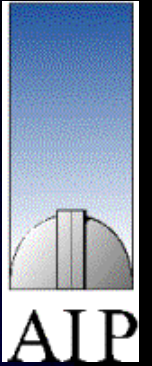


RHESSI



Workshop



Electron Acceleration by DC Electric Fields in the Flaring Corona

6th RHESSI Workshop
– Group V: Theoretical Implications –

H. Önel, G.Mann

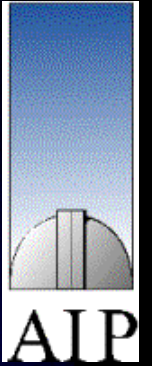
April 6, 2006

4th – 8th April 2006 – Meudon, France

RHESSI



Workshop



Electron Acceleration by DC Electric Fields in the Flaring Corona

6th RHESSI Workshop

– Group I: Electron Acceleration and Propagation –

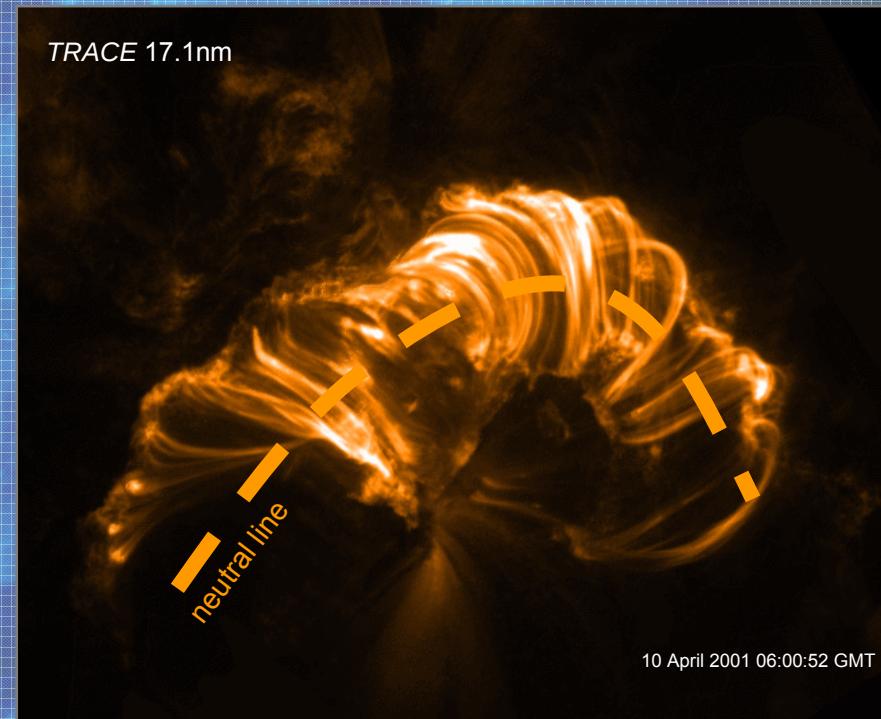
H. Önel, G. Mann

April 6, 2006

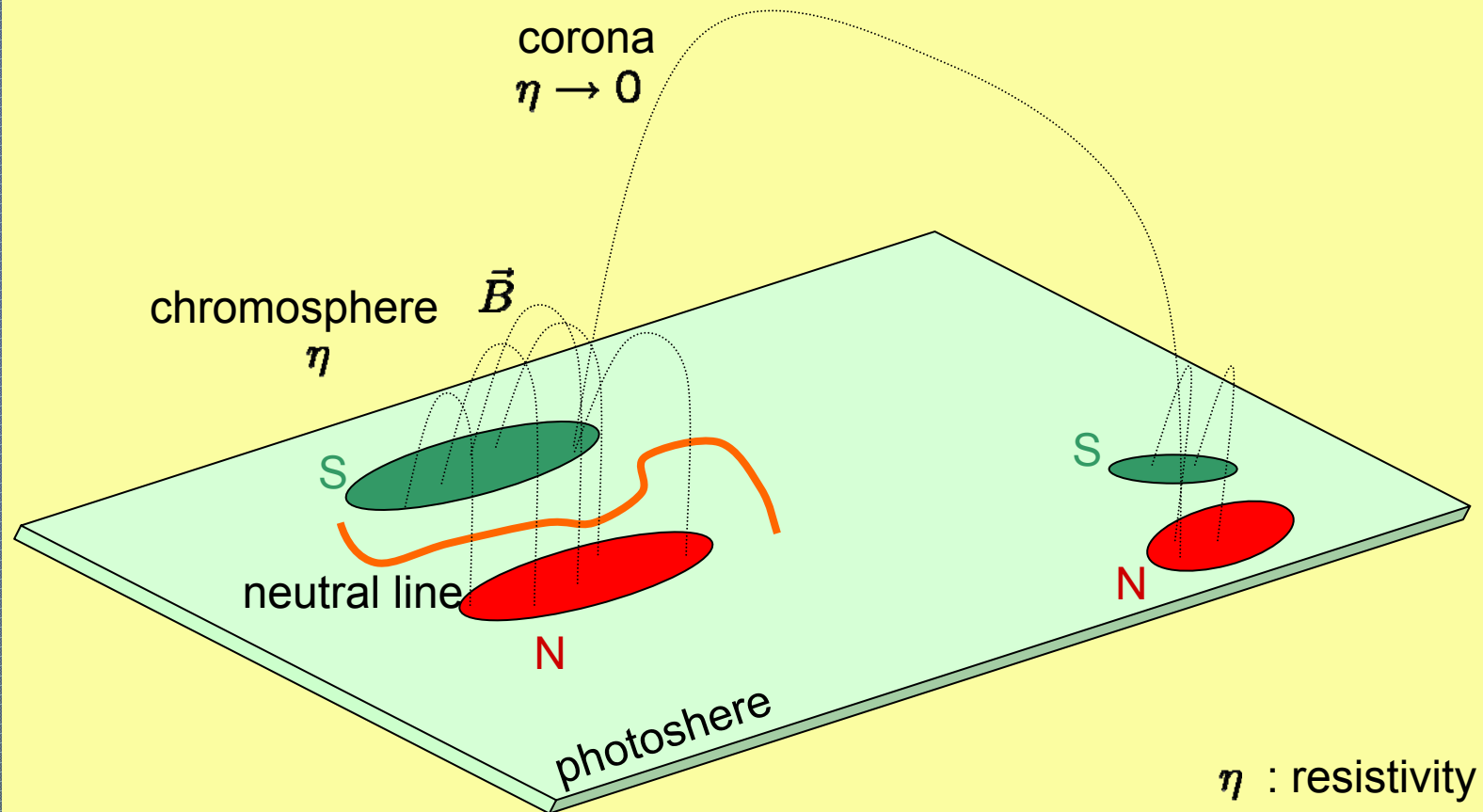
4th – 8th April 2006 – Meudon, France

Introduction

- **arc-like structures of magnetic fields can be observed at active regions**
- **different active regions (temporarily) become magnetically connected with each other**



Current Model (I)



Resistivities (I)

$$\eta = \frac{\mu \nu [N, v^{-3}]}{N q^2}$$

Resistivities are almost independent from the particle density.

$$\eta \approx \frac{\mu \nu [\cancel{N}, v^{-3}]}{\cancel{N} q^2}$$

Estimation

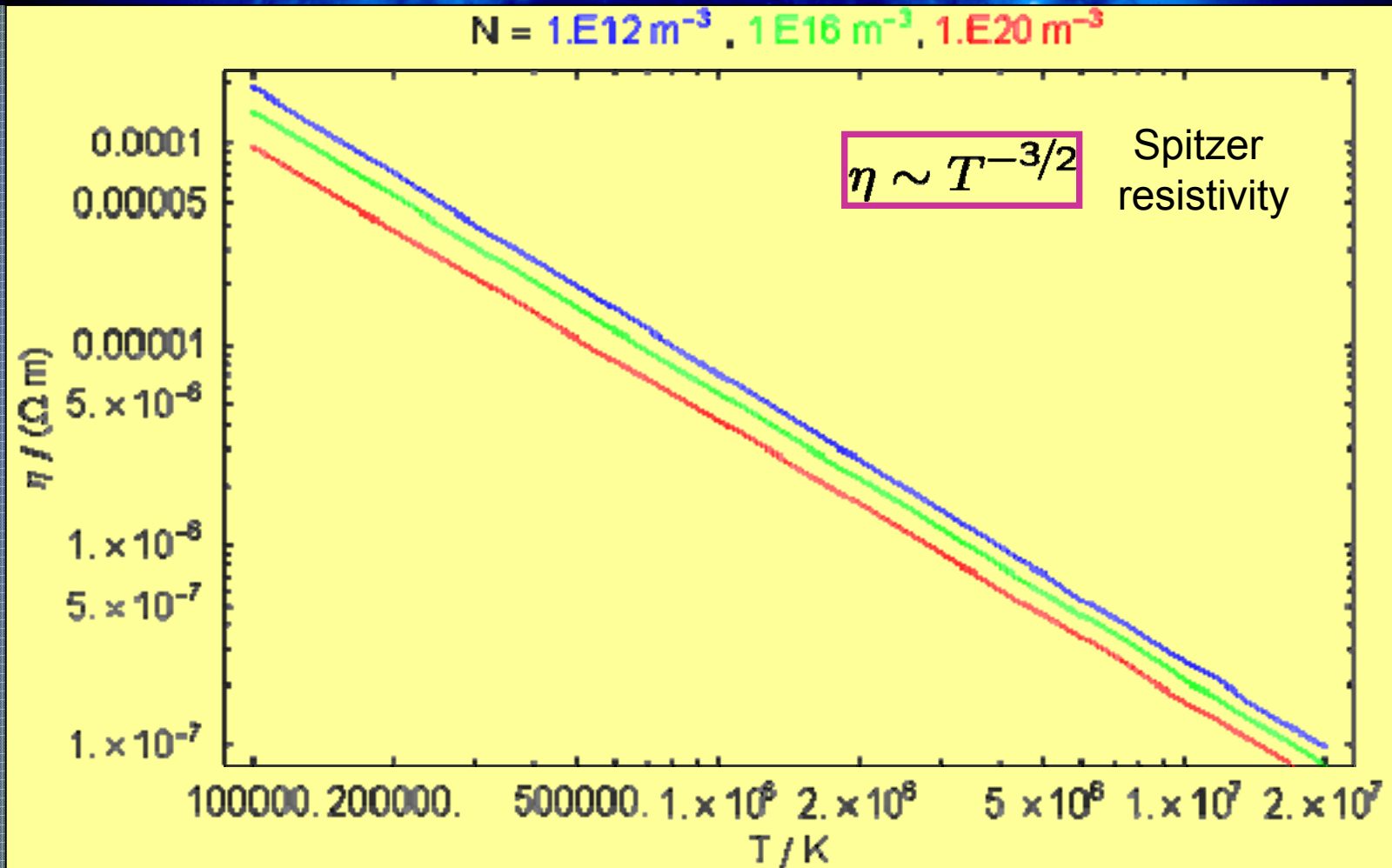
$$v^{-3} \rightarrow v_{\text{th}}^{-3} = \sqrt{\frac{\mu^3}{k_B^3} \cdot T^{-3/2}}$$

