Cluster Population Statistics: Basic Model + Some Recent Results

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elements of this talk

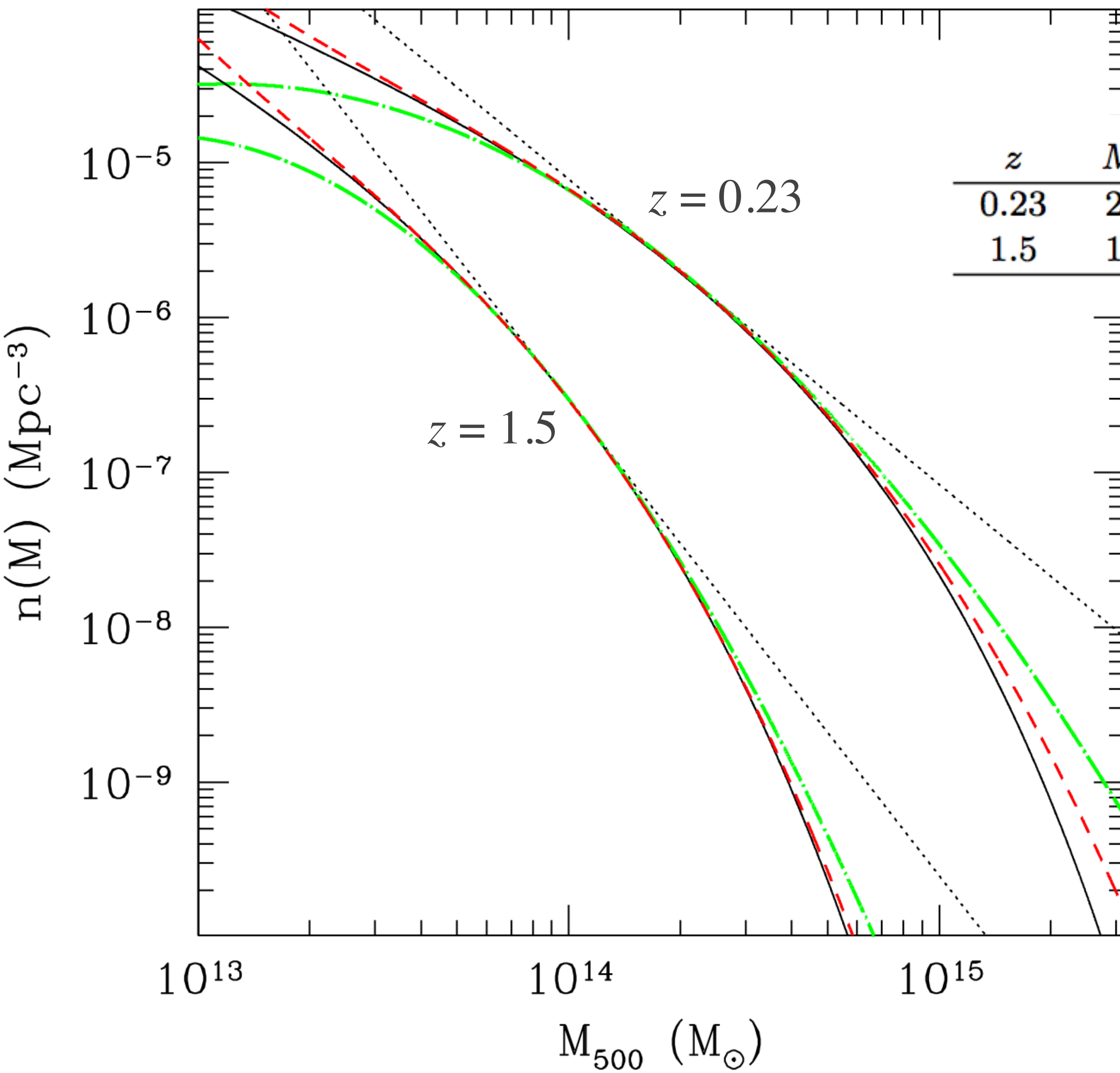
- FAST (analytic) expressions for **halo** multi-property statistics
  - multi-property space density and conditional probabilities
  - useful for: exposing degeneracies, speedier MCMC analysis (high S/N obs.)
- stacked dynamical mass estimates of redMaPPer clusters
  - simulation calibration exercise + application to SDSS
- early X-ray follow-up of DES redMaPPer clusters
  - X-ray temperature scaling with optical richness
- lies, damn lies and …
  - Planck-CLASH mass bias analysis

a fast model for multi-property halo statistics

Evrard, Arnault, Huterer, Farahi (2014)

1403.1456

simple idea : shape of the mass function is ~polynomial in log-space



z	M_p	A	β_1	β_2	β_3
0.23	2.0	1.944	1.97	0.70	0.40
1.5	1.0	0.293	3.07	1.20	0.73

A in units of 10^{-6} Mpc^{-3}

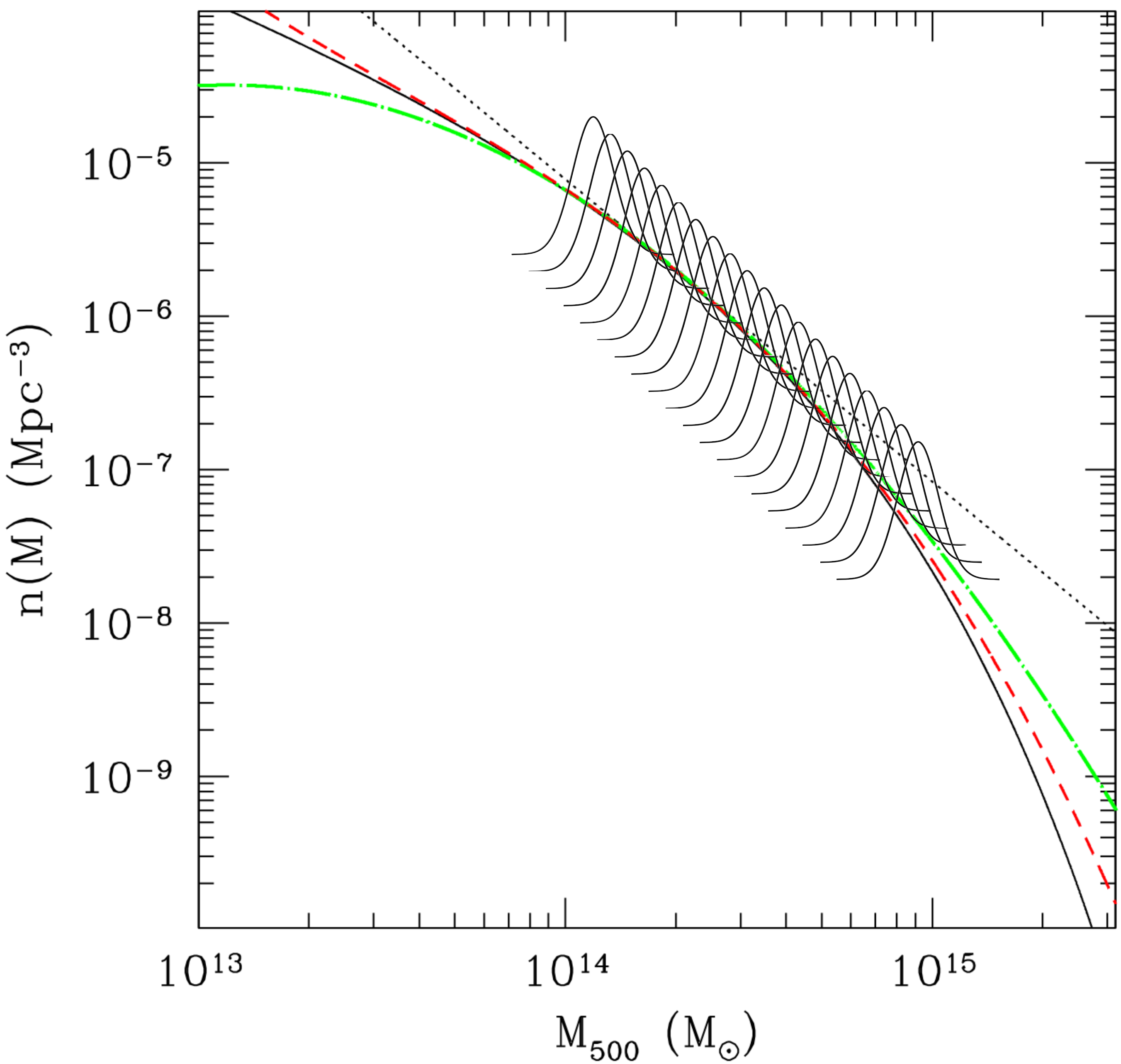
MF data from HMFcalc

(black lines)

Murray et al (2013)

ongoing work:

extend to arbitrary z by fitting A and beta's to polynomials in z benefit: closed-form expressions for cluster property likelihoods



local convolution is now analytic:

* **exact** for quadratic

* approx. for cubic

useful model outputs (closed form to 2nd order)

joint, multi-observable space density

$\mu = \ln M_{true}$

$s_i = \ln(\text{observable}_i)$

$$n_1(\mathbf{s}) = A_1' \exp\left[-\frac{1}{2}\left((\mathbf{s}-\boldsymbol{\pi})^T \mathbf{C}^{-1}(\mathbf{s}-\boldsymbol{\pi}) - \frac{\langle\mu|\mathbf{s}\rangle_1^2}{\sigma^2_{\mu|\mathbf{s},1}}\right)\right]$$

$$A_1' = A\ \sigma_{\mu|\mathbf{s},1}\ \left((2\pi)^{N-1}|\mathbf{C}|\right)^{-1/2}$$

$$\langle\mu|\mathbf{s}\rangle_1 = \left[\boldsymbol{\alpha}^T\mathbf{C}^{-1}(\mathbf{s}-\boldsymbol{\pi}) - \beta_1\right]\sigma^2_{\mu|\mathbf{s},1}$$

mean selected mass

$$\sigma^2_{\mu|\mathbf{s},1} = (\boldsymbol{\alpha}^T\mathbf{C}^{-1}\boldsymbol{\alpha})^{-1}.$$

variance in selected mass

<u>scaling relation inputs:</u>

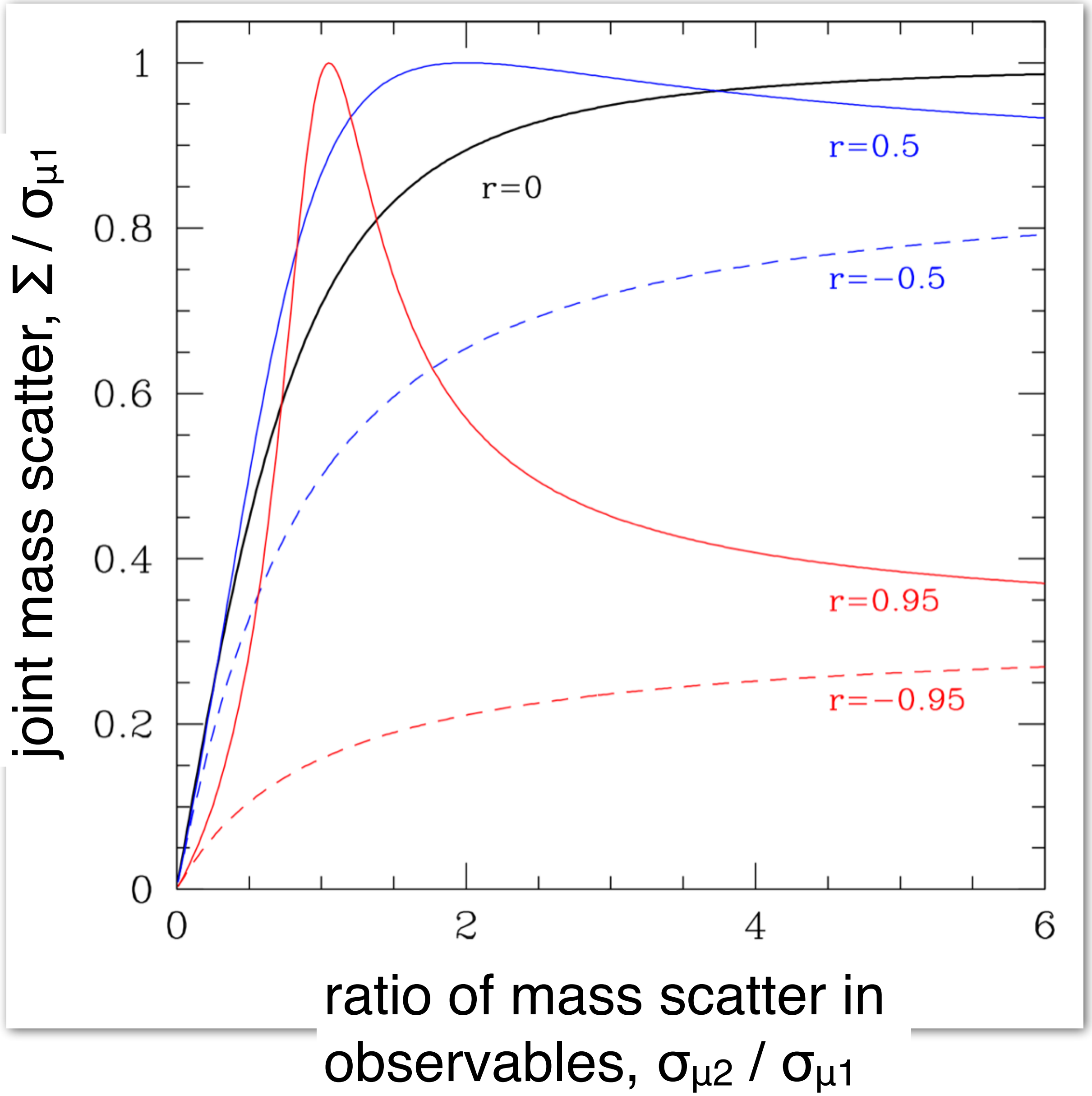
$$\langle\mathbf{s}|\mu\rangle = \boldsymbol{\pi} + \boldsymbol{\alpha}\mu$$

