization and explains the observed statistical data.

- SOC is violated when the system is over-driven (large filling factor) and the average amplitude of perturbations of the magnetic field are not small compared to the prescribed critical threshold. Over driven systems deviate from power law behaviour.

- Developing MHD consistent CA models we can estimate the physical parameters (current, $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$ etc). Based on these models we can estimate the local sub-resolution heating and acceleration of particles.

- Particle acceleration and diffusion in a set of randomly distributed DC electric fields have been developed in the past (see Vlahos (1996); Anastasiadis et al. (1997)). When the electric fields associated with the unstable current sheets are distributed in a well defined fractal geometry (see more in McIntosh et al. (2002) for the fractal topologies of the unstable sites developed during the SOC state), a new type of acceleration environment is developed (see Fig. 6). This problem is currently under intensive study (Vlahos & Isliker (2002b))

Models along these lines have been proposed (Vlahos et al. (1984); Vlahos (1993, 1994) see also Fig. 7) in the mid 80's and the beginning of the 90's and remain undeveloped due to the lack of tools for the global analysis of the active region till recently.

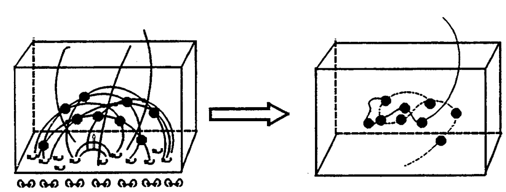

*Figure 7. Current sheets developed intermittently at random positions. Particles are accelerated by the stochastic DC E-field appearing along their orbit (Vlahos (1993))*

The old cartoons for the magnetic energy release based on "monolithic" and isolated current sheets inside a single flux tube or above a specific arcade cannot explain the statistical properties of solar flares presented in this review. The fact that they represent a reasonable explanation for several well observed events does not elevate them on the status of the "standard" model. These models fail also to explain the important and still open question of particle acceleration (see review by Peter Cargill in this volume).

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