Spitzer resistivity $\eta \sim T^{-3/2}$

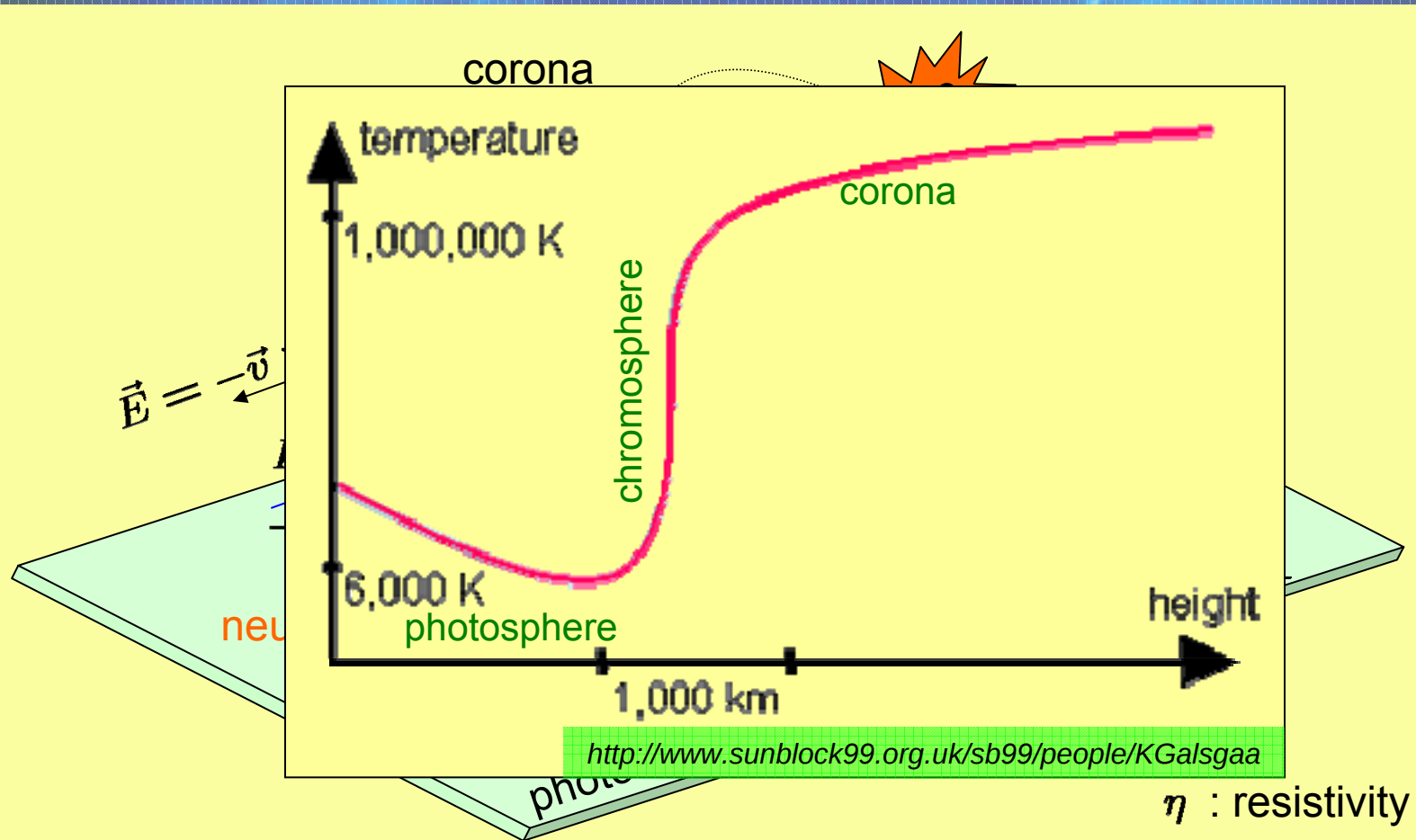
$$\begin{aligned} \nu_j &= \frac{4\pi C_j^2 \ln [\Lambda_j] N_j}{\mu_j^2 v_j^3} \\ \mu_j &= \left(\frac{1}{m_e} + \frac{1}{m_j} \right)^{-1} \\ v_j &= |v_j - v_e| \\ C_j &= \frac{(-e) \cdot q_j}{4\pi\epsilon_0} \\ \Lambda_j &= \sqrt{\frac{\lambda_D^2 + b_{0,j}^2}{2b_{0,j}^2}} \\ \lambda_D &= \sqrt{\frac{\epsilon_0 k_B T}{N e^2}} \\ b_{0,j} &= \frac{C_j}{\mu_j v_j^2} \end{aligned}$$

Resistivity (II)

– Temperature Dependence –



Current Model (II)



Frozen Field Lines

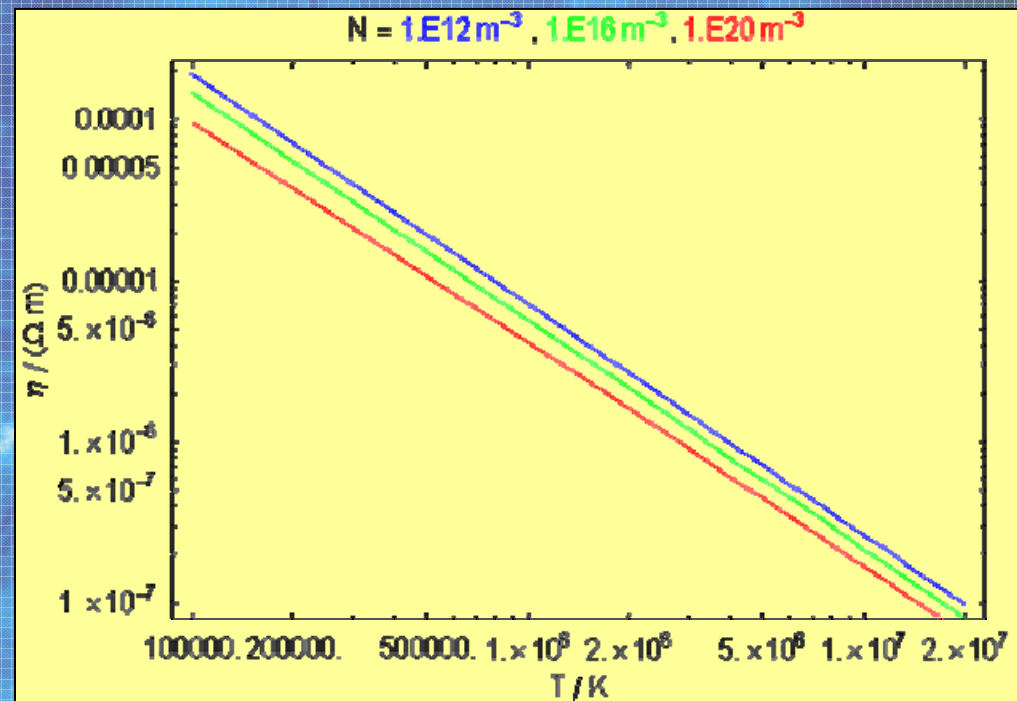
Condition for frozen field lines:

- Infinite conductivity
→ Vanishing resistivity

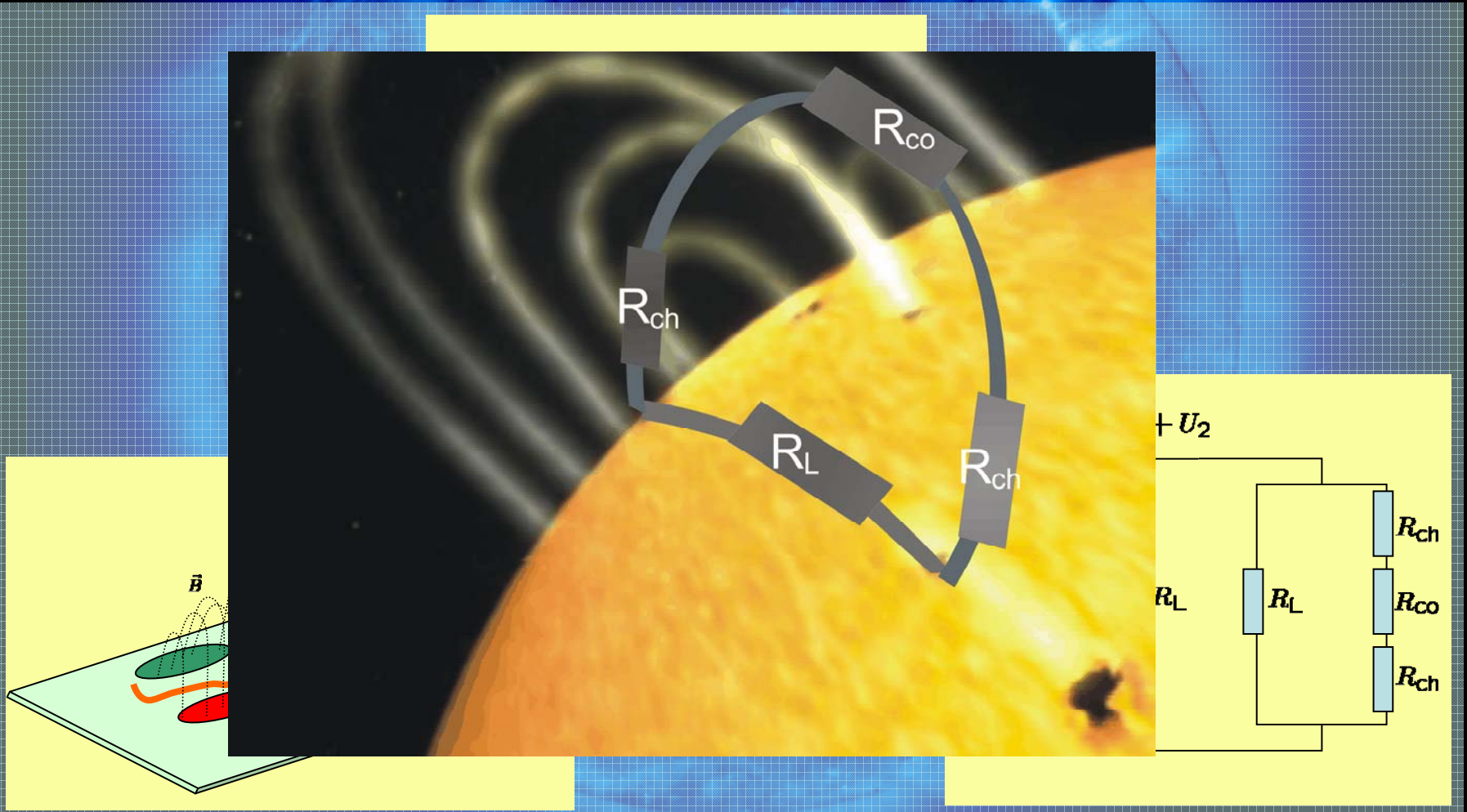
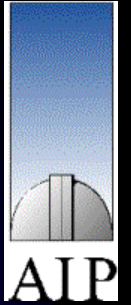
$$\eta \longrightarrow 0$$

Thus:

- Frozen Fields exist in the Corona, but not in the Photosphere



Replacing the Current Model by an Electrical Circuits



Resisto

For Comparison: Large Flares

Energy release: 10^{25} J

Power: 10^{22} W

=====

Timescale: ≈ 10 Minutes

	R_i	R_L	R_{ch}	R_{co}
L	26 Mm	10 Mm	2.6 Mm	120 Mm
r	2.6 Mm	2.6 Mm	2 Mm	5.2 Mm
η	$4 \cdot 10^{-3} \Omega m$	$4 \cdot 10^{-3} \Omega m$	$4 \cdot 10^{-3} \Omega m$	$4 \cdot 10^{-3} \Omega m$
T	6000 K	6000 K	6000 K	6000 K
N	$4 \cdot 10^{19} m^{-3}$	$4 \cdot 10^{19} m^{-3}$	$4 \cdot 10^{19} m^{-3}$	$4 \cdot 10^{19} m^{-3}$
R	$4.8 \cdot 10^{-9} \Omega$	$1.8 \cdot 10^{-9} \Omega$	$5.9 \cdot 10^{-10} \Omega$	$1.1 \cdot 10^{-10} \Omega$