log-means (vector)

$$C_{ab} = \langle(s_a - \langle s_a|\mu\rangle)(s_b - \langle s_b|\mu\rangle)\rangle$$

covariance of log-observables deltas

mass scatter for joint selection in two observables



mass variance w/ joint selection

$$\Sigma^2 = \frac{1-r^2}{\sigma_{\mu 1}^{-2} + \sigma_{\mu 2}^{-2} - 2r\sigma_{\mu 1}^{-1}\sigma_{\mu 2}^{-1}}$$

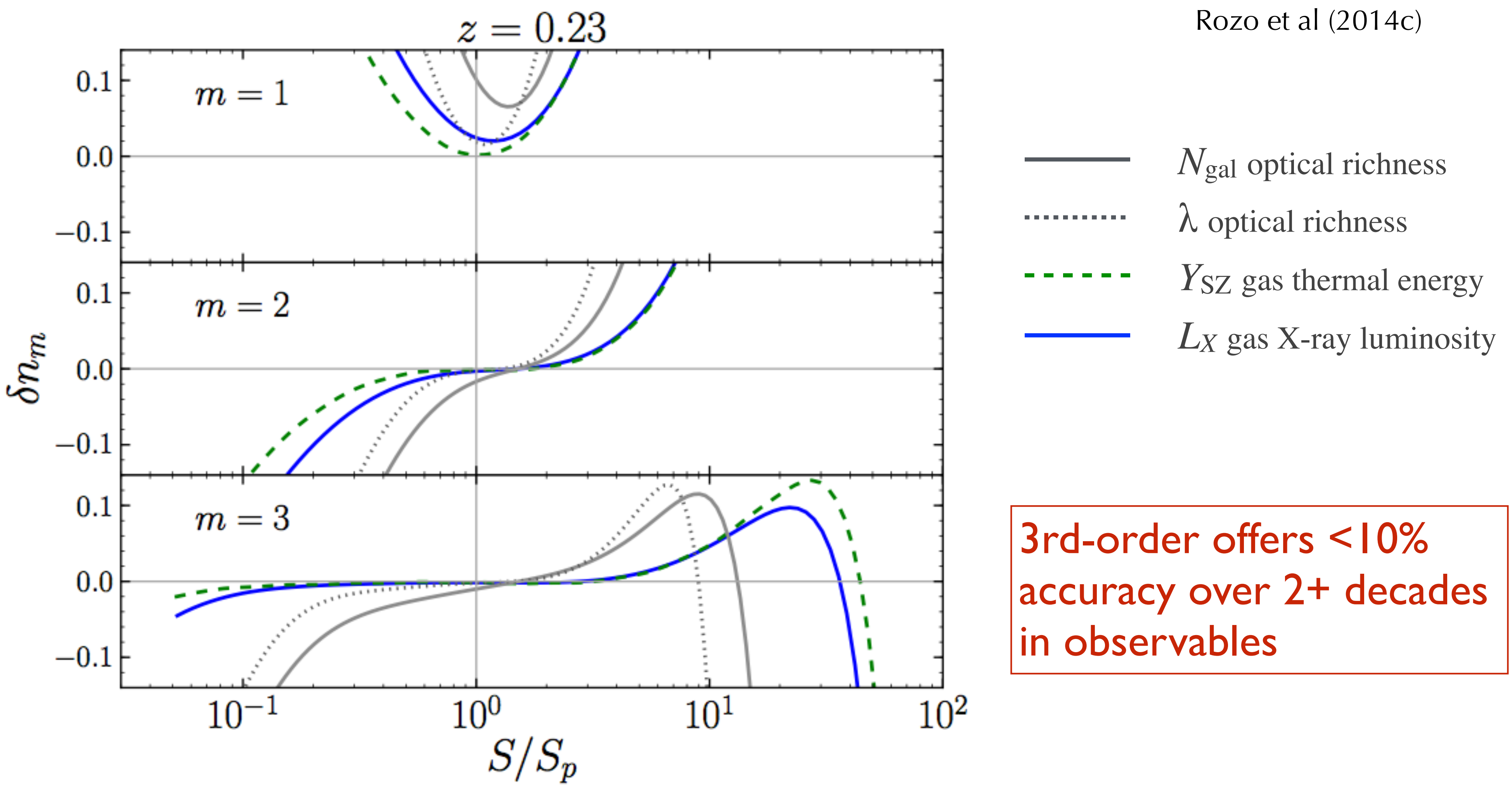
- joint selection always lowers mass scatter
- **anti-correlated observables** offer most improvement

  (r is correlation coefficient of properties 1 and 2 at fixed true mass)

optical/X-ray/SZ counts : accuracy relative to local Tinker convolution

S	$S_p = e^{\pi_s}$	α_s	$\sigma_{\ln S}$	$\sigma_{\mu\|s}$
L_X	0.61	1.55	0.39	0.252
$D_A^2 Y_{SZ}$	0.62	1.71	0.15	0.088
$N_{\rm gal}\,(\lambda)$	37	0.94	0.42 (0.21)	0.45 (0.23)

Rozo et al (2014c)



3rd-order offers <10% accuracy over 2+ decades in observables

more model output: scaling behavior of property-selected samples

log-normal PDF, $P(s_b \mid s_a)$, for s_a-selected (complete) samples

$$
\begin{aligned}
\langle s_b | s_a \rangle_1 &= \pi_b + \alpha_b [\, \langle \mu | s_a \rangle_1 + \beta_1 \, r_{ab} \, \sigma_{\mu|a,1} \, \sigma_{\mu|b,1} \,] \\
\sigma^2_{b|a,1} &= \alpha_b^2 \, [\, \sigma^2_{\mu|a,1} + \sigma^2_{\mu|b,1} - 2 r_{ab} \, \sigma_{\mu|a,1} \sigma_{\mu|b,1} \,]
\end{aligned}
$$

mean s_b at fixed s_a

variance ”

$\mu = \ln M_{true}$

$s_i = \ln(\text{property}_i)$

These are the measurable* quantities in survey samples.

* with (astrophysical and instrumental) noise … :(

stacked dynamical mass estimates of redMaPPer clusters

Farahi, Evrard, Rozo, Rykoff, Wechsler (2016)

1601.05773

simulation of stacked spectroscopic mass estimates (SDSS, soon DES+DESI)

Motivation: SDSS redMaPPer spectroscopic analysis

redMaPPer IV: Photometric Membership Identification of Cluster Galaxies with 1% Precision

E. Rozo[1,2,3], E. S. Rykoff[1,2], M. Becker[1,2], R. M. Reddick[1,2,4], R. H. Wechsler[1,2,4],

[1] *Kavli Institute for Particle Astrophysics and Cosmology, P.O. Box 2450, Stanford, CA 94305*

[2] *SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, CA 94025*

[3] *Department of Physics, University of Arizona, 1118 E 4th St, Tucson, AZ 85721*

[4] *Department of Physics, Stanford University, 382 Via Pueblo Mall, Stanford, CA 94305*

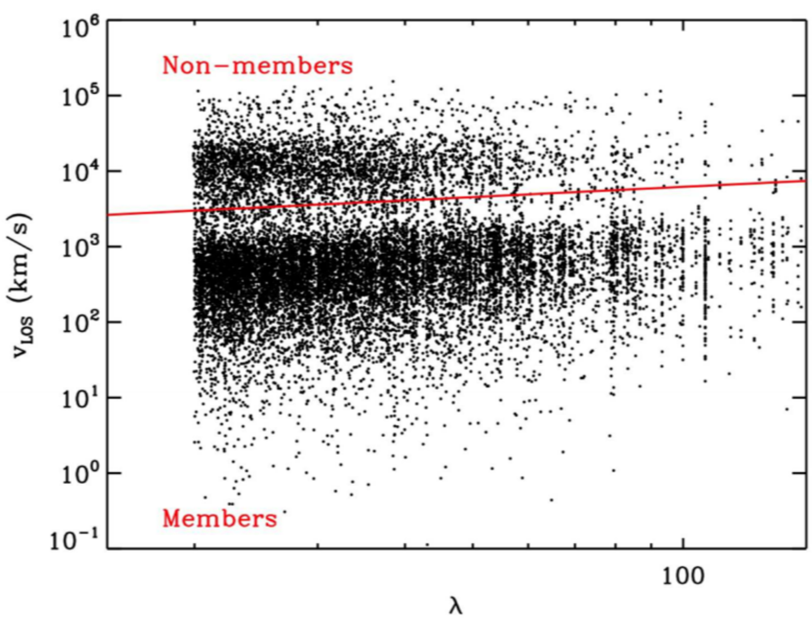


Figure 2. Line-of-sight velocities of spectroscopic central–satellite pairs of redMaPPer galaxy clusters. The red line shows the initial split into spectroscopic members and non-cluster members used to initialize the calibration of the velocity dispersion of the clusters.

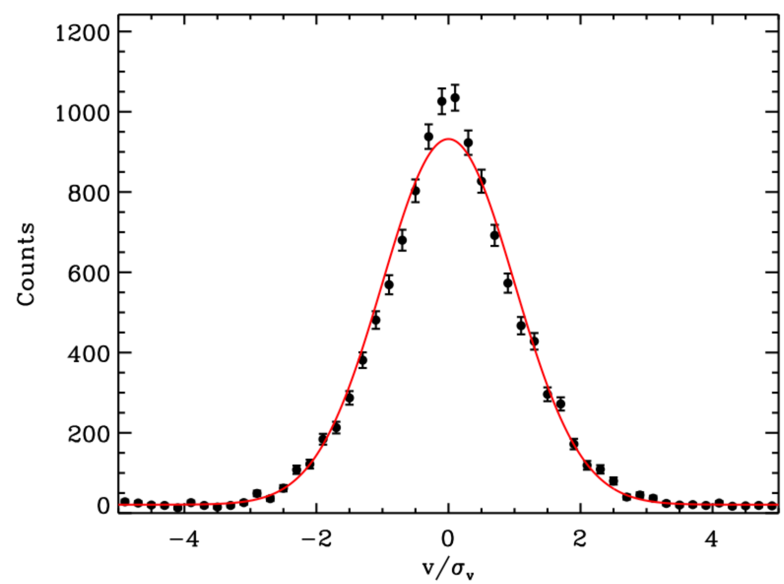


Figure 3. Distribution of the line-of-sight velocity v, normalized by the expected velocity dispersion σ_v of the cluster, for all central–satellite pairs in our sample. The red curve is our best fit model (i.e. a gaussian of zero mean and width unity, and a flat background of non cluster galaxies). The amplitude is set by demanding that the total number of galaxy counts in the model curve exactly match the total number of counts in the data.

real data

synthetic and real galaxy cluster images from DECam

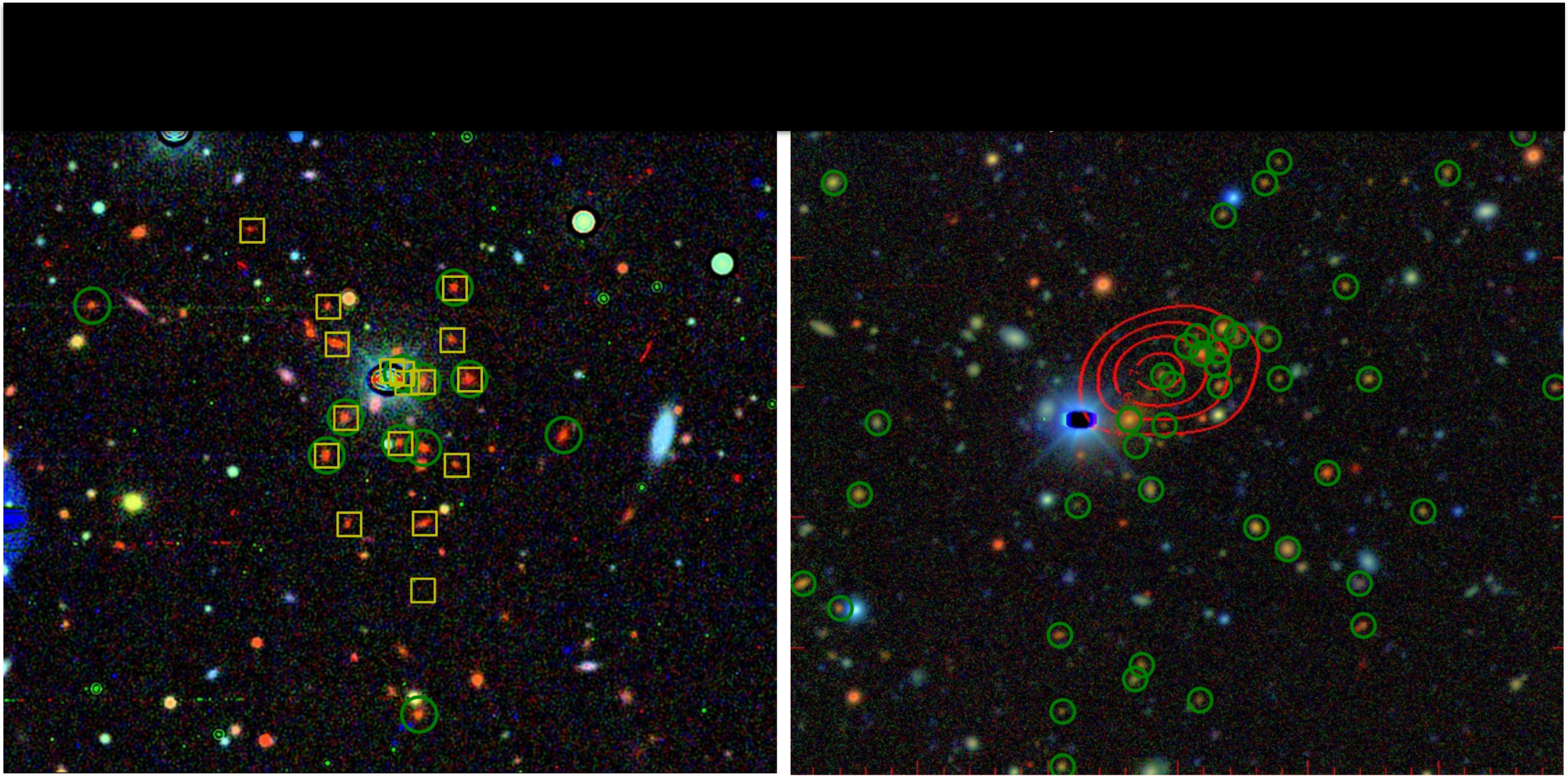
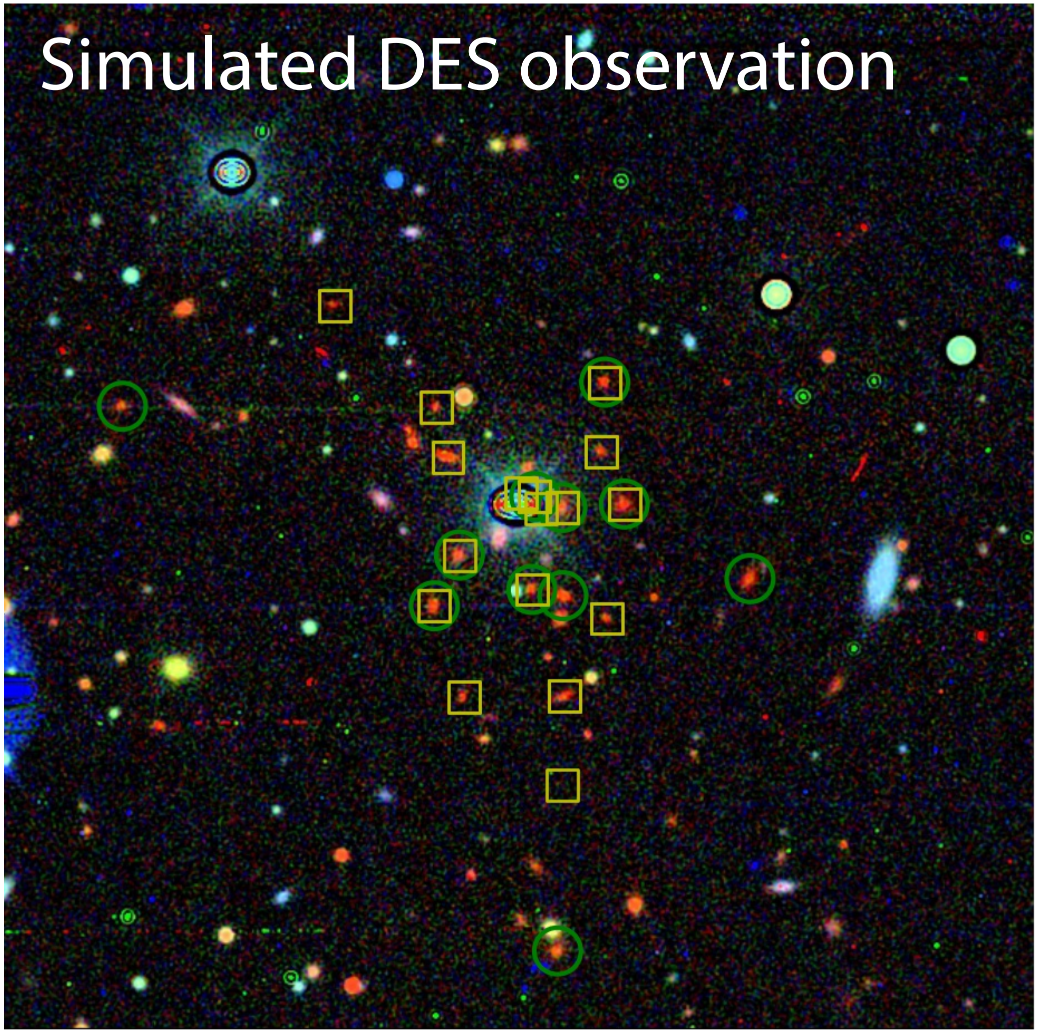


image credit: Chris Miller (Michigan)

synthetic and real galaxy cluster images from DECam



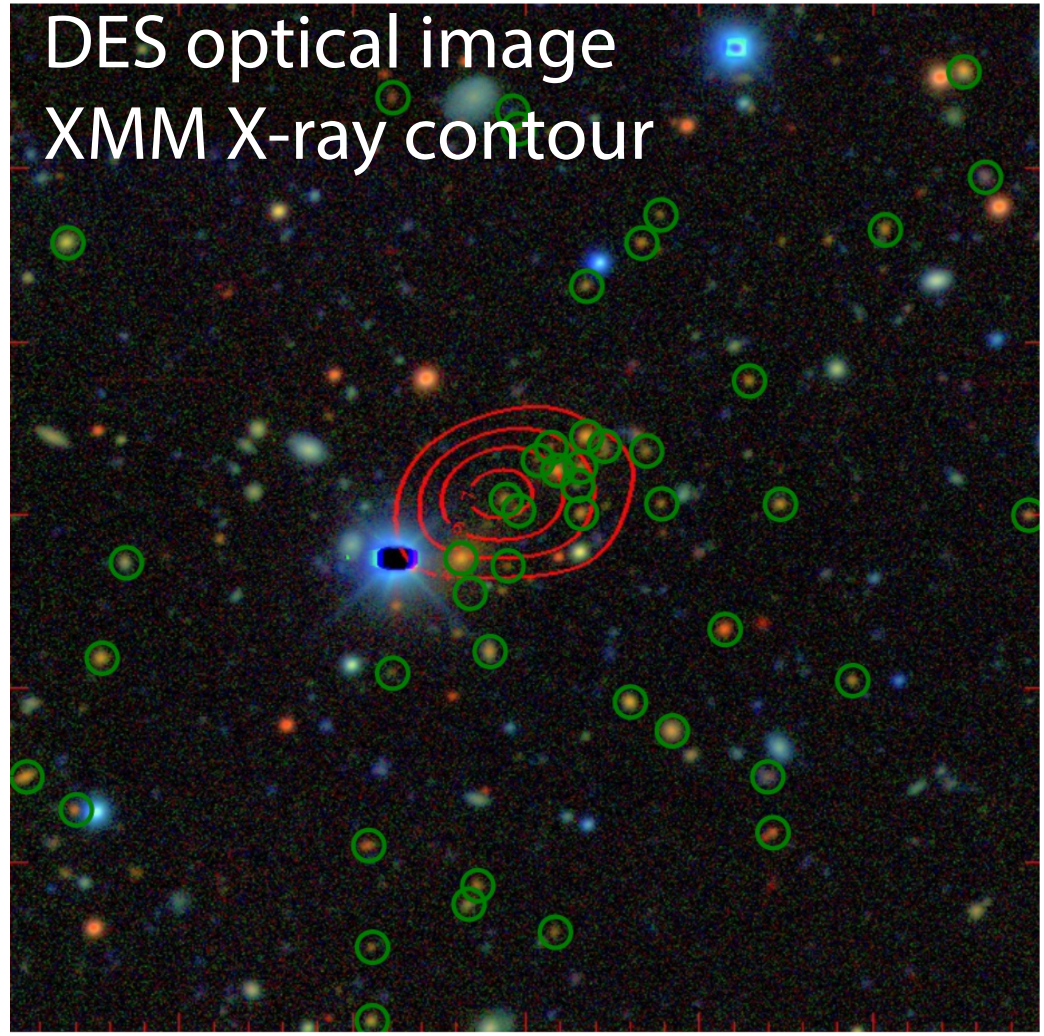


image credit: Chris Miller (Michigan)

application of redMaPPer to synthetic LCDM sky survey

red sequence in massive halos

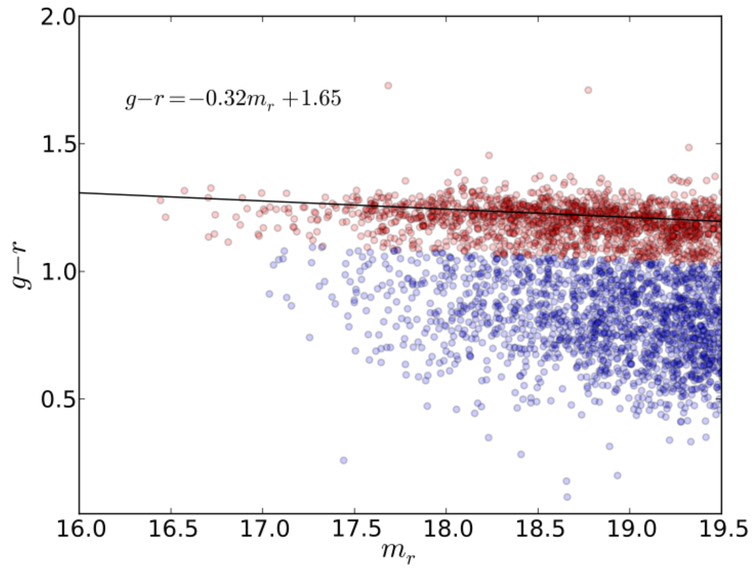


Figure 1. Color–magnitude diagram for Aardvark simulation galaxies occupying halos of mass $M_{200c} > 10^{14}\, h^{-1}\,\mathrm{M}_\odot$ in the redshift interval $0.19 < z < 0.21$. The line indicates the red sequence ridge-line, $g - r = 1.65 - 0.32\, m_r$; 78% of galaxies brighter than $m_i = 19$ lie within 0.2 mag of this ridge-line.