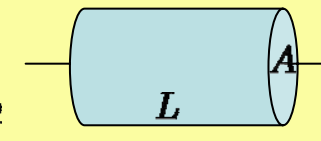
DC-Acceleration

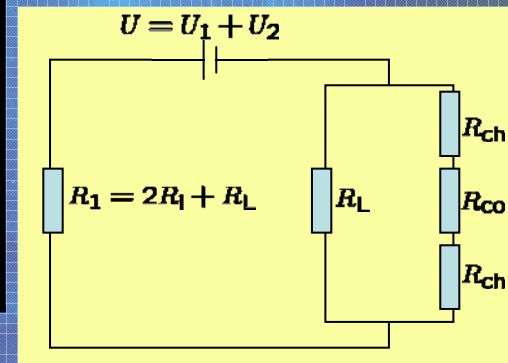
$$U_{co} \approx 326 \text{ kV}$$

$$E_{co} \approx -2.7 \text{ mV/m}$$

macroscopic conductor

$$R = \frac{\eta L}{A}$$

$$A = \pi r^2$$




Induction

$$v = 0.5 \text{ kms}^{-1}$$

$$B = 0.05 \text{ T (= 500 G)}$$

$$U_1 = U_2 = vBL[R_i] = 0.65 \cdot 10^9 \text{ V}$$

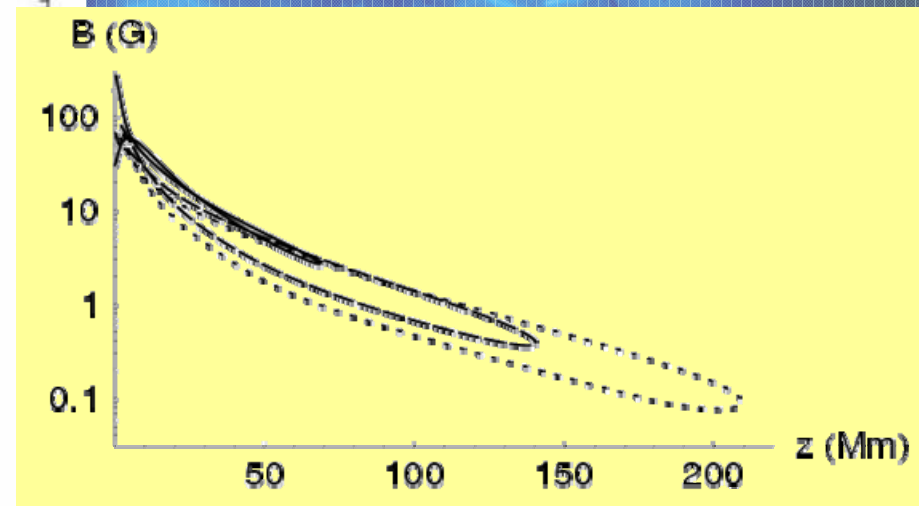
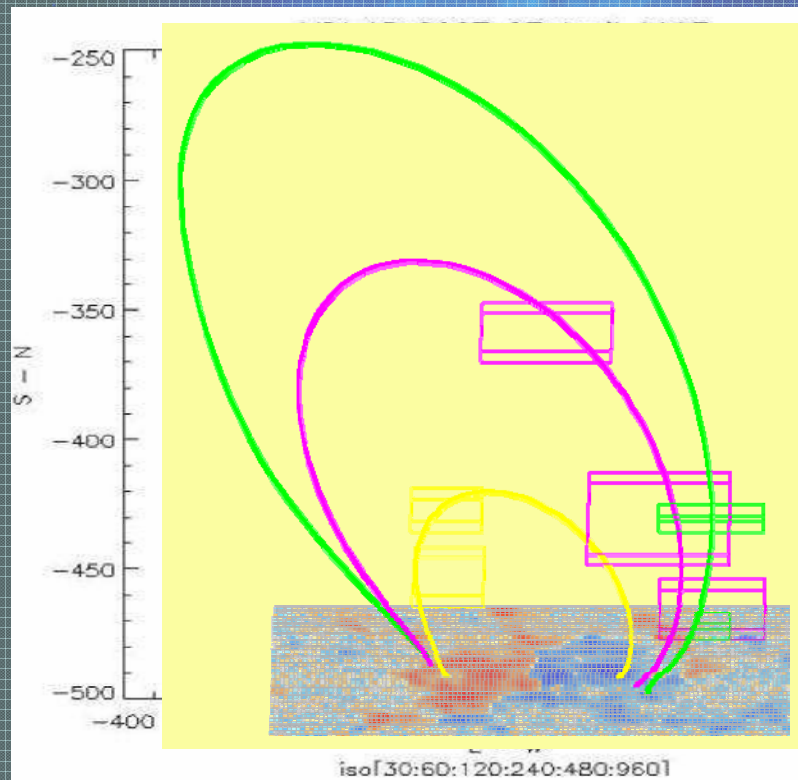
Power

$$P_{co+ch} \approx 2.3 \cdot 10^{27} \text{ W}$$

$$P_{co} \approx 4.8 \cdot 10^{20} \text{ W}$$

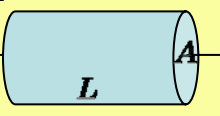
Resistors

– Estimation of Parameters –



macroscopic conductor

$$R = \frac{\eta L}{A}$$

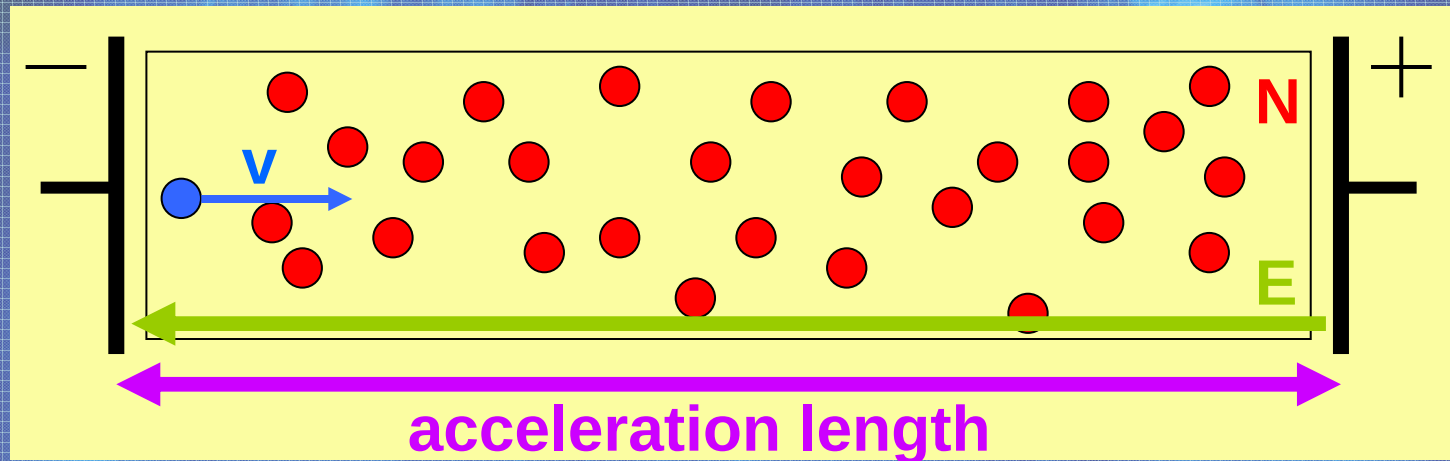
$$A = \pi r^2$$


	R_i	R_L	R_{ch}	R_{co}
L	26 Mm	10 Mm	2.6 Mm	120 Mm
r	2.6 Mm	2.6 Mm	2 Mm	5.2 Mm

Reference: Aurass et al. (2005A&A...435.1137A)

DC-Field Electron Acceleration

– Condenser –



Setup for Calculations

- 1D calculation
- full relativistic
- initial particle distribution corresponds to a κ – distribution

Relativistic 1D-Acceleration of Electrons with DC Fields

Law of Motion

$$\frac{dp}{dt} = m_e c \gamma^3 \frac{d\beta}{dt} = -eE - \text{sign}[\beta] (D_e + D_p)$$

Dreicer Velocity & Dreicer Field

$$\exists! \beta_{\text{Dreicer}} \in [0, 1) \cap \mathbf{R} : \left. \frac{dp}{dt} \right|_{\beta_{\text{Dreicer}}} = 0$$

$$\Rightarrow E_{\text{Dreicer}} := E|_{\beta_{\text{Dreicer}}} \sim \frac{1}{\beta_{\text{Dreicer}}^2}$$

Coulomb Collisions (Coulomb Friction)

$$D_j \sim \frac{Z_j^2 N_j}{\mu_j^2} \cdot \beta^{-2}$$

$$\lim_{\beta \rightarrow \tilde{\beta} \gg \beta_{\text{Dreicer}}} [D_j] \sim 0$$

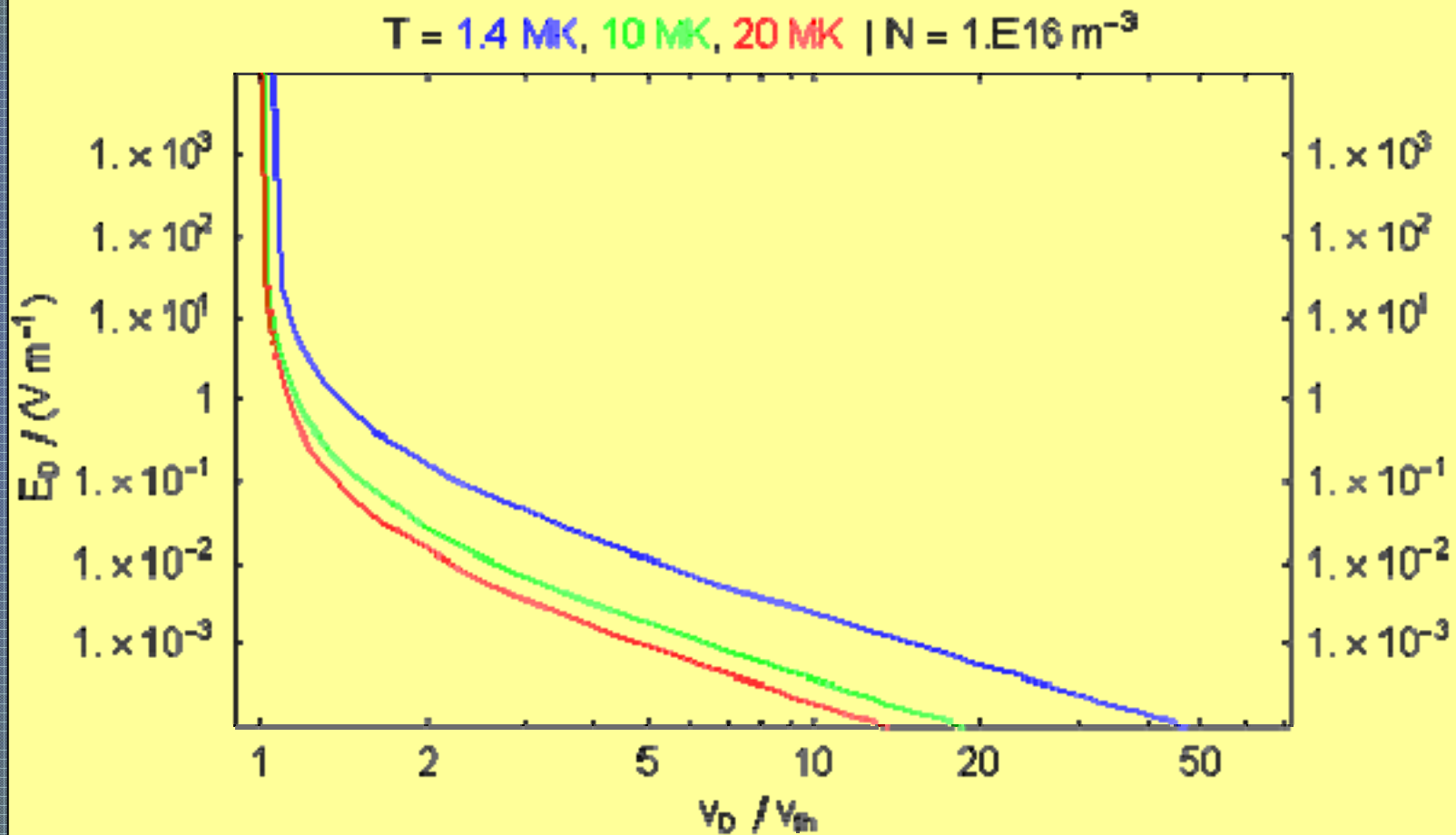
Brief Summary

$$\beta_{\text{end}}[t] = \beta_{\text{end}}[\beta_{\text{initial}}, E, t]$$

$$\begin{aligned} \beta &= v/c \\ \gamma &= (1 - \beta^2)^{-1/2} \\ \mu_j &= \frac{m_e m_j}{m_e + m_j} \\ D_j &\sim \frac{Z_j^2 N_j}{\mu_j^2} \cdot \frac{1}{\beta^2} \\ v_D &= c \beta_{\text{Dreicer}} \end{aligned}$$

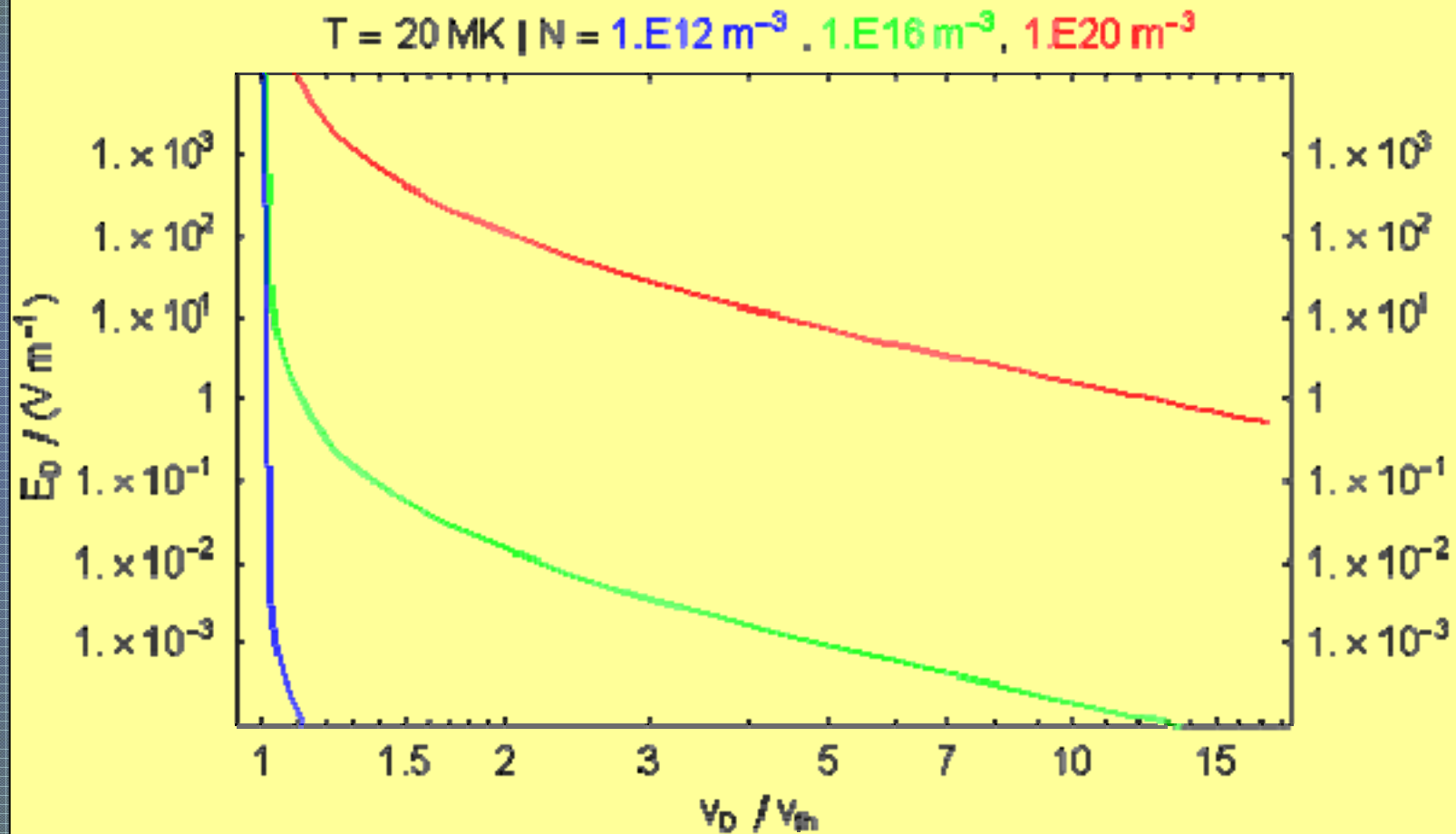
Dreicer Field

– Temperature Dependence –

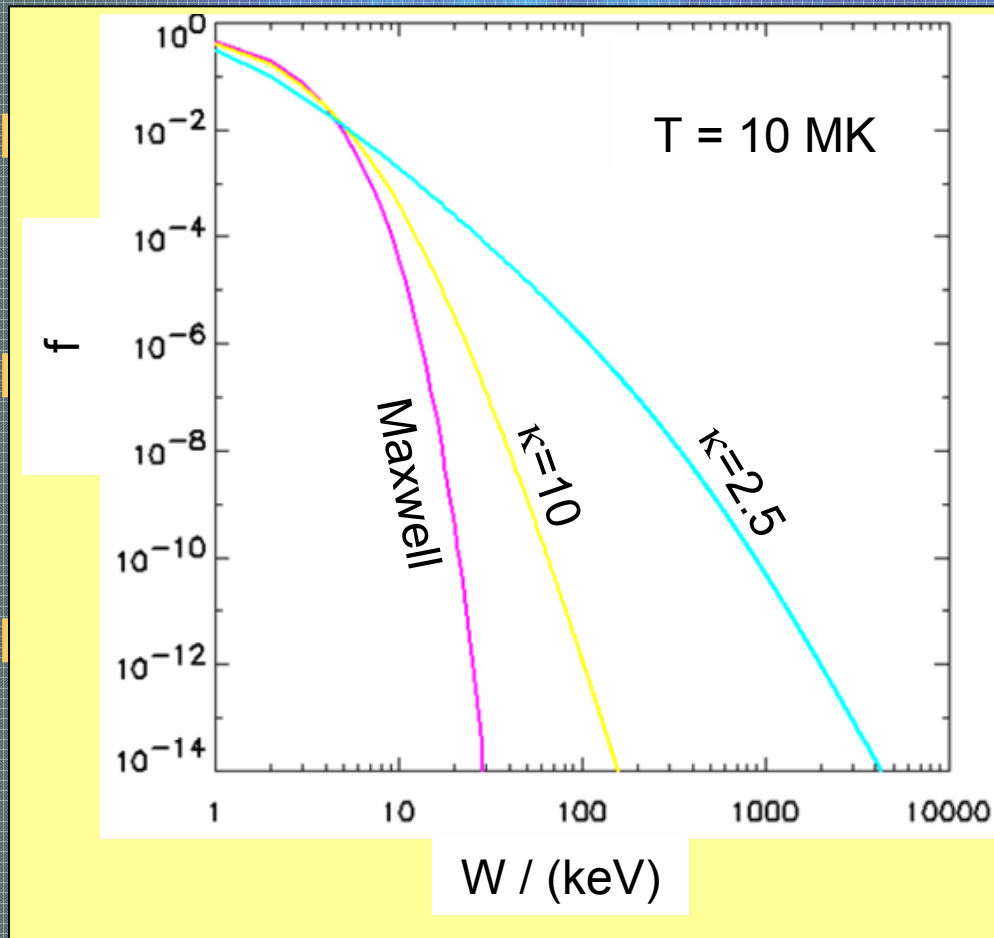


Dreicer Field

– Density Dependence –



Kappa Distribution



Brief Summary:

At energies much higher than the thermal energy the distribution does not decrease exponentially (as Maxwell Distribution) but corresponding to a power law.

$$= \frac{1}{\sqrt{2\pi\kappa\epsilon_{\kappa}c^2}} \frac{\Gamma[\kappa + 1]}{\Gamma[\kappa + 1/2]} \text{ if } \epsilon_{\kappa} \ll 1$$

ture

$$\Rightarrow \epsilon_{\kappa} = \frac{(\kappa - 1/2)}{\kappa} \epsilon_{\text{therm}}$$

$$\epsilon = \frac{W_{\text{kin}}}{m_e c^2} = \gamma - 1$$

Distribution Function and the Flux for the Accelerated Electrons

Distribution Function

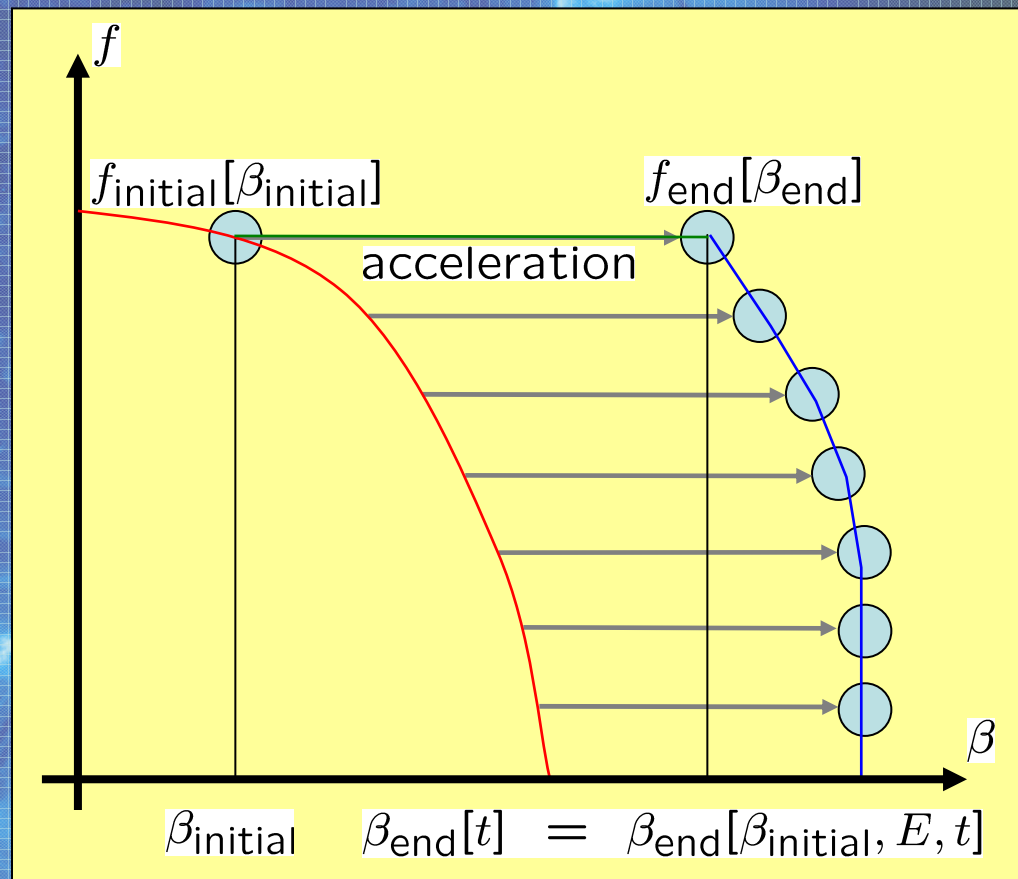
- single particle acceleration
- shifted distribution function
- extended condenser

Total Flux

$$\Phi = N_0 c^2 \int_0^{\infty} d\varepsilon \cdot \left(\frac{d\beta}{d\varepsilon} \cdot \beta f[\varepsilon] \right)$$

Differential Flux

$$j = \frac{d\Phi}{dW_{\text{kin}}} = \frac{d\Phi}{d\varepsilon} \cdot \frac{d\varepsilon}{dW_{\text{kin}}}$$



Flux

Total Flux

$$\Phi = N_0 c^2 \int_0^{\infty} d\varepsilon \cdot \left(\frac{d[\beta^2/2]}{d\varepsilon} f[\varepsilon] \right)$$

Brief Summary:

At energies much higher than the thermal energy the differential flux does not decrease exponentially but corresponding to a power law.