comparison of cluster counts

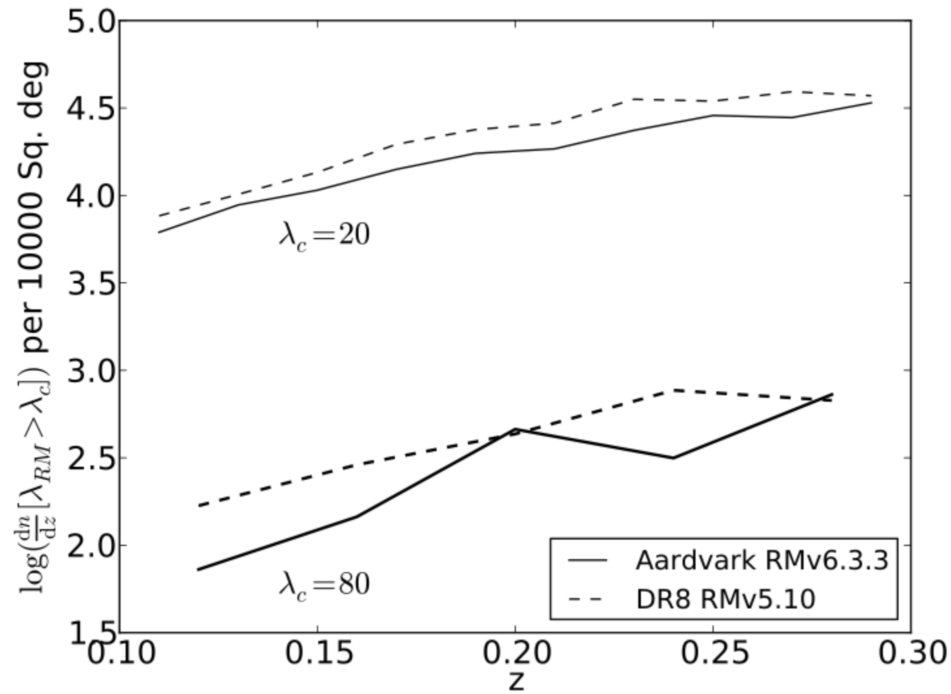


Figure 2. Differential sky number counts per 10,000 square degree of clusters with richness, $\lambda > 20$ (thin lines) and 80 (bold lines) are shown for the Aardvark simulated galaxy catalog run with RMv6.3.3 (solid) and SDSS DR8 run with RMv5.10 (dashed, Rozo et al. 2015b) samples.

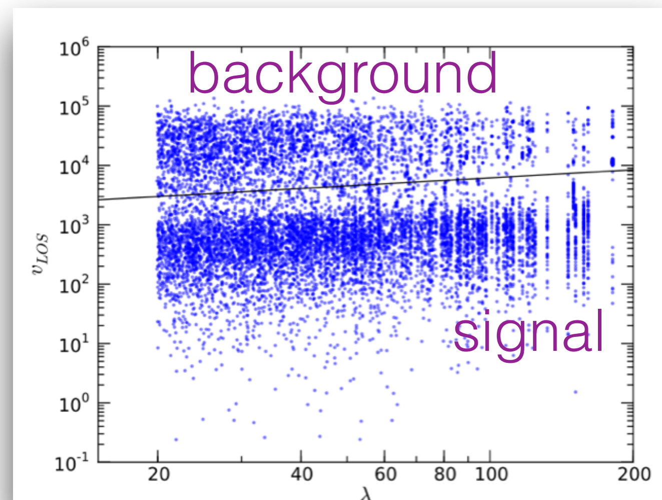
Stacked spectroscopic mass estimates: model ingredients + results

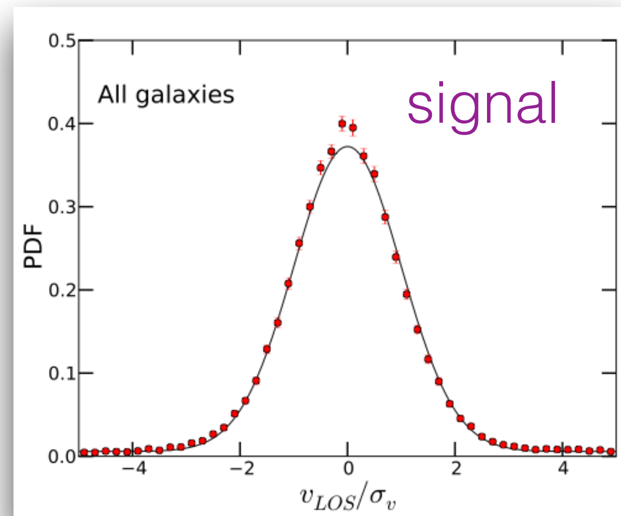
Signal velocity dispersion scaling: $$\sigma_v(\lambda, z_{\rm cen}) = \sigma_p \left(\frac{1+z_{\rm cen}}{1+z_p}\right)^{\beta} \left(\frac{\lambda}{\lambda_p}\right)^{\alpha}$$

Central-satellite LOS velocity likelihood:
Gaussian + flat background $$\mathcal{L}_i = pG(v_i) + (1-p)\frac{1}{2v_{\rm max}}$$

sample	σ_p [km s^{-1}]	p	α	β
FS	585 ± 2	0.885 ± 0.002	0.387 ± 0.007	0.83 ± 0.07
CS	547 ± 2	0.919 ± 0.002	0.405 ± 0.008	0.87 ± 0.08
Bolshoi	535 ± 4	0.884 ± 0.003	0.295 ± 0.010	-
RMIV	598 ± 6	0.916 ± 0.004	0.435 ± 0.020	0.54 ± 0.19

redMaPPer applied to 10,000 sq deg simulation



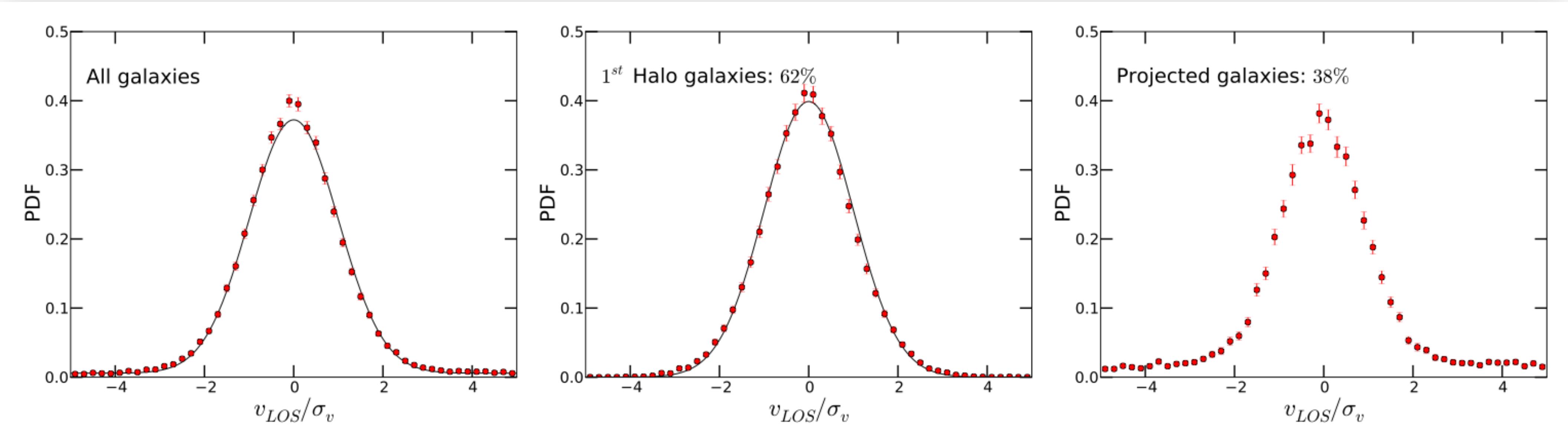


simulation

Stacked spectroscopic mass estimates: model ingredients

In the simulations we know in which halos the cluster members reside.

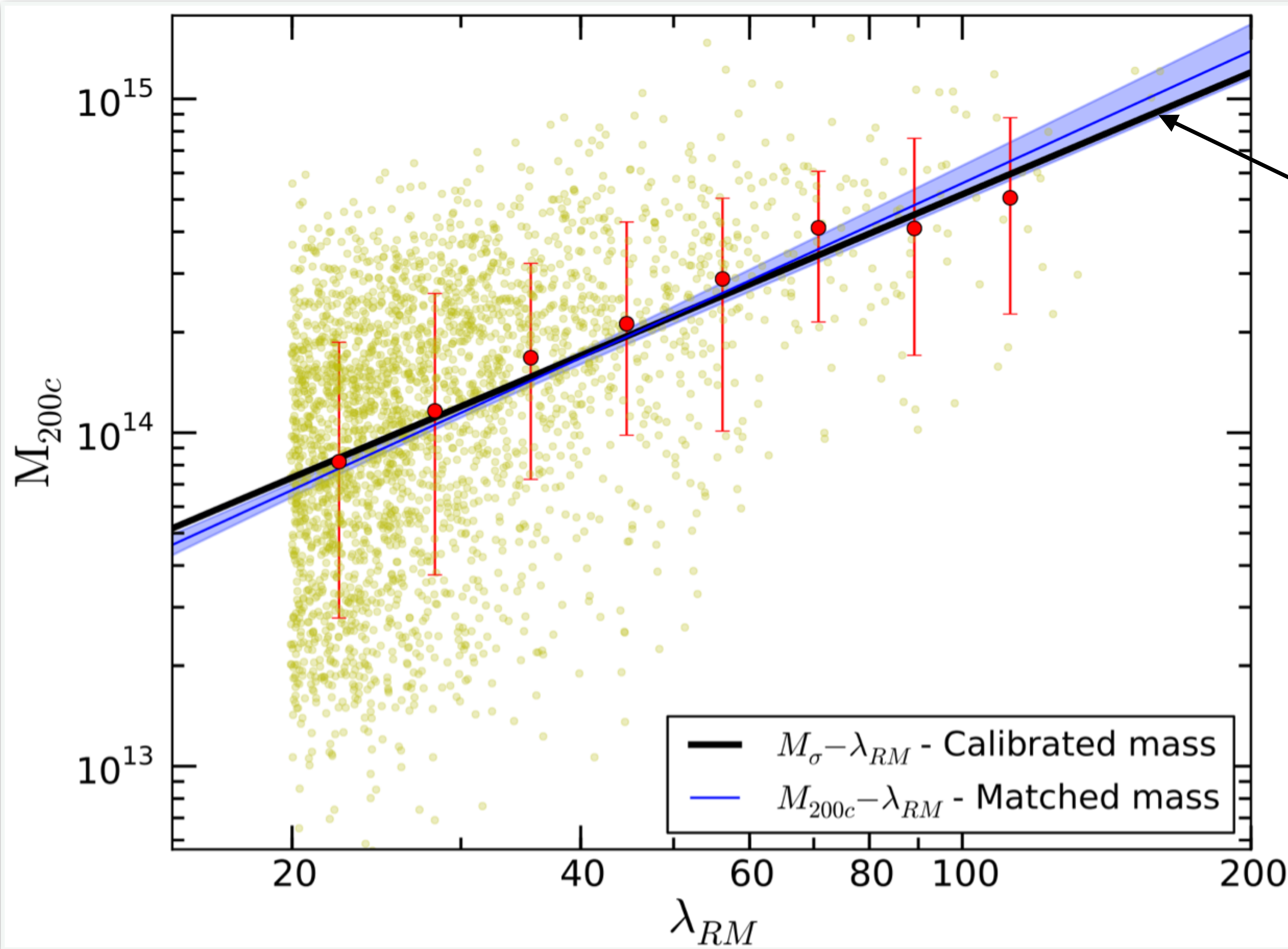
All | top-ranked halo: 62% | lower-rank halos: 38%



match clusters to halos by maximizing membership contribution: (member ***strength***, *S*)

$$S_{\alpha,i} = \frac{1}{\lambda_\alpha} \sum_{n \in G_\alpha} P_{mem,\alpha}(n) P_{halo,i}(n)$$