Differential Flux

$$j = \frac{d\Phi}{dW_{\text{kin}}} = \frac{d\Phi}{d\varepsilon} \cdot \frac{d\varepsilon}{dW_{\text{kin}}}$$

Example: Non-Accelerated κ -Distribution Flux

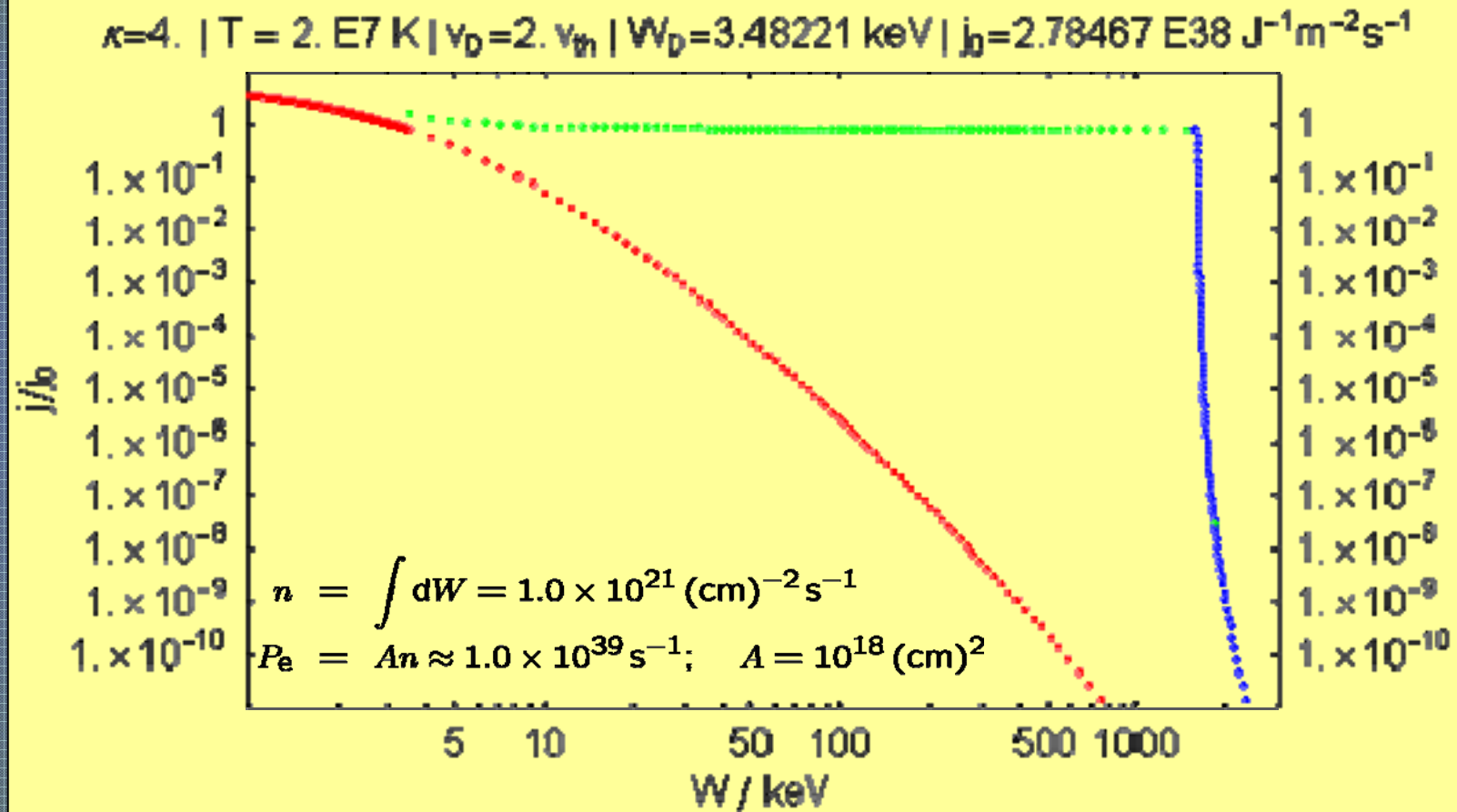
$$j = \frac{j_0}{\sqrt{2\pi\kappa\varepsilon_\kappa}} \cdot \frac{\Gamma[\kappa + 1]}{\Gamma[\kappa + 1/2]} \cdot \frac{1}{(1 + \varepsilon)^3} \cdot \left(1 + \frac{\varepsilon}{\kappa\varepsilon_\kappa} \right)^{-\kappa-1}$$

$$j_0 = \frac{N_0}{m_e c}$$



Calculation (I)

Electric Field: -13.53 mV m^{-1} $j_0 = 4.46 \text{ E18 (keV)}^{-1} (\text{cm})^{-2} \text{ s}^{-1}$



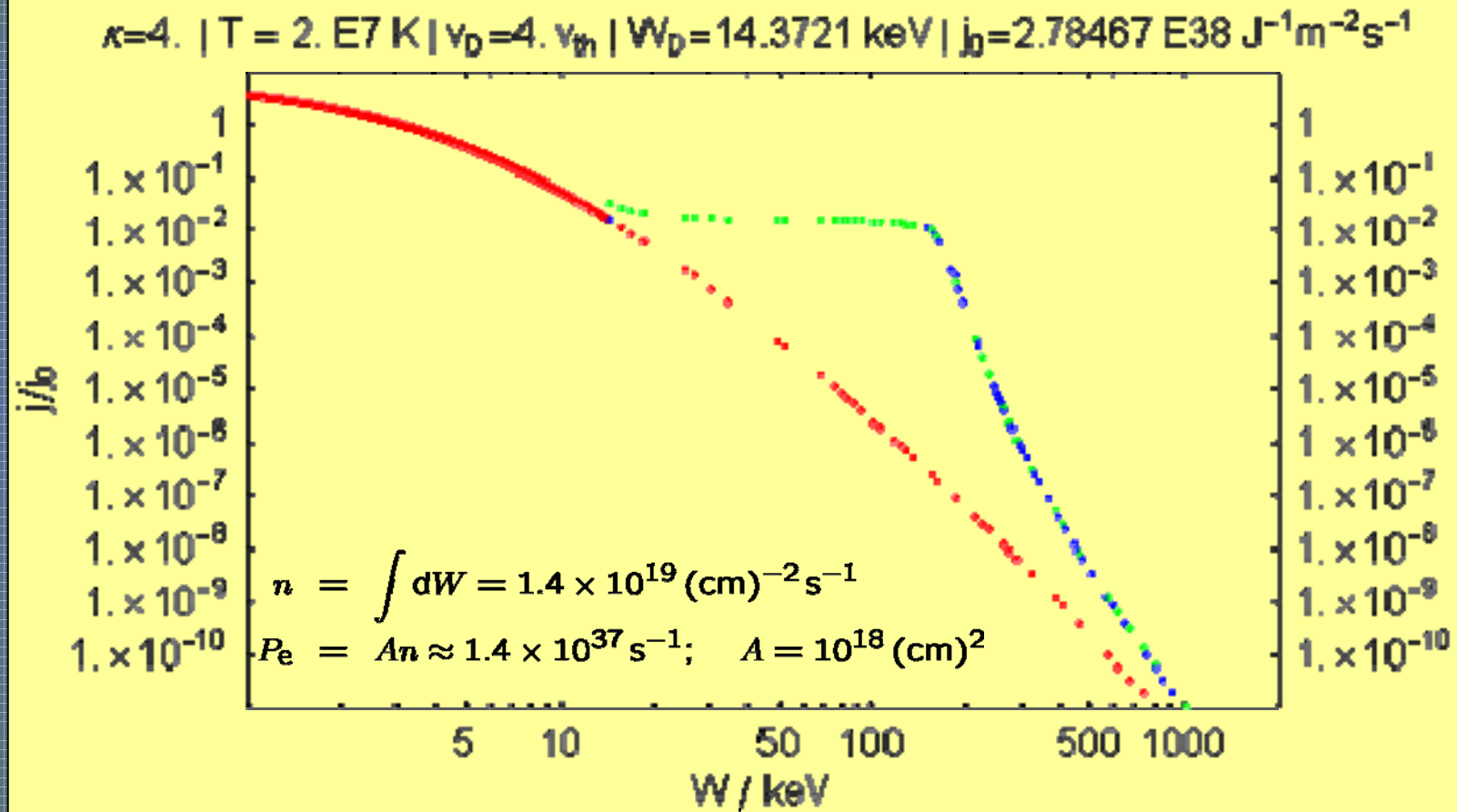
Acceleration length: 120 Mm

Electron density: 10^{16} m^{-3}



Calculation (II)

Electric Field: -1.60 mV m^{-1} $j_0 = 4.46 \text{ E18 (keV)}^{-1} \text{ (cm)}^{-2} \text{ s}^{-1}$ AIP



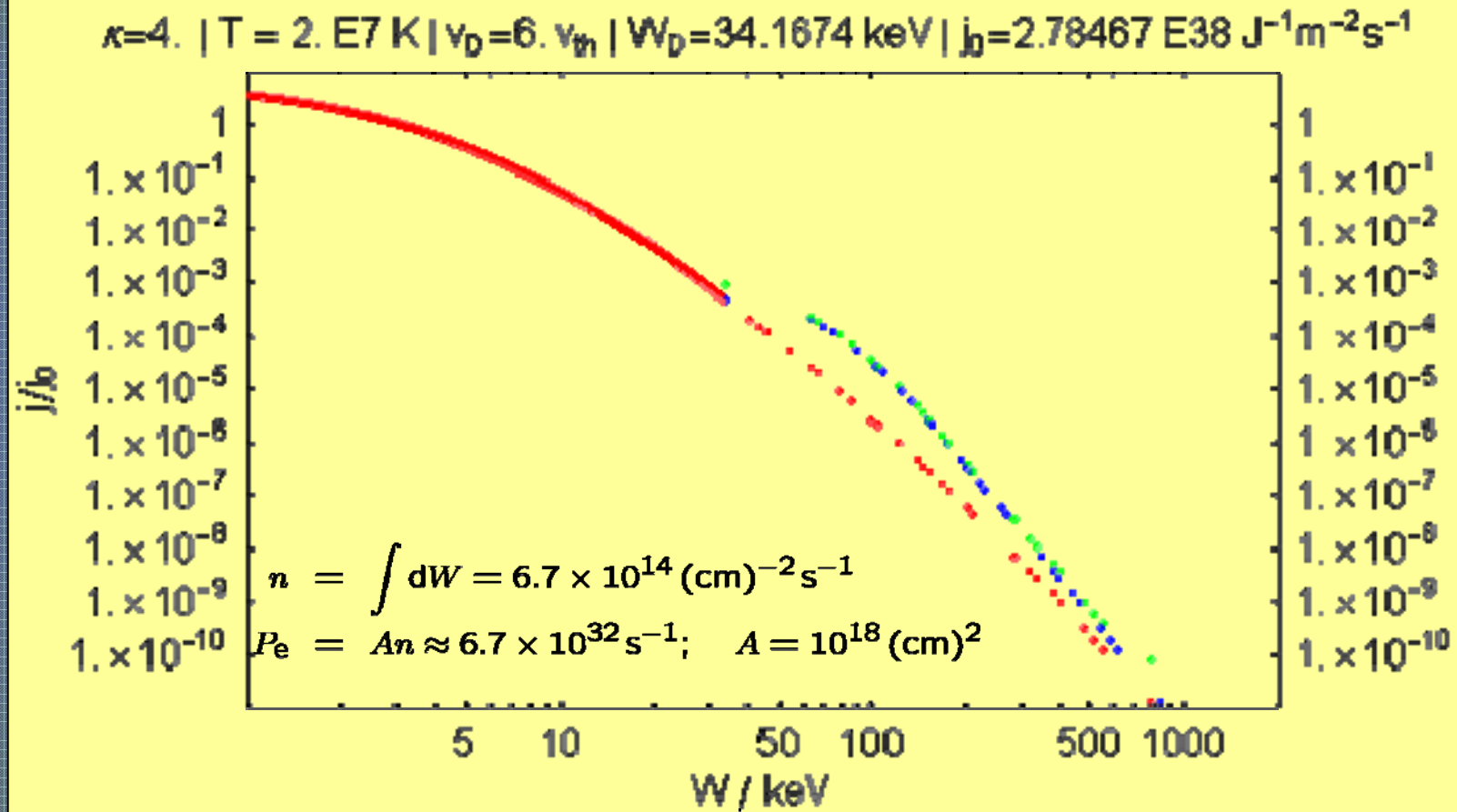
Acceleration length: 120 Mm

Electron density: 10^{16} m^{-3} 20



Calculation (III)