Simulation of stacked spectroscopic mass estimates (SDSS, soon DES+DESI)



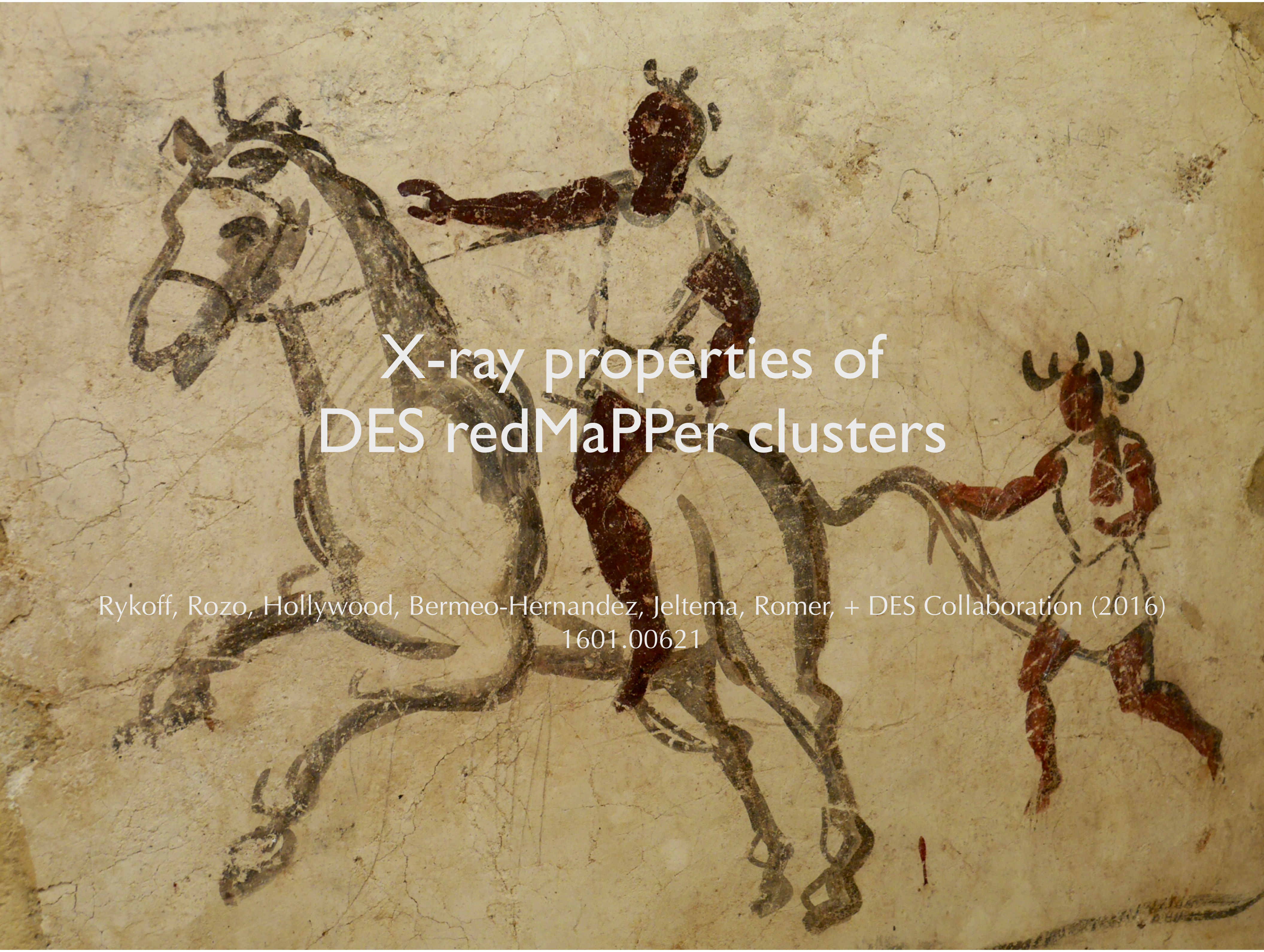
Mass estimate from **virial velocity scaling** recovers the log-mean membership matched mass (yellow, red + blue band) to within a few %.

Apply this method to the SDSS sample with assumptions about ***velocity bias*** of galaxies.

Result: (10^{14} M_{sun})

Quoted mass is $\langle \ln(M_{200c}) | \lambda \rangle$

$$M_{200,p}(\lambda_p = 30, z_p = 0.2) = 1.47 \pm 0.33.$$



X-ray properties of DES redMaPPer clusters

Rykoff, Rozo, Hollywood, Bermeo-Hernandez, Jeltema, Romer, + DES Collaboration (2016)
1601.00621

DES redMaPPer SVA1 cluster sample

Rykoff, Rozo + DES, 2016

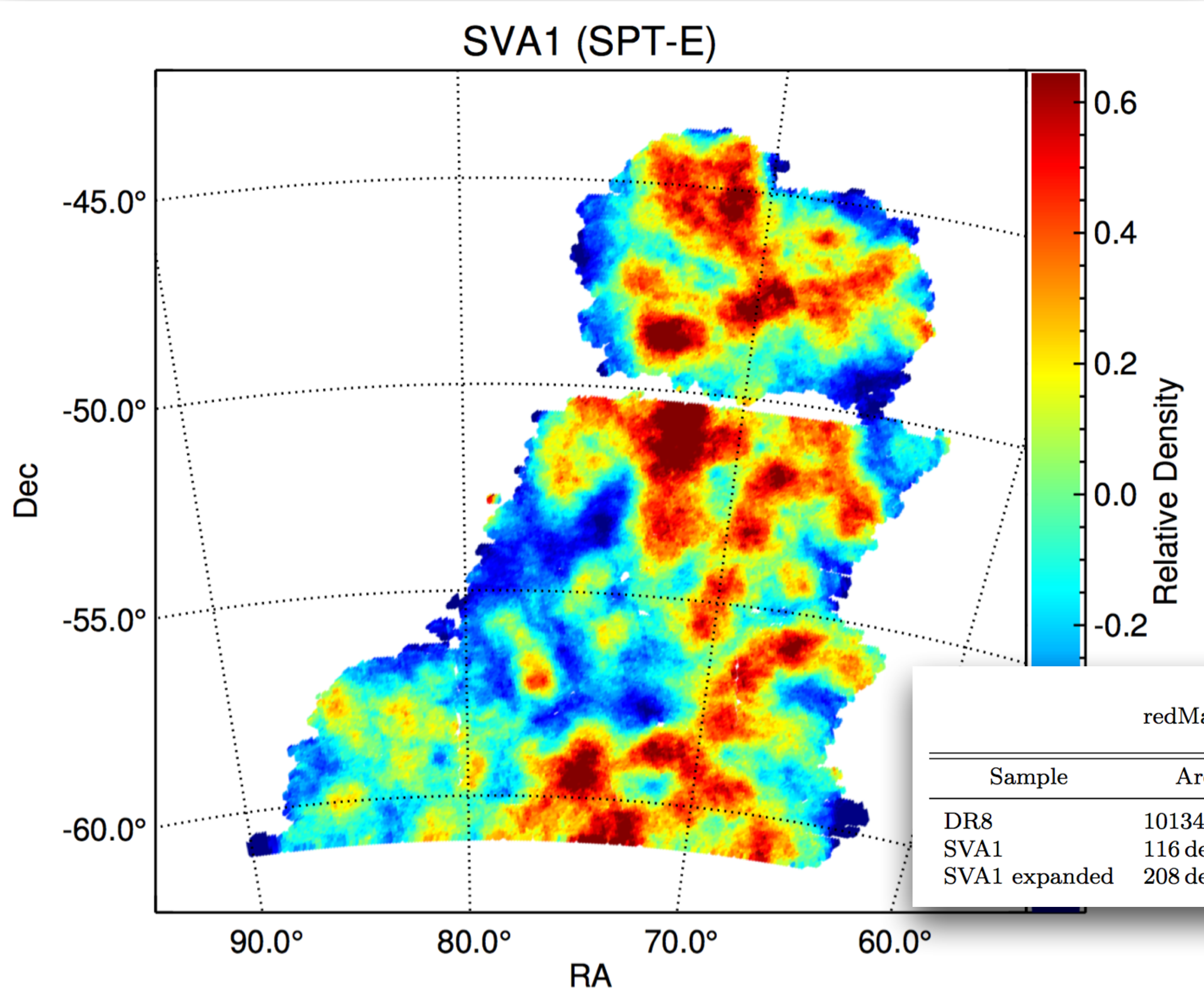


Table 1
redMaPPer Cluster Samples

Sample	Area	Redshift Range	No. of Clusters
DR8	$10134\,\mathrm{deg}^2$	$0.08 < z_\lambda < 0.6$	26308
SVA1	$116\,\mathrm{deg}^2$	$0.2 < z_\lambda < 0.9$	804
SVA1 expanded	$208\,\mathrm{deg}^2$	$0.2 < z_\lambda < 0.9$	1414

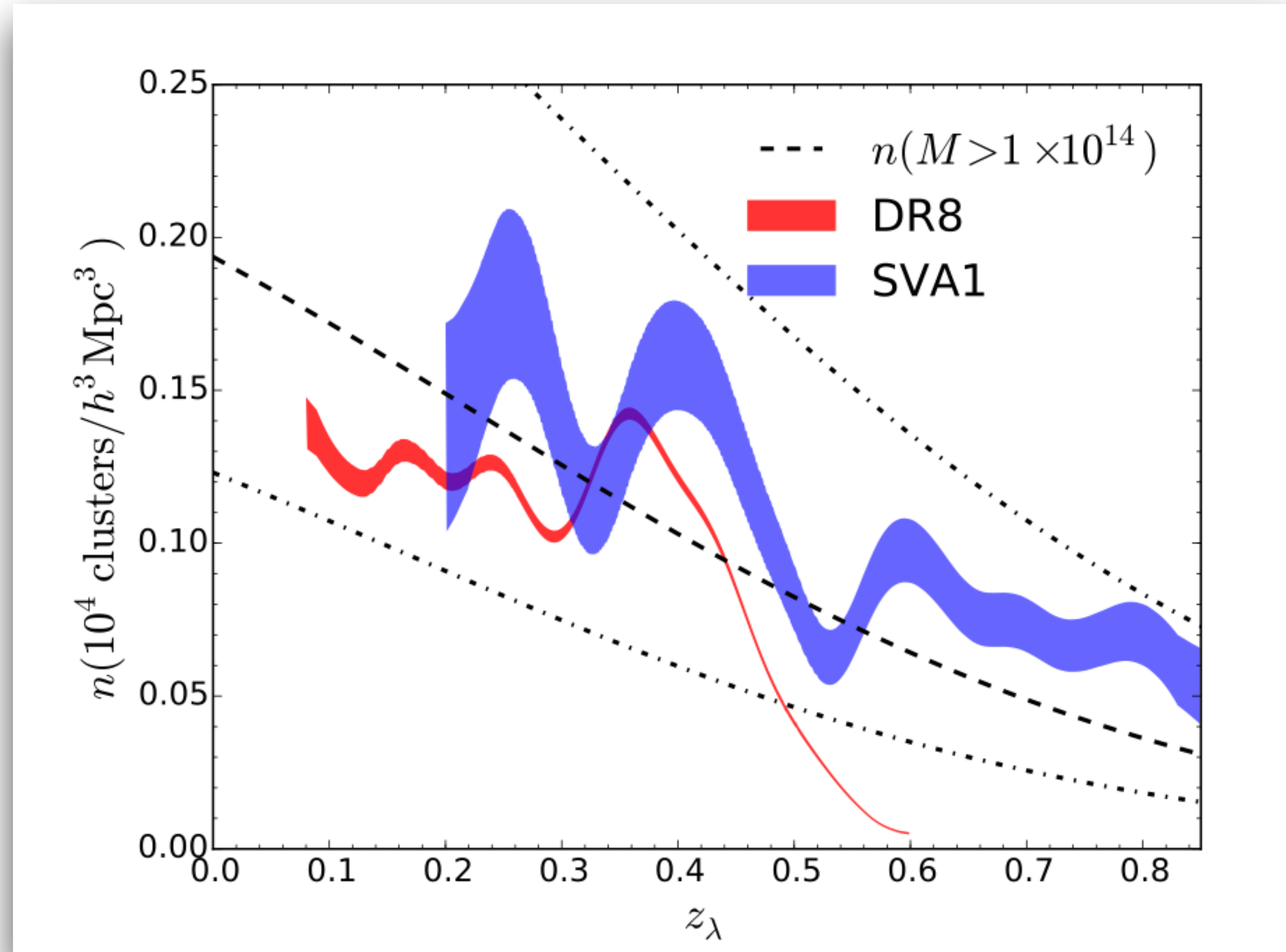
redMaPPer is a matched-filter likelihood method

a small training set of spectroscopic redshifts is used to map out the location of the red sequence (red+dead galaxy colors) with redshift