Electric Field: -0.59 mV m^{-1} $j_0 = 4.46 \text{ E18 (keV)}^{-1} (\text{cm})^{-2} \text{ s}^{-1}$



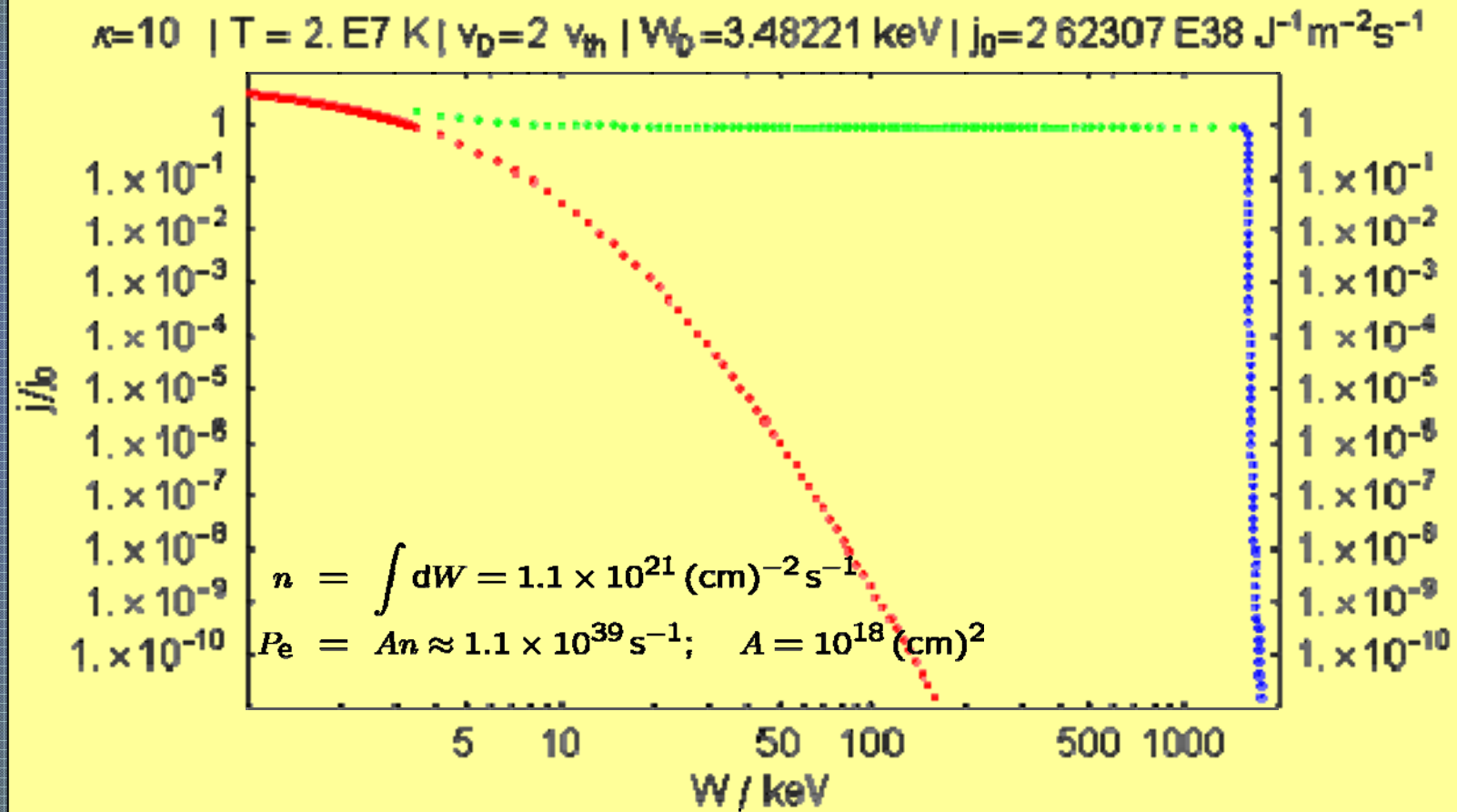
Acceleration length: 120 Mm

Electron density: 10^{16} m^{-3}



Calculation (IV)

Electric Field: -13.53 mV m^{-1} $j_0 = 4.20 \text{ E18 (keV)}^{-1} (\text{cm})^{-2} \text{ s}^{-1}$ AIP



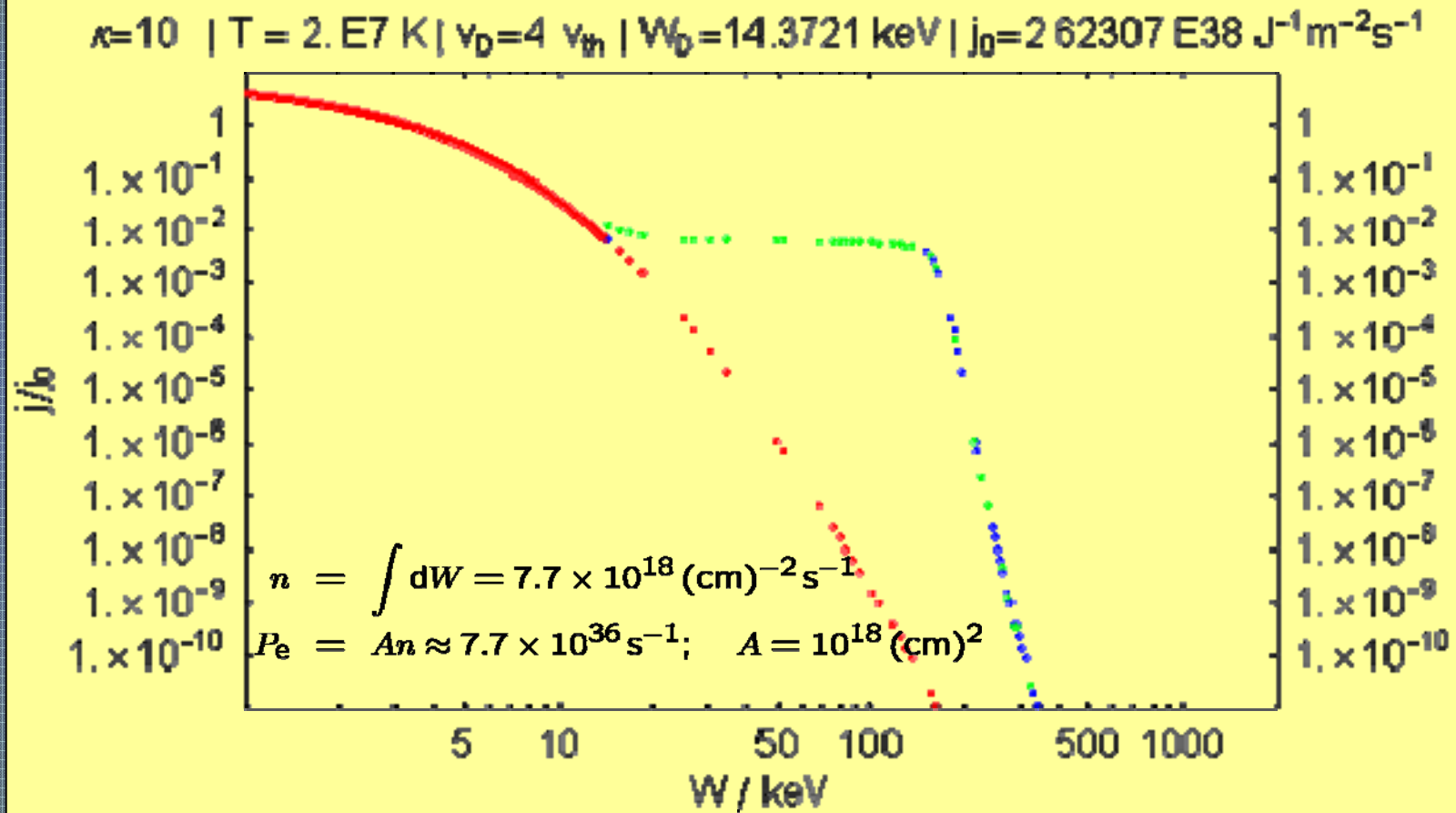
Acceleration length: 120 Mm

Electron density: 10^{16} m^{-3} 22



Calculation (V)

Electric Field: -1.60 mV m^{-1} $j_0 = 4.20 \text{ E18 (keV)}^{-1} (\text{cm})^{-2} \text{ s}^{-1}$ AIP