DES SVA samples arriving soon!

redMaPPer: DES-SVA1 + SDSS-DR8 cluster number densities

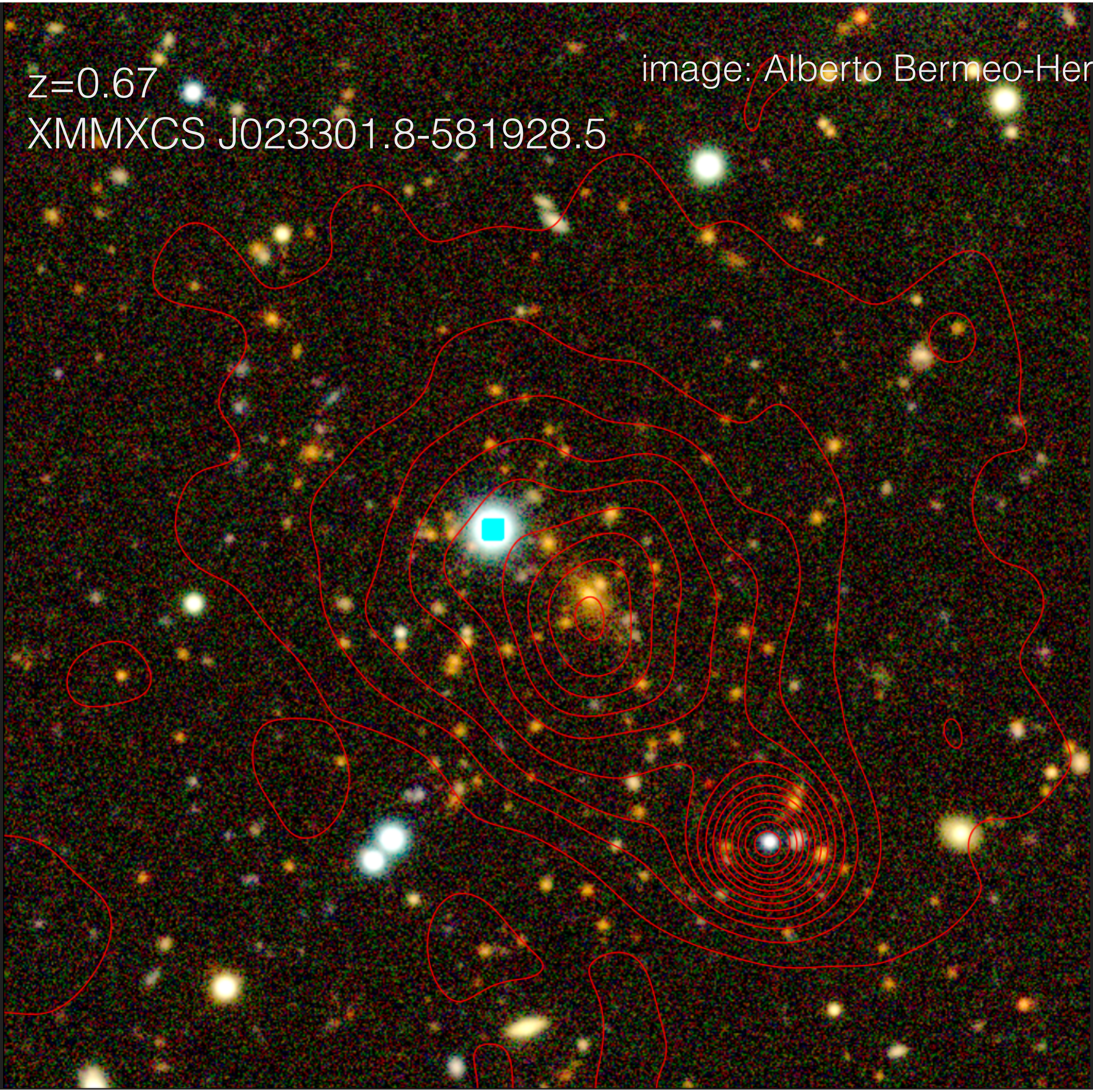
Rykoff, Rozo + DES, 2016

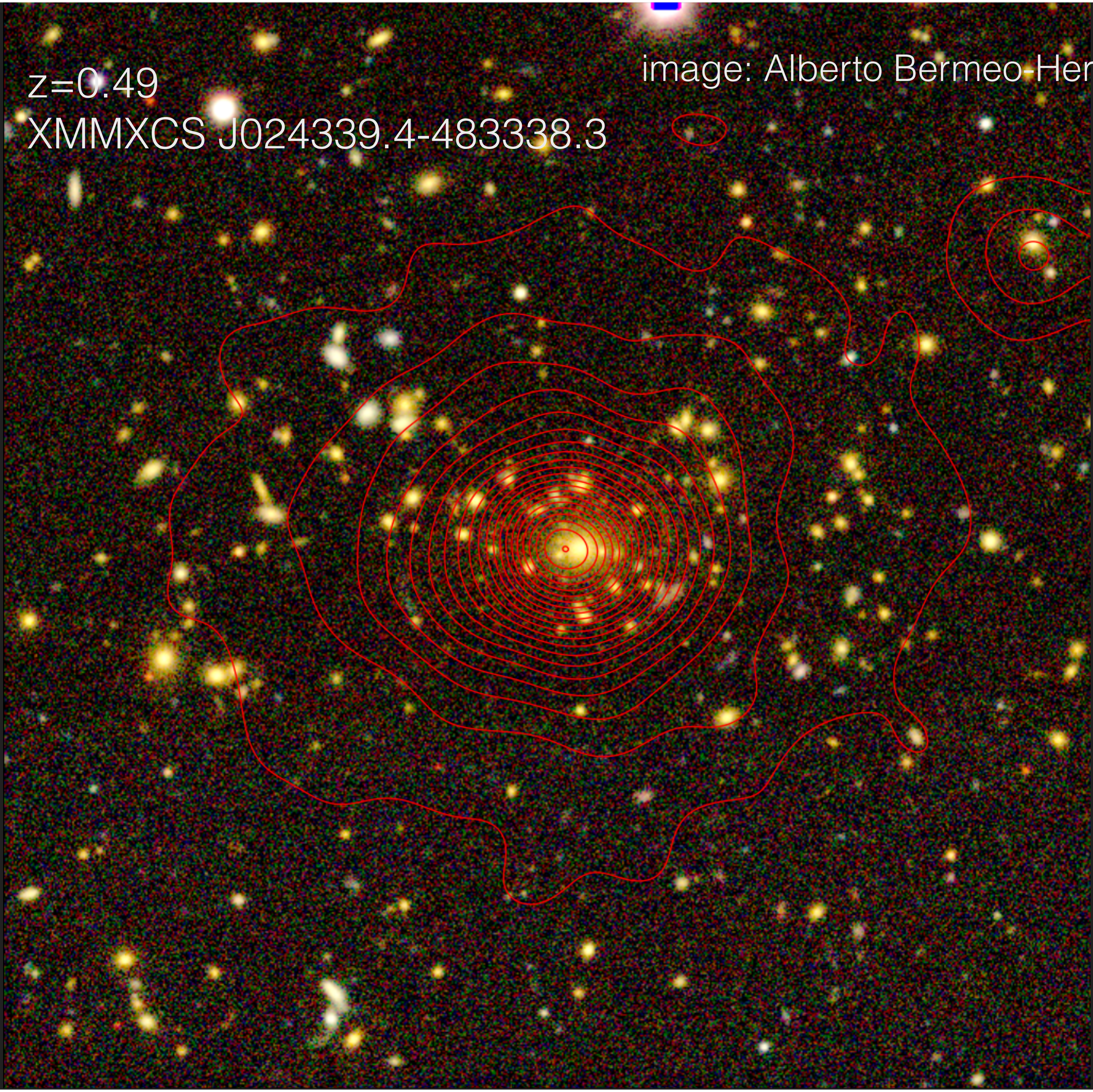


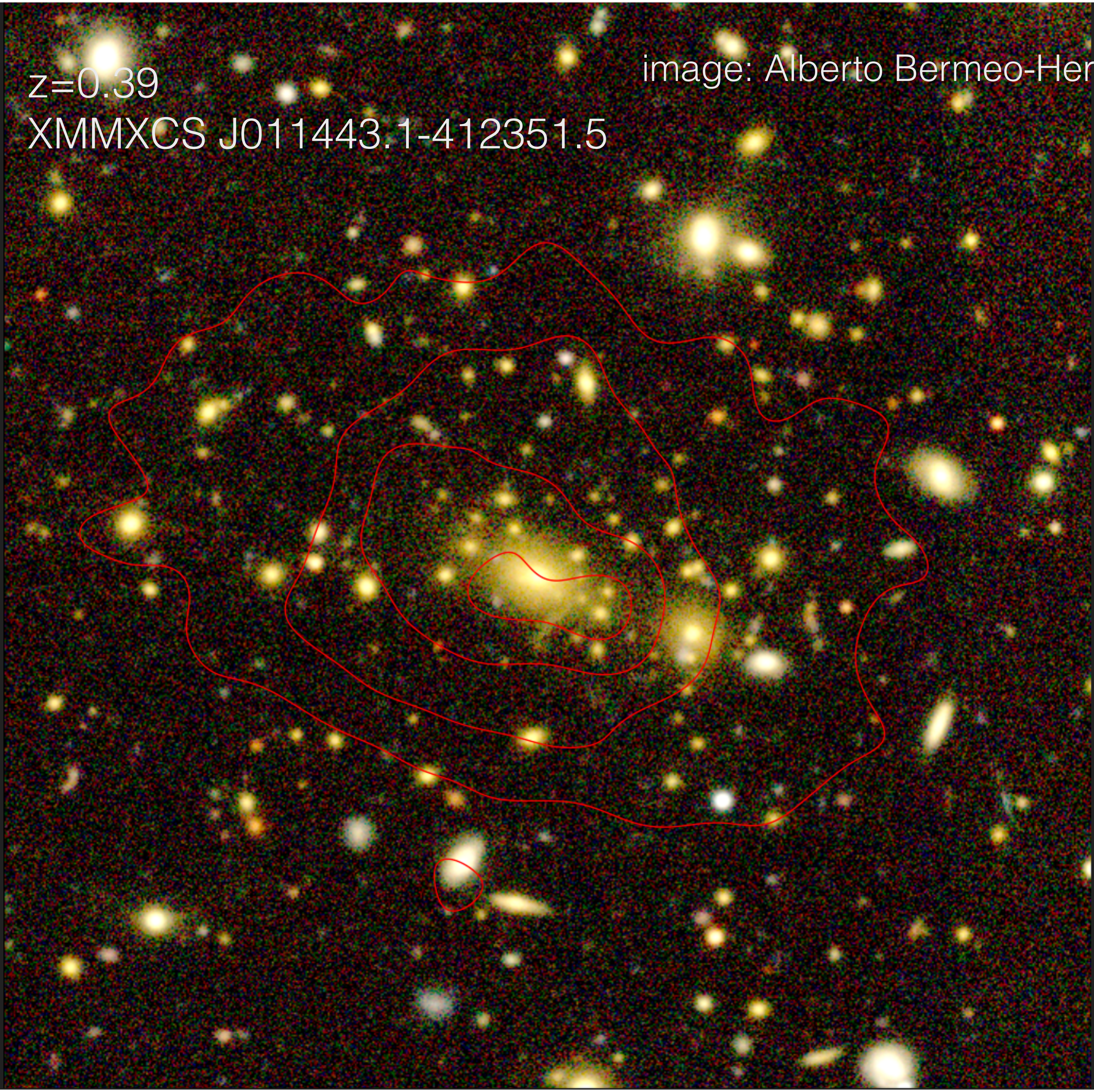
space density of clusters with $\lambda>20$