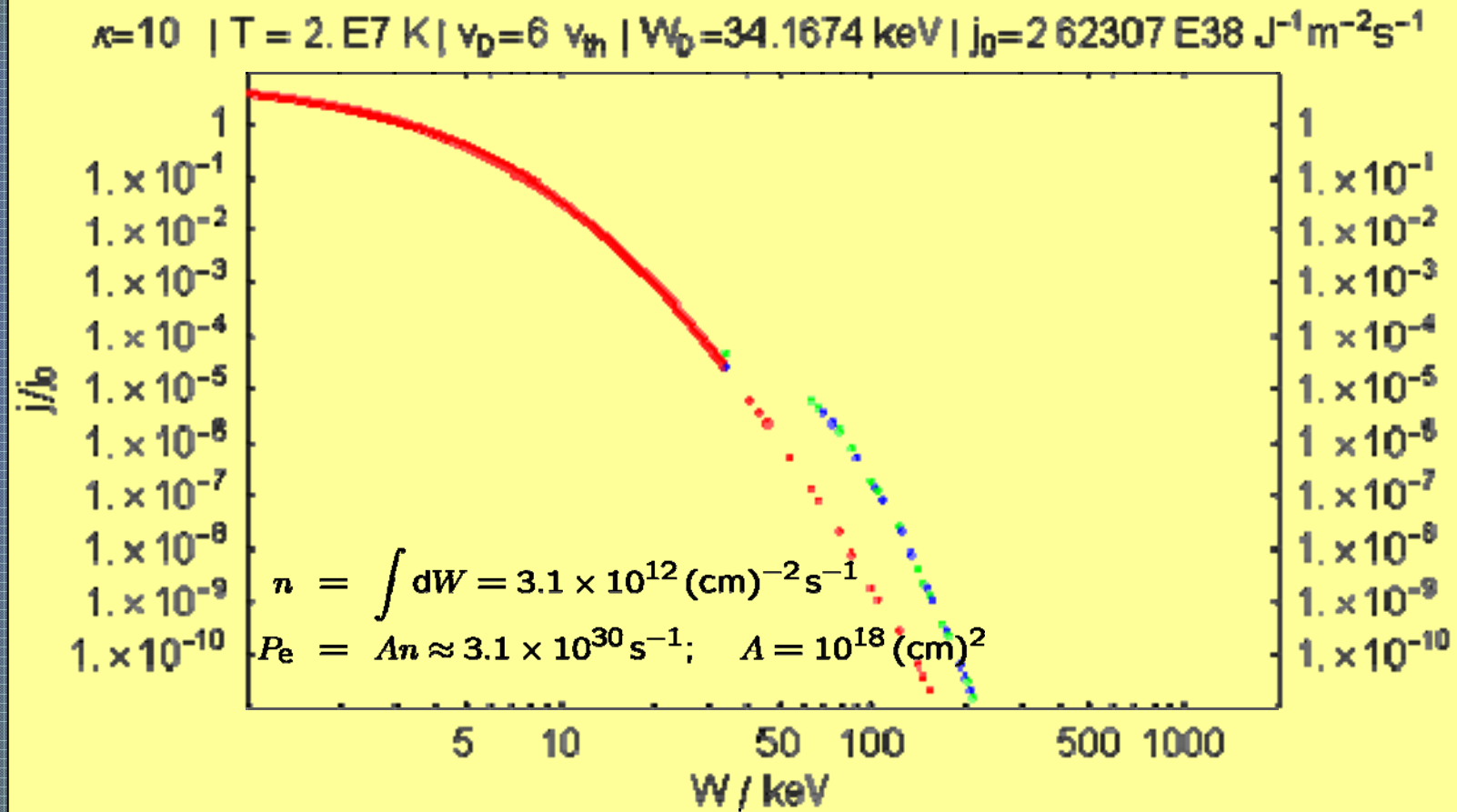


Acceleration length: 120 Mm Electron density: 10^{16} m^{-3} 23



Calculation (VI)

Electric Field: -0.59 mV m^{-1} $j_0 = 4.20 \text{ E18 (keV)}^{-1} (\text{cm})^{-2} \text{ s}^{-1}$ AIP

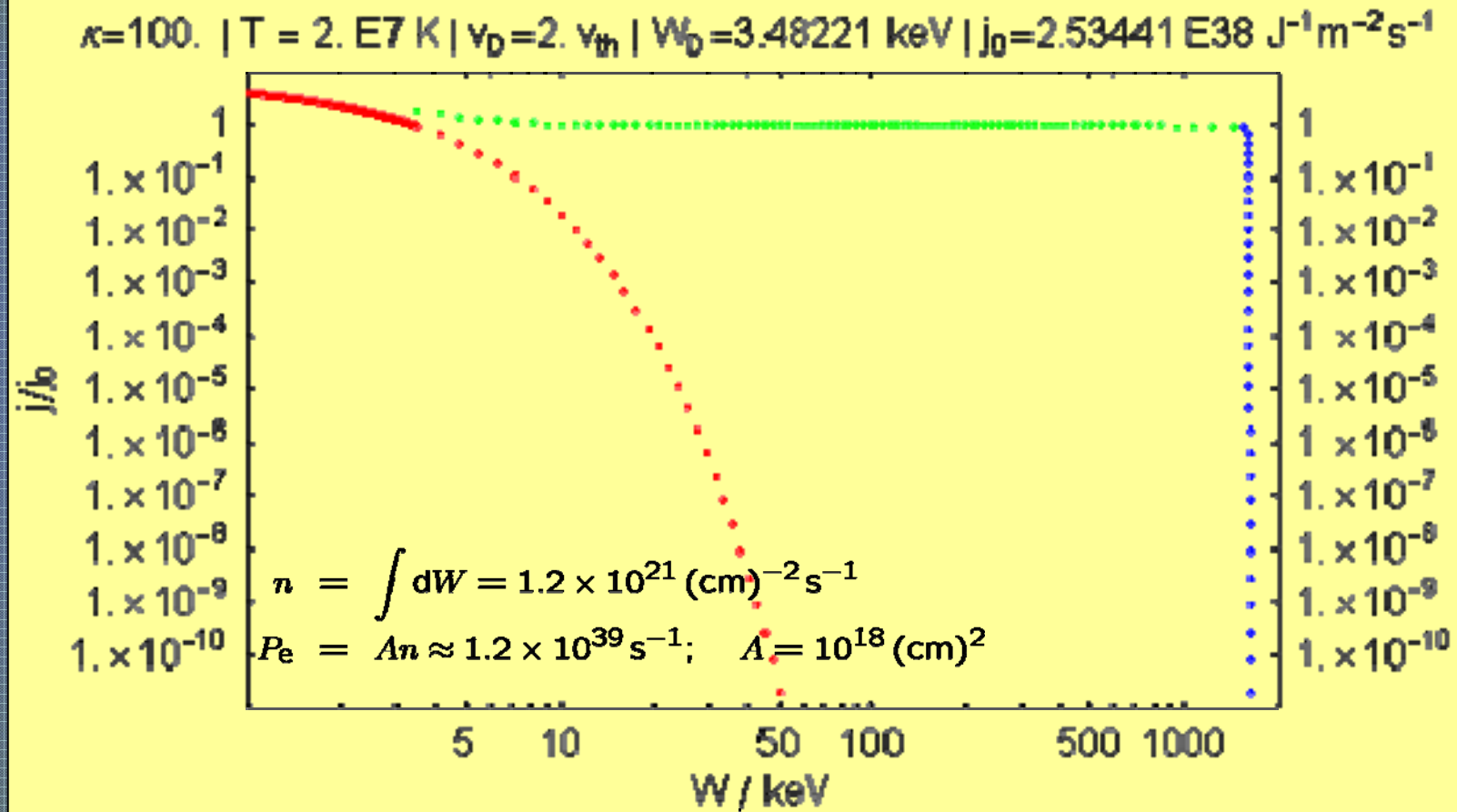


Acceleration length: 120 Mm Electron density: 10^{16} m^{-3} ₂₄



Calculation (VII)

Electric Field: -13.53 mV m^{-1} $j_0 = 4.06 \text{ E18 (keV)}^{-1} (\text{cm})^{-2} \text{ s}^{-1}$ AIP



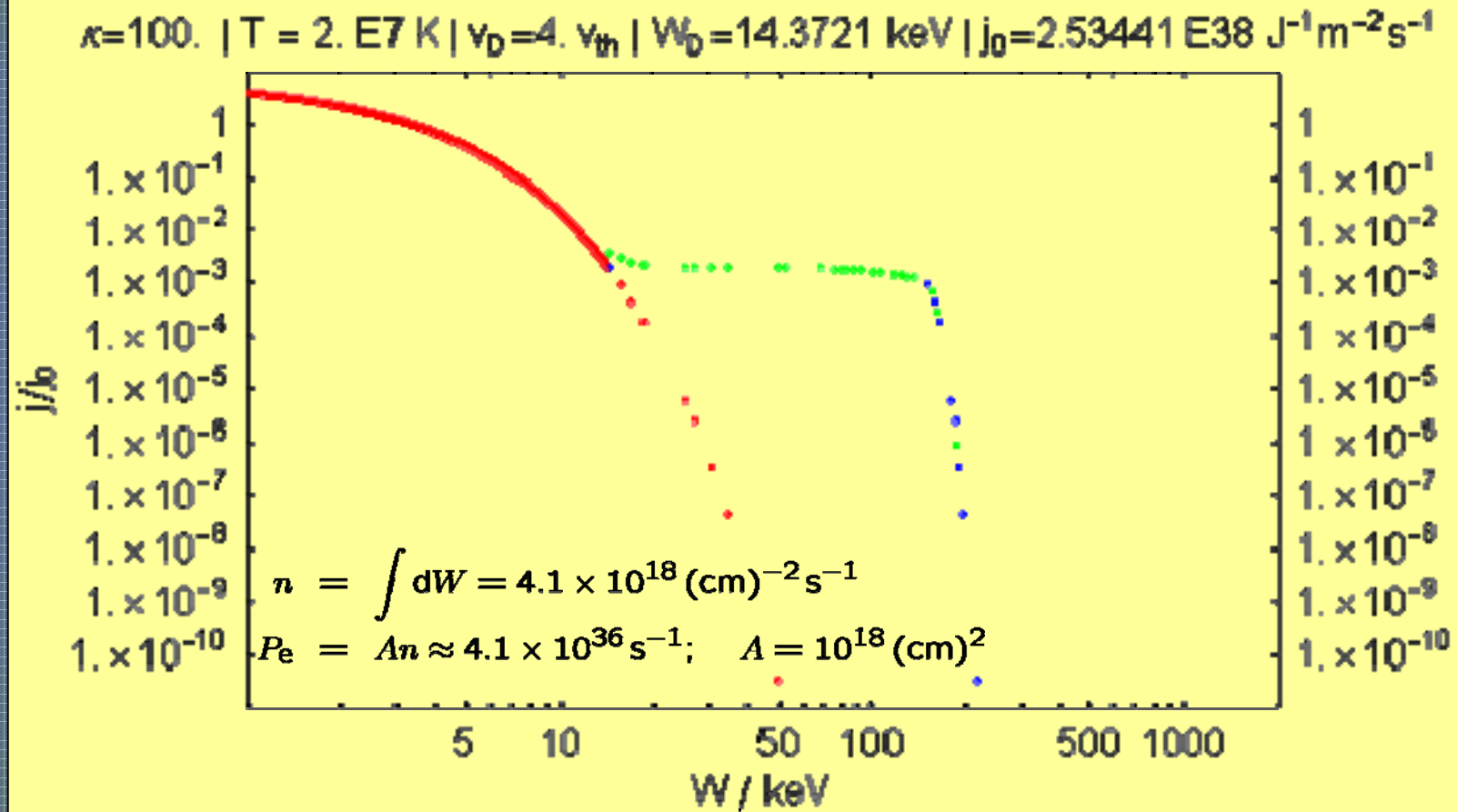
Acceleration length: 120 Mm

Electron density: 10^{16} m^{-3} 25



Calculation (VIII)