~follow the space density of 10^{14} Msun halos in LCDM cosmology (not a design feature)

local variations from cosmic variance (DES) and color-space (Malmquist) bias


z=0.67
XMMXCS J023301.8-581928.5
image: Alberto Bermeo-Hernandez


z=0.49
XMMXCS J024339.4-483338.3
image: Alberto Bermeo-Hernandez


z=0.39
XMMXCS J011443.1-412351.5
image: Alberto Bermeo-Hernandez

DES redMaPPer SVA1 + XMM/Chandra matches

Rykoff, Rozo + DES, 2016

$$\ln(T_X) = \alpha + \beta \ln(\lambda/50) + \gamma \ln[E(z)/E(0.4)],$$

(γ=2/3 fixed)

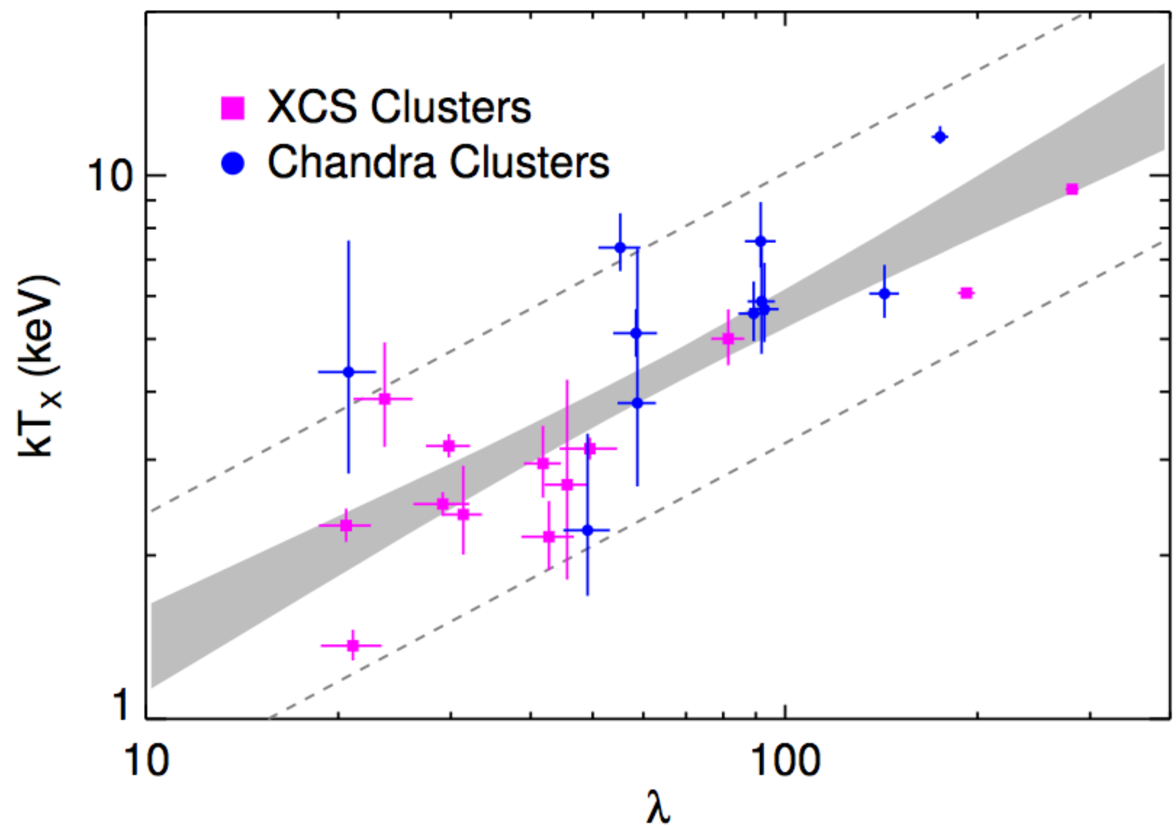


Figure 10. T_X–λ scaling relation derived from XCS (magneta squares) and *Chandra* (blue circles) clusters. All *Chandra* temperatures have been corrected according to Eqn. 6. The gray band shows the best fit ($\pm 1\sigma$) scaling relation, and the dashed gray lines show $2\sigma_{\rm int}$ intrinsic scatter constraints.

$$\alpha = 1.31 \pm .07,$$
$$\beta = 0.60 \pm 0.09,$$
$$\sigma_{\ln T|\lambda} = 0.28^{+0.07}_{-0.05}.$$

based on cross-matching DES-SV redMaPPer with X-ray catalogs

more to come!


lies, damn lies and …
Calibrating the Planck Cluster Mass Scale with CLASH
Penna-Lima, Bartlett, Rozo, Melin, Merten, Evrard, Postman, Rykoff, +…
in prep.

analysis of 21 clusters common to CLASH and Planck

Calibrating the *Planck* Cluster Mass Scale with CLASH

M. Penna-Lima[1], J. G. Bartlett[1,2], E. Rozo[3], J.-B. Melin[4], J. Merten[5], A E. Evrard[6,7], M. Postman[8], E. Rykoff[9,10], and
...

ABSTRACT.

… We compare the lensing masses to the *Planck* Sunyaev-Zeldovich (SZ) mass proxy for 21 clusters in common, employing a Bayesian analysis to simultaneously fit a parameterized CLASH selection function and the distribution between the measured observables and true cluster mass. In the case of an assumed constant bias, b_{SZ}, between true cluster mass, M_{500}, and the *Planck* mass proxy, M_{PL}, our analysis constrains $1 - b_{SZ} = 0.73 \pm 0.07$. …

illuminating approach to likelihood of mass measurements

allow for freedom in BOTH weak lensing and SZ (X-ray…) mass proxies

$$\langle \ln M_{\rm SZ} | \ln M_{500} \rangle = \ln(1 - b_{\rm SZ}) + \alpha_{\rm SZ} \ln\left(\frac{M_{500}}{M_0}\right)$$
$$\langle \ln M_{\rm L} | \ln M_{500} \rangle = \ln(1 - b_{\rm L}) + \alpha_{\rm L} \ln\left(\frac{M_{500}}{M_0}\right).$$

include PDF of measurement errors, e.g., P($M_{\rm PL}$ | $M_{\rm SZ}$)

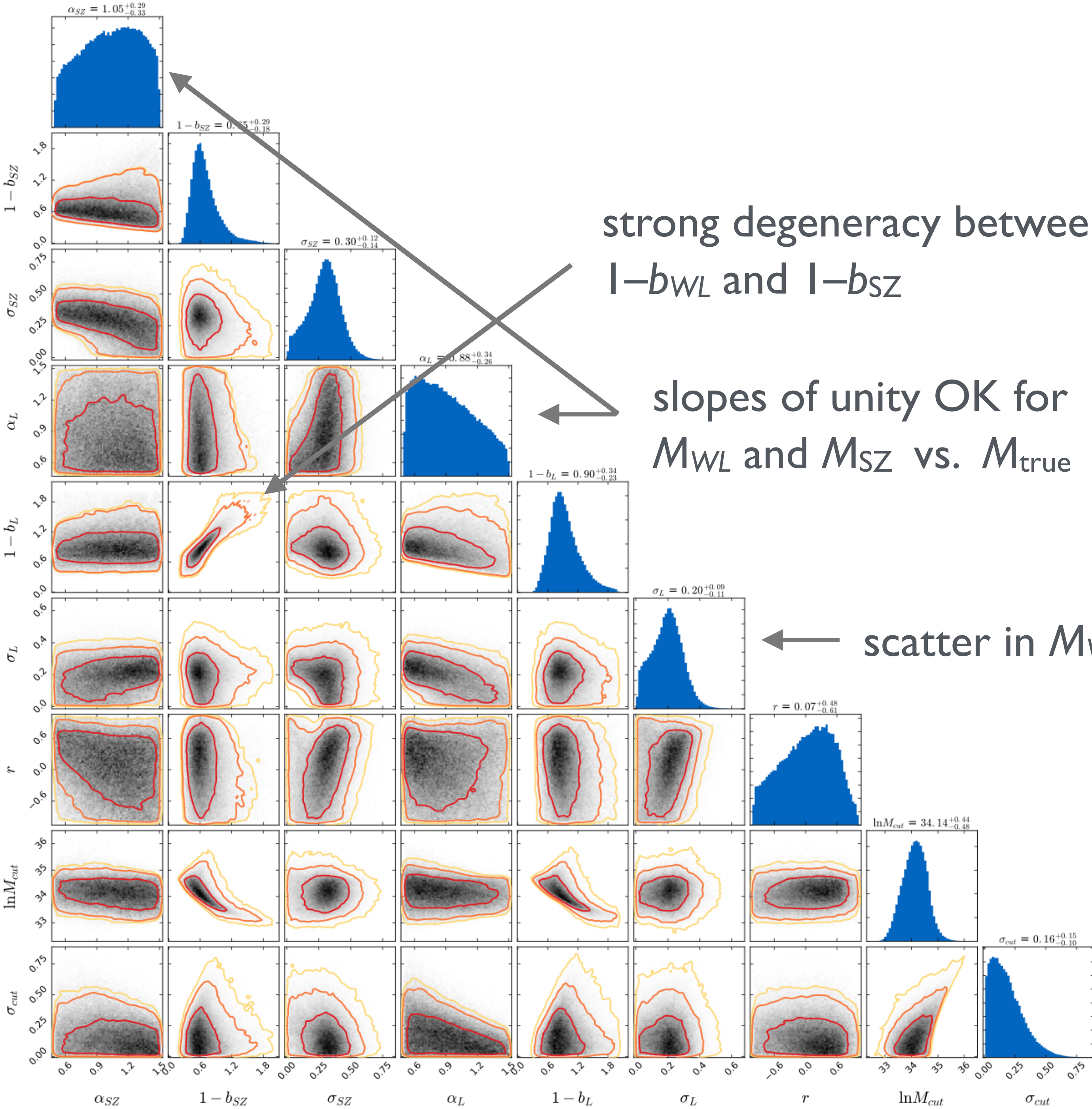
The probability of a CLASH cluster having data $(M_{\rm PL}, M_{\rm CL}, z_{\rm spec})$ is

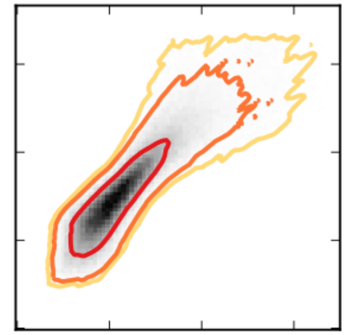
$$P(M_{\rm PL}, M_{\rm CL} | M_{500}, \mathbf{p}) P(z_{\rm spec}|z) dM_{\rm PL} dM_{\rm CL} dz_{\rm spec},$$

where

$$P(M_{\rm PL}, M_{\rm CL} | M_{500}, \mathbf{p}) = \int dM_{\rm SZ} dM_{\rm L}\ P(M_{\rm PL}|M_{\rm SZ}) P(M_{\rm CL}|M_{\rm L}) \times P(M_{\rm SZ}, M_{\rm L} | M_{500}; \mathbf{p}), \quad (3)$$