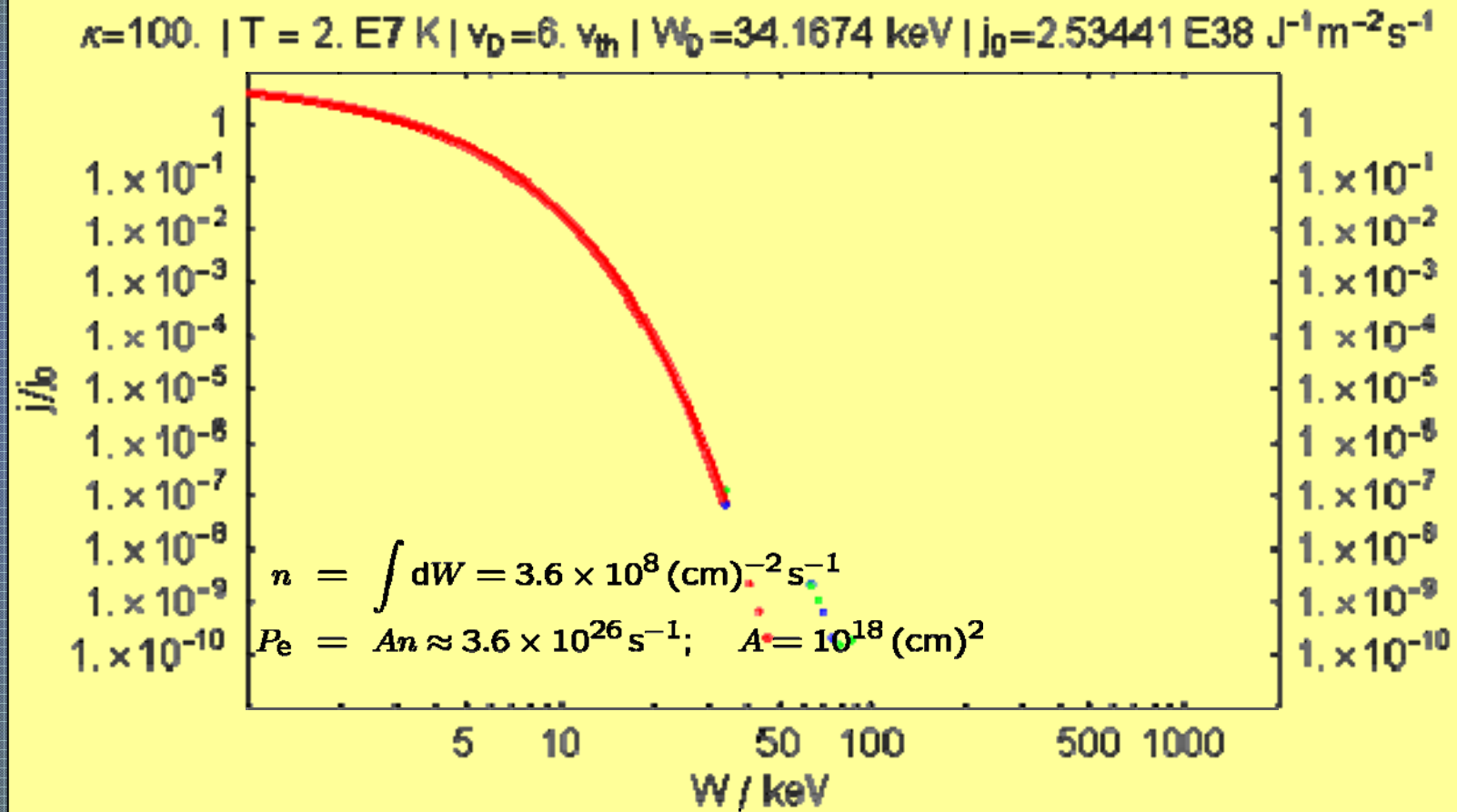
Electric Field: -1.60 mV m^{-1} $j_0 = 4.06 \text{ E18 (keV)}^{-1} (\text{cm})^{-2} \text{ s}^{-1}$ AIP



Acceleration length: 120 Mm Electron density: 10^{16} m^{-3} 26

Calculation (IX)

Electric Field: -0.59 mV m^{-1} $j_0 = 4.06 \text{ E18 (keV)}^{-1} (\text{cm})^{-2} \text{ s}^{-1}$

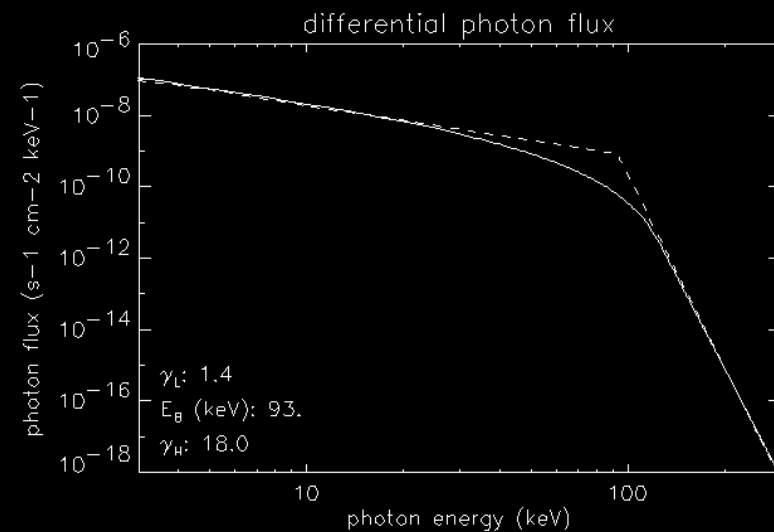
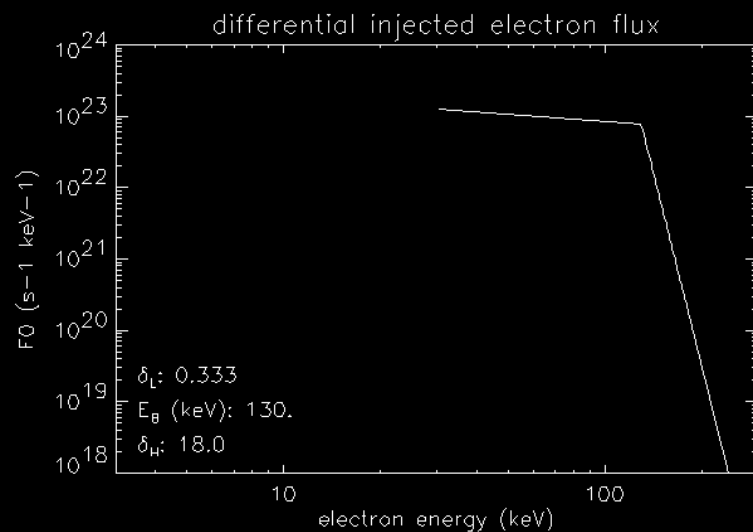


Acceleration length: 120 Mm

Electron density: 10^{16} m^{-3} 27

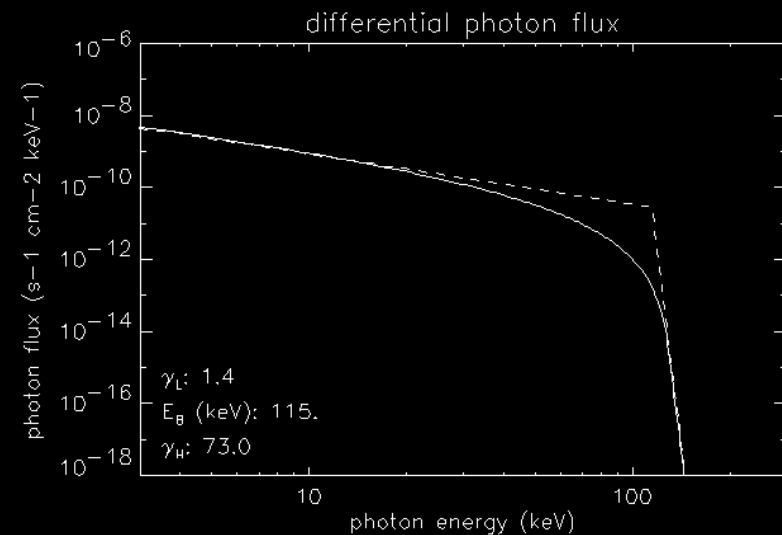
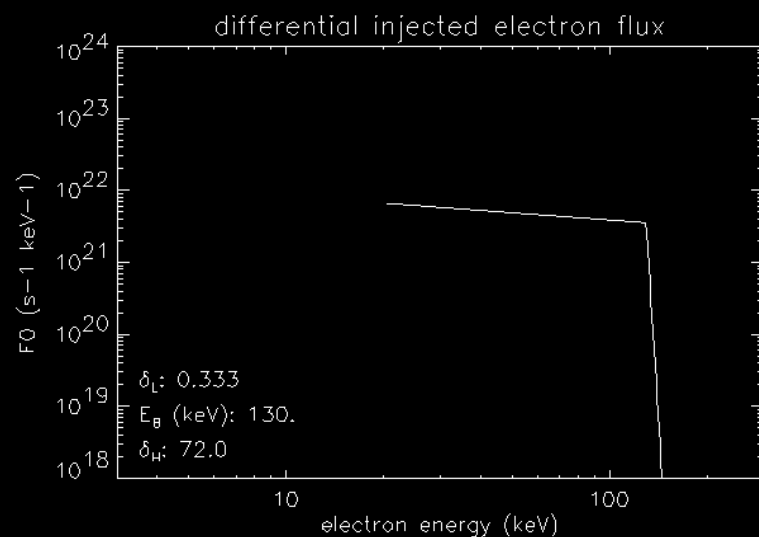
X-Ray Flux for Calculation (II)

$\kappa=4. \quad T = 2. \text{E}7 \text{K}$



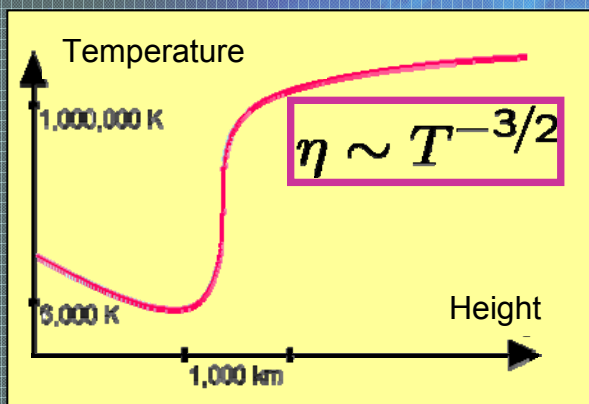
X-Ray Flux for Calculation (VIII)

$\kappa=100.$ | $T = 2. E7 K$

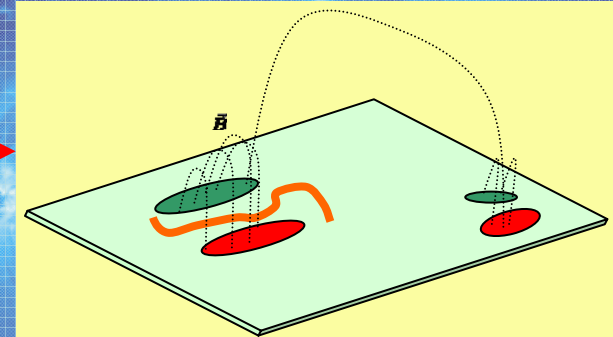


Summary

Electron Acceleration by DC Fields

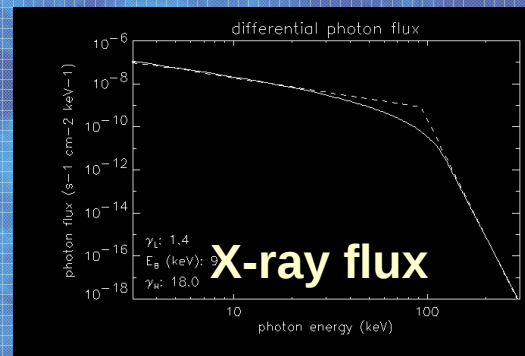
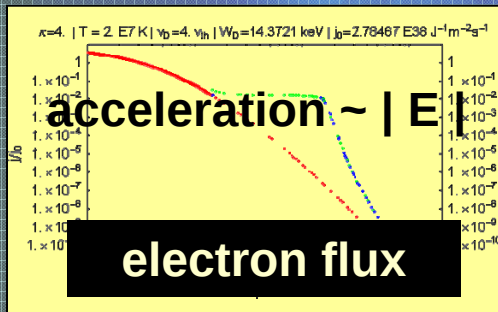
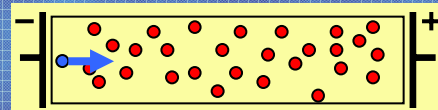


short circuited



electric field induction by
Photospheric footpoint motion

acceleration
by DC field



$$U = U_1 + U_2$$

Circuit Estimations

$$P_{\text{co+ch}} \sim 10^{27} \text{ W}$$

$$P_{\text{co}} \sim 10^{20} \text{ W}$$

$$U_{\text{co}} \sim 300 \text{ kV}$$

$$E_{\text{co}} \sim -3 \text{ mV m}^{-1}$$