results with uninformative priors





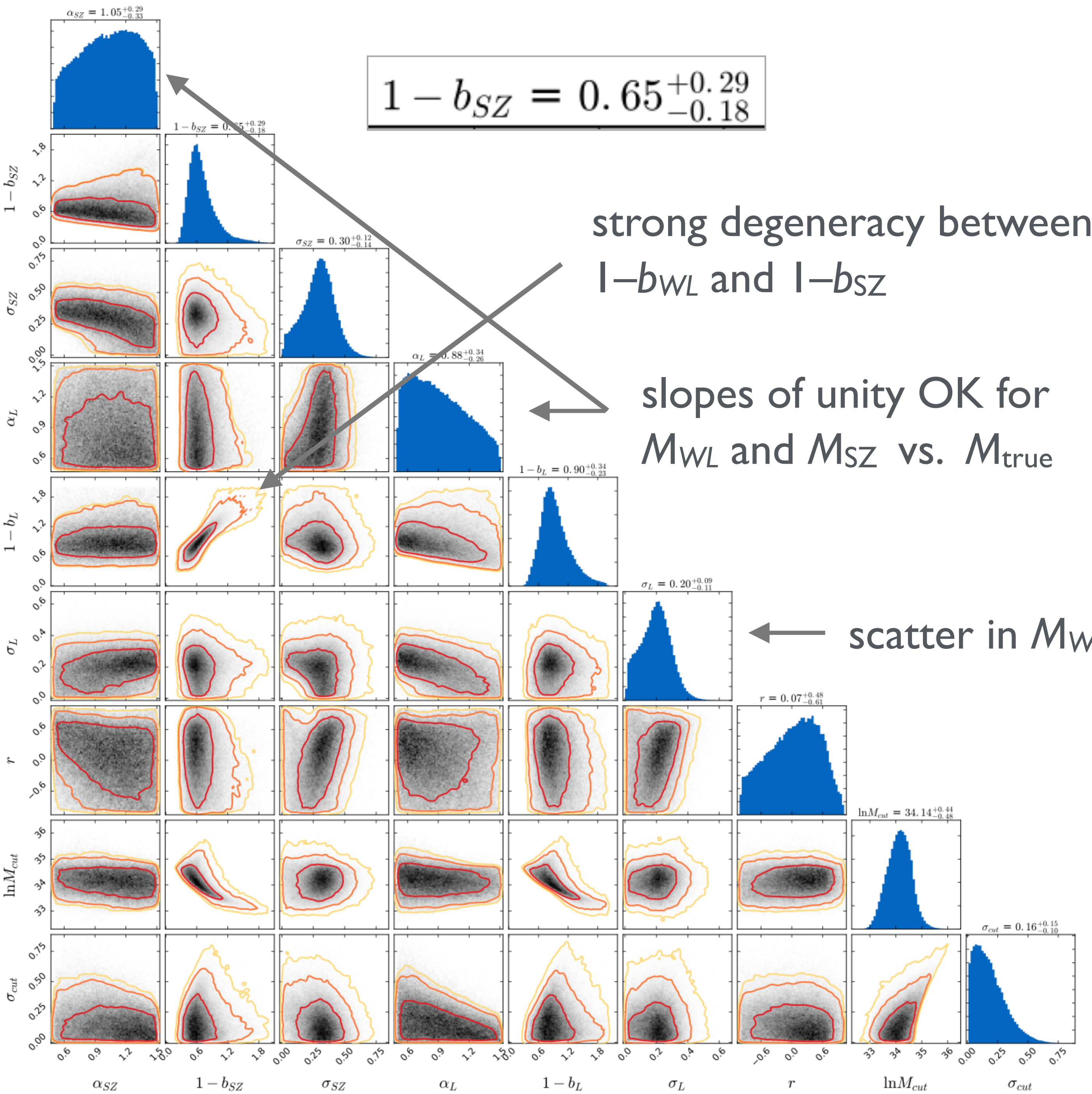
strong degeneracy between $1\!-\!b_{WL}$ and $1\!-\!b_{SZ}$

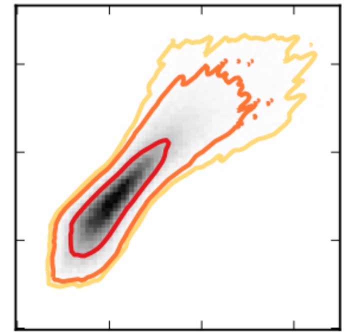
slopes of unity OK for M_{WL} and M_{SZ} vs. M_{true}

scatter in M_{WL} is bounded

results with uninformative priors

$$1 - b_{SZ} = 0.65^{+0.29}_{-0.18}$$



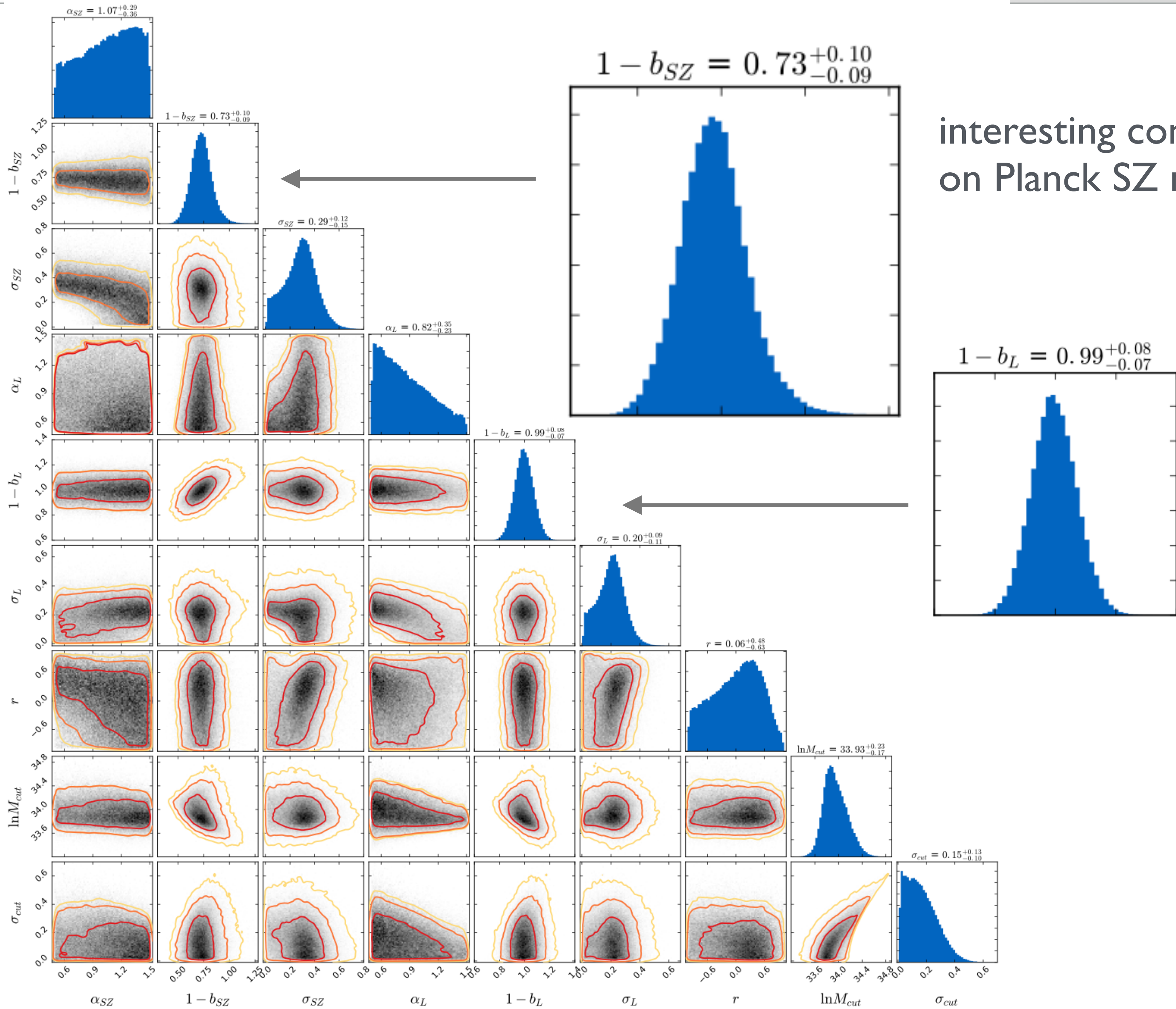


strong degeneracy between $1 - b_{WL}$ and $1 - b_{SZ}$

slopes of unity OK for M_{WL} and M_{SZ} vs. M_{true}

scatter in M_{WL} is bounded

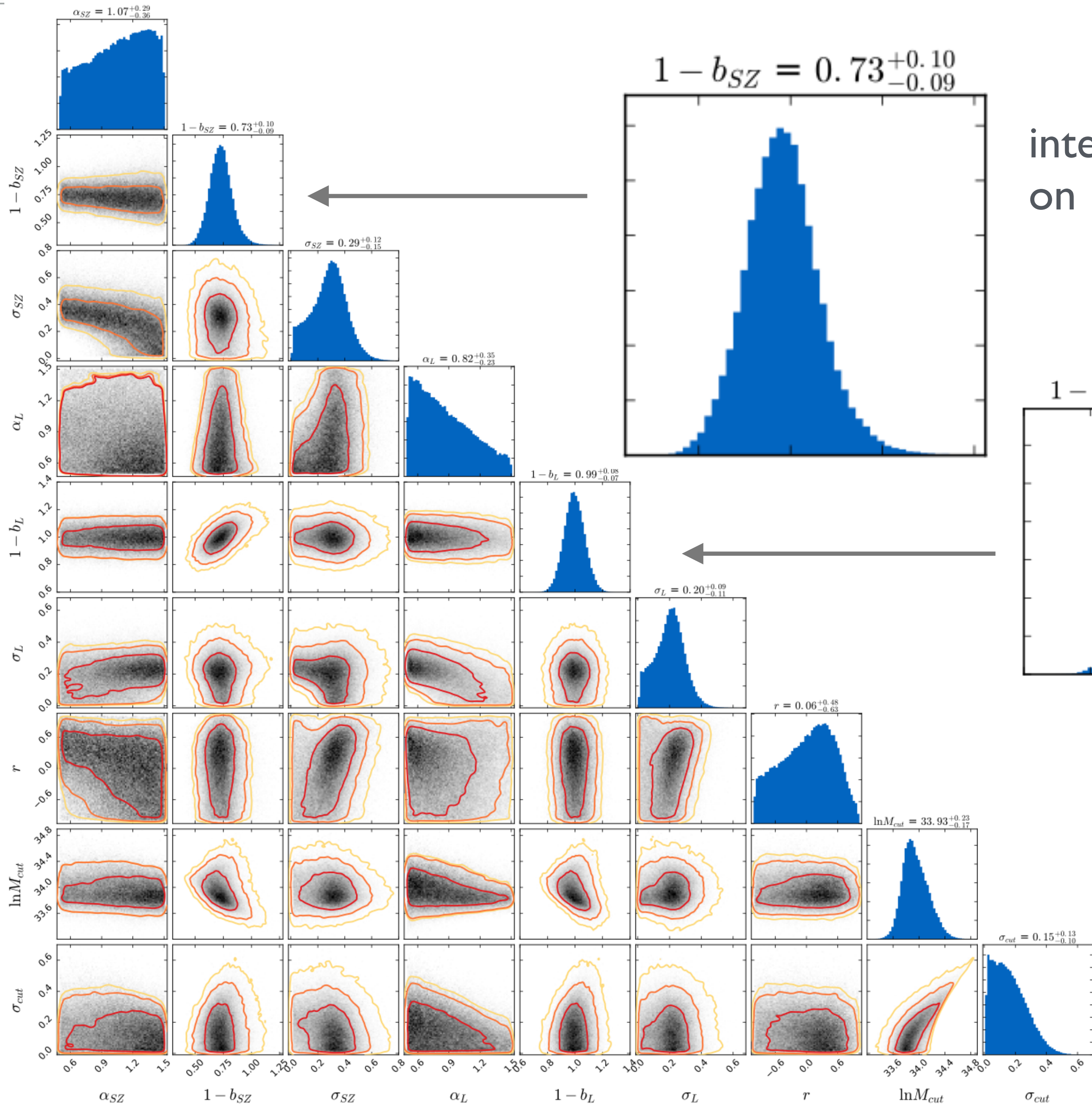
Gaussian prior of 1±0.08 on $1\!-\!b_{WL}$



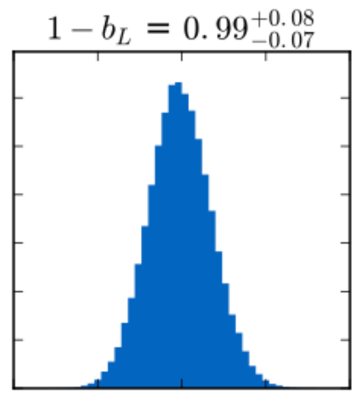
interesting constraint on Planck SZ mass bias

lensing posteriors reflect priors

Gaussian prior of 1±0.08 on $1-b_{WL}$



interesting constraint on Planck SZ mass bias



lensing posteriors reflect priors

More analysis framed in this manner would help make explicit what the data are telling us.


Let’s keep working to understand
the physics and phenomenology
of massive halos in our universe!
Thank